Physics in Aperiomics B

Electromagnetic Induction

Closing the switch

In this model opening the switch has -𝕆D×e𝕪 kinetic work and +𝕆D×e𝕒 potential work moving through the wire. In the left coil this has an increase in -𝕆D×e𝕪 kinetic work which induces -𝕆D×e𝕪 kinetic work in the right coil. That creates a -𝕆D kinetic difference on the right and a e𝕪/-𝕠d kinetic current making the meter move towards the negative side.

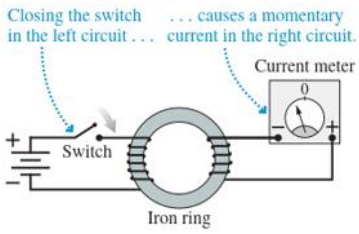
30.1 Induced Currents

Oersted's 1820 discovery that a current creates a magnetic field generated enormous excitement. One question scientists hoped to answer was whether the converse of Oersted's discovery was true: that is, can a magnet be used to create a current?

The breakthrough came in 1831 when the American science teacher Joseph Henry and the English scientist Michael Faraday each discovered the process we now call *electromagnetic induction*. Faraday—whom you met in Chapter 22 as the inventor of the concept of a *field*—was the first to publish his findings, so today we study Faraday's law rather than Henry's law.

Faraday's 1831 discovery, like Oersted's, was a happy combination of an unplanned event and a mind that was ready to recognize its significance. Faraday was experimenting

FIGURE 30.1 Faraday's discovery.

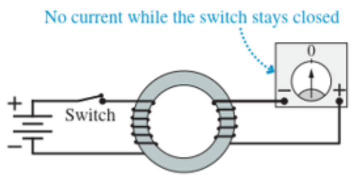


Keeping the switch closed

When the switch is closed there is no change, it is like a filled capacitor. With no -𝕆D×e𝕪 kinetic work being done there is no kinetic current. The constructive and destructive interference increases as the switch remains open, this changes the -𝕆D×e𝕪 kinetic work being done on the electrons until they reach a maximum -𝕆D kinetic difference. Then the current stops as the switch remains closed, it moves through a loop so there is no overall -𝕆D kinetic torque in the wire.

with two coils of wire wrapped around an iron ring, as shown in **FIGURE 30.1**. He had hoped that the magnetic field generated in the coil on the left would induce a magnetic field in the iron, and that the magnetic field in the iron might then somehow create a current in the circuit on the right.

Like all his previous attempts, this technique failed to generate a current. But Faraday happened to notice that the needle of the current meter jumped ever so slightly at the instant he closed the switch in the circuit on the left. After the switch

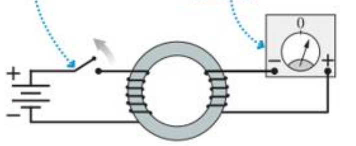


Opening the switch

When the switch is opened there is a reaction by the +𝕆D potential difference in the wire as the -𝕆D×e𝕪 kinetic work is stopped, that makes the meter move towards the positive side.

was closed, the needle immediately returned to zero. The needle again jumped when he later opened the switch, but this time in the opposite direction. Faraday recognized that the motion of the needle indicated a current in the circuit on the right, but a momentary current only during the brief interval when the current on the left was starting or stopping.

Faraday's observations, coupled with his mental picture of field lines, led him to suggest that a current is generated only if the magnetic field through the coil is *changing*. This explains why all the previous attempts to generate a current with static magnetic fields had been unsuccessful. Faraday set out to test this hypothesis.



Opening and closing the switch

When the switch is opened below, the upper coil does -𝕆D×e𝕪 kinetic work which induces -𝕆D×e𝕪 kinetic work in the lower coil. That moves the meter towards the negative side. Closing the switch has a reaction with +𝕆D×e𝕒 potential work towards the positive side of the meter.

Moving the bar magnet

The bar magnet changes the constructive and destructive interference in the coiled wire as it moves. This changes the e𝕪/-𝕠d kinetic velocity of the electrons making the meter move.

Pushing and pulling the coil

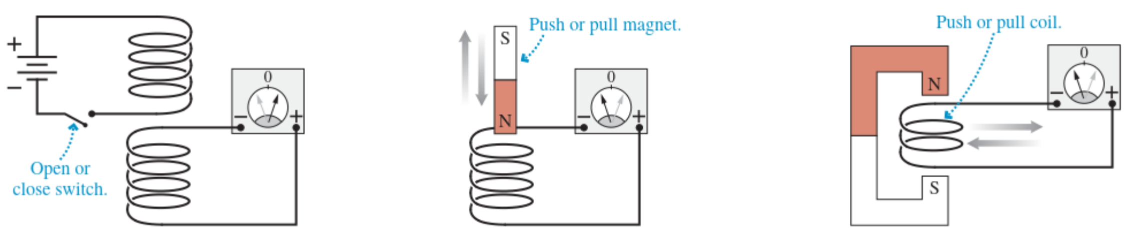
Pushing and pulling the coil into the magnet also changes the constructive and destructive interference, that makes the meter move.

Faraday investigates electromagnetic induction

Faraday placed one coil directly above the other, without the iron ring. There was no current in the lower circuit while the switch was in the closed position, but a momentary current appeared whenever the switch was opened or closed.

He pushed a bar magnet into a coil of wire. This action caused a momentary deflection of the current-meter needle, although *holding* the magnet inside the coil had no effect. A quick withdrawal of the magnet deflected the needle in the other direction.

Must the magnet move? Faraday created a momentary current by rapidly pulling a coil of wire out of a magnetic field. Pushing the coil *into* the magnet caused the needle to deflect in the opposite direction.



Opening or closing the switch creates a momentary current.

Pushing the magnet into the coil or pulling it out creates a momentary current.

Pushing the coil into the magnet or pulling it out creates a momentary current.

A changing magnetic field

When the magnetic field is changing then so is the -𝕆D kinetic difference and probability. This changing probability is also a torque in the wire loop, that makes the electrons moving around it in a current. When it stops changing there is no longer a changing kinetic probability, the electrons have already moved to their most probable positions to be measured in.

Loops and torque

This also occurs in loops because of the -𝕆D kinetic torque, as the -𝕆D kinetic probability changes then so does the -𝕆D kinetic torque moving the electrons in a loop.

Probability in any circuit

It is like closing the switch in a normal circuit without a coil, the +𝕆D potential probability or difference and the -𝕆D kinetic probability or difference change where the electrons are likely to be measured. That creates a kinetic current.

Faraday found that there is a current in a coil of wire if and only if the magnetic field passing through the coil is *changing*. This is an informal statement of what we'll soon call *Faraday's law*. The current in a circuit due to a changing magnetic field is called an **induced current.** An induced current is not caused by a battery; it is a completely new way to generate a current.

Three orthogonal degrees of freedom

In this model there are three orthogonal degrees of freedom, the first is where the magnet does -𝕆D×e𝕪 kinetic work on the conductor. The second is where this creates a +𝕆D potential and -𝕆D kinetic difference in where the charges are likely to be measured. These are two directions of spin, the third orthogonal direction cannot also be spin because then there can be no motion. This third direction is then a E𝕐/-𝕠d kinetic impulse moving the conductor with a e𝕪/-𝕠d kinetic velocity.

A potential and kinetic spin

The magnet does -𝕆D×e𝕪 kinetic work with kinetic spin from the electron, the conductor does +𝕆D×e𝕒 potential work as a reaction so the spin is different as a reaction. This would also resist the change from the magnet to conserve all forces. There are no other spin directions in this model except for the neutrino, so there is a E𝕐/-𝕠d kinetic impulse. There would also be a E𝔸/+𝕠d potential impulse being done as the inverse of the +𝕆D×e𝕒 potential work being done, that comes from the particle/wave duality.

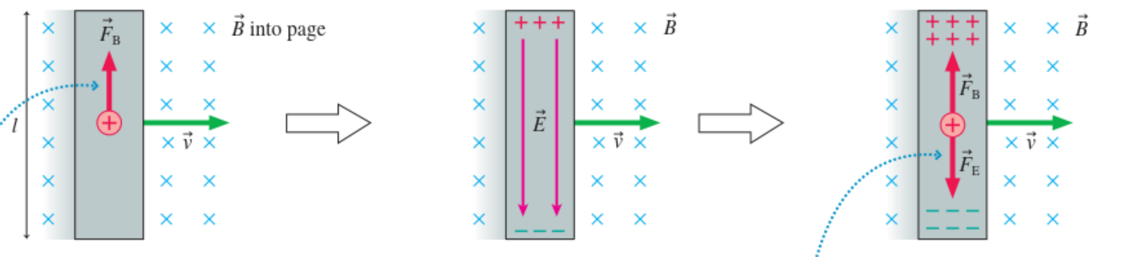
Each force affects the others

Because there are three orthogonal forces, each can affect the others. The potential and kinetic differences in the conductor would have their +𝕆D×e𝕒 potential work react against the E𝕐/-𝕠d kinetic impulse of its motion. This would also react against the -𝕆D×e𝕪 kinetic work of the magnet, if that was an electromagnet then that would slow the e𝕪/-𝕠d kinetic velocity of its current. If it was a bar magnet, then this would push against the constructive interference of the aligned electrons doing -𝕆D×e𝕪 kinetic work.

30.2 Motional emf

We'll start our investigation of electromagnetic induction by looking at situations in which the magnetic field is fixed while the circuit moves or changes. Consider a conductor of length l that moves with velocity $\vec{v}$ through a perpendicular uniform magnetic field $\vec{B}$, as shown in **FIGURE 30.2**. The charge carriers inside the wire—assumed to be positive—also move with velocity $\vec{v}$, so they each experience a magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$ of strength $F_B = qvB$. This force causes the charge carriers to move, separating the positive and negative charges. The separated charges then create an electric field inside the conductor.

FIGURE 30.2 The magnetic force on the charge carriers in a moving conductor creates an electric field inside the conductor.



Charge carriers in the conductor experience a force of magnitude $F_B = qvB$. Positive charges are free to move and drift upward.

The resulting charge separation creates an electric field in the conductor. $\vec{E}$ increases as more charge flows.

The charge flow continues until the electric and magnetic forces balance. For a positive charge carrier, the upward magnetic force $\vec{F}_B$ is equal to the downward electric force $\vec{F}_E$.

Here F_E = -𝕠d×𝕖𝕪/-𝕠d times E which is 𝕖𝕪 or its inverse 1/-𝕠d, that comes from the particle/wave duality. This gives two forces, -𝕠d×𝔼𝕐/-𝕠d is observed as a square kinetic displacement force with Planck's constant. -𝕠d×𝕖𝕪/-𝕆D is measured as a square probability or torque with Boltzmann's constant. It depends on whether there is an observation of electrons as particles with power, or a measurement of voltage. Here E as 𝕖𝕪 is v as the 𝕖𝕪/-𝕠d kinetic velocity times B as -𝕠d to give 𝕖𝕪.

The charge carriers continue to separate until the electric force $F_E = qE$ exactly balances the magnetic force $F_B = qvB$, creating an equilibrium situation. This balance happens when the electric field strength is

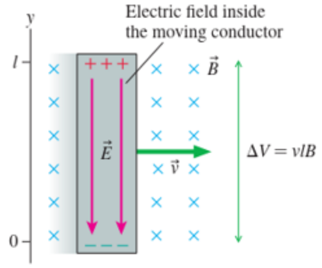
$$E = vB \qquad (30.1)$$

In (30.2) V_{top} would be the +𝕆D potential difference, from this is subtracted the -𝕆D kinetic difference. This is an integral because +𝕆D×𝕖𝕒 potential work and -𝕆D×𝕖𝕪 kinetic work measure areas from their Pythagorean Triangles. E_y here is being integration with respect to a distance in between the potential and kinetic difference, that would be 𝕖𝕒 from +𝕆D×𝕖𝕒 potential work and 𝕖𝕪 from -𝕆D×𝕖𝕪 kinetic work.

That is also integrating 𝕖𝕪/-𝕠d × -𝕠d to give vlB, this is the 𝕖𝕪 kinetic electric charge times a distance l. That can be regarded as the 𝔼𝕐/-𝕠d kinetic impulse where 𝔼𝕐=𝕖𝕪×l, that induces the -𝕆D kinetic difference as the electrons are impelled to one end of the conductor. This is also integrating 𝕖𝕒/+𝕠d × +𝕠d as V-B to give vlB where 𝔼𝔸=𝕖𝕒×l. The electrons are impelled away from the positive charges with a 𝔼𝔸/+𝕠d potential impulse creating a +𝕆D potential difference.

FIGURE 30.3 Generating an emf.

(a) Magnetic forces separate the charges and cause a potential difference between the ends. This is a motional emf.



In other words, **the magnetic force on the charge carriers in a moving conductor creates an electric field $E = vB$ inside the conductor.**

The electric field, in turn, creates an electric potential difference between the two ends of the moving conductor. **FIGURE 30.3a** defines a coordinate system in which $\vec{E} = -vB\,\hat{j}$. Using the connection between the electric field and the electric potential,

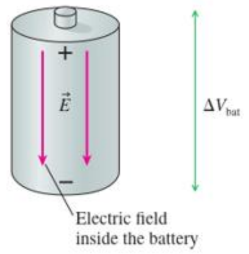
$$\Delta V = V_{top} - V_{bottom} = -\int_0^l E_y\,dy = -\int_0^l (-vB)\,dy = vlB \qquad (30.2)$$

Thus **the motion of the wire through a magnetic field *induces* a potential difference vlB between the ends of the conductor.** The potential difference depends on the strength of the magnetic field and on the wire's speed through the field.

There's an important analogy between this potential difference and the potential difference of a battery. **FIGURE 30.3b** reminds you that a battery uses a nonelectric

Here the difference between the +𝕆D potential and -𝕆D kinetic difference is ΔV as $\mathcal{E}$. A chemical $\mathcal{E}$ is where +𝕆D×𝕖𝕒 potential work and -𝕆D×𝕖𝕪 kinetic work create the potential and kinetic differences. A motional emf or $\mathcal{E}$ comes from the 𝔼𝕐/-𝕠d kinetic impulse of the moving conductor. This would tend to slow as it produces the potential and kinetic difference, otherwise it would be a perpetual motion machine. The difference could run a second circuit moving another conductor and so on.

(b) Chemical reactions separate the charges and cause a potential difference between the ends. This is a chemical emf.



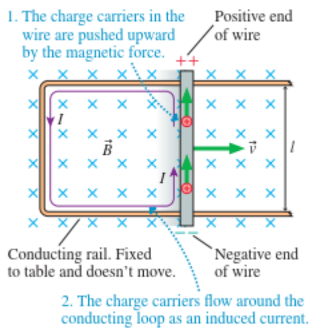
force—the charge escalator—to separate positive and negative charges. The emf $\mathcal{E}$ of the battery was defined as the work performed per charge (W/q) to separate the charges. An isolated battery, with no current, has a potential difference $\Delta V_{bat} = \mathcal{E}$. We could refer to a battery, where the charges are separated by chemical reactions, as a source of *chemical emf.*

The moving conductor develops a potential difference because of the work done by magnetic forces to separate the charges. You can think of the moving conductor as a "battery" that stays charged only as long as it keeps moving but "runs down" if it stops. The emf of the conductor is due to its motion, rather than to chemical reactions inside, so we can define the **motional emf** of a conductor moving with velocity $\vec{v}$ perpendicular to a magnetic field $\vec{B}$ to be

$$\mathcal{E} = vlB \tag{30.3}$$

In this model the circuit is a loop, so the +𝕆D potential and -𝕆D kinetic torque can move electrons around it. The resistance in the circuit would come from the +𝕆D×e𝕒 potential work done by protons in it. If it was an insulator for example, the electrons would be more bound in its atoms and the -𝕆D kinetic probability could not change their positions as easily.

FIGURE 30.5 A current is induced in the circuit as the wire moves through a magnetic field.



Induced Current in a Circuit

The moving conductor of Figure 30.2 had an emf, but it couldn't sustain a current because the charges had nowhere to go. It's like a battery that is disconnected from a circuit. We can change this by including the moving conductor in a circuit.

FIGURE 30.5 shows a conducting wire sliding with speed v along a U-shaped conducting rail. We'll assume that the rail is attached to a table and cannot move. The wire and the rail together form a closed conducting loop—a circuit.

Suppose a magnetic field $\vec{B}$ is perpendicular to the plane of the circuit. Charges in the moving wire will be pushed to the ends of the wire by the magnetic force, just as they were in Figure 30.2, but now the charges can continue to flow around the circuit. That is, the moving wire acts like a battery in a circuit.

The current in the circuit is an *induced current.* In this example, the induced current is counterclockwise (ccw). If the total resistance of the circuit is R, the induced current is given by Ohm's law as

$$I = \frac{\mathcal{E}}{R} = \frac{vlB}{R} \tag{30.4}$$

Impulse as the pulling force

In this model the pulling force comes from a E𝕐/-𝕠d kinetic impulse, that creates +𝕆D×e𝕒 potential work in an orthogonal direction. The -𝕆D×e𝕪 kinetic work done by the magnet is in the third orthogonal direction. The magnetic drag is where the +𝕆D×e𝕒 potential work reacts against the E𝕐/-𝕠d kinetic impulse moving the wire.

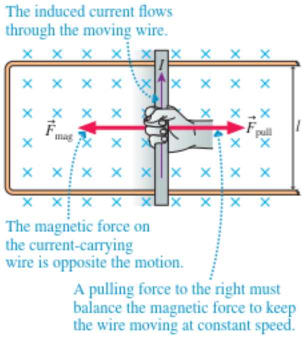
The loop area

In (30.5) l^2 is the area of the loop, as this changes it includes more of the -𝕆D×e𝕪 kinetic work from the magnet. In this model that would be the same as the squared -𝕆D kinetic probability, the inverse of this is the E𝕐/-𝕠d kinetic impulse where E𝕐 is a squared straight Pythagorean Triangle side as a vector. This is equivalent to an area, but here a straight Pythagorean Triangle side can only be a linear or a squared force vector. It cannot be an area because that would be a work integral.

The strength of the magnet as B²

The B² variable would be -𝕆D as the kinetic probability or torque, this can be varied like the area +𝕠d×e𝕒/+𝕆D by changing the strength of the magnet or the e𝕪 distance it is from the conducting wire. The e𝕪/-𝕠d kinetic velocity as v would be the E𝕐/-𝕠d kinetic impulse, that is the amount of force to move the wire from stationary and maintain it against the magnetic drag.

FIGURE 30.6 A pulling force is needed to move the wire to the right.



In this situation, the induced current is due to magnetic forces on moving charges.

We've assumed that the wire is moving along the rail at constant speed. It turns out that we must apply a continuous pulling force $\vec{F}_{pull}$ to make this happen. **FIGURE 30.6** shows why. The moving wire, which now carries induced current I, is in a magnetic field. You learned in Chapter 29 that a magnetic field exerts a force on a current-carrying wire. According to the right-hand rule, the magnetic force $\vec{F}_{mag}$ on the moving wire points to the left. This "magnetic drag" will cause the wire to slow down and stop *unless* we exert an equal but opposite pulling force $\vec{F}_{pull}$ to keep the wire moving.

The magnitude of the magnetic force on a current-carrying wire was found in Chapter 29 to be $F_{mag} = IlB$. Using that result, along with Equation 30.4 for the induced current, we find that the force required to pull the wire with a constant speed v is

$$F_{pull} = F_{mag} = IlB = \left(\frac{vlB}{R}\right)lB = \frac{vl^2B^2}{R} \qquad (30.5)$$

Power as the number of electrons per second

Here power is a force F times a velocity. F=ma which is -𝕠d×e𝕪/-𝕆D, that would be -𝕆D×e𝕪 kinetic work in this model as e𝕪/-𝕆D. Power is defined as the amount of ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy passing through a wire per second as 1/-𝕠d. The unit of kinetic energy here would be an -𝕠d and e𝕪 Pythagorean Triangle, the number passing through a wire per second does not accelerate the Pythagorean Triangles, so it is not a force.

Power and impulse

As the -𝕆D×e𝕪 kinetic work increases in a wire there is a stronger +𝕆D potential and -𝕆D kinetic difference, this increases the number of -𝕠d and e𝕪 Pythagorean Triangles per second. It can also be regarded as a E𝕐/-𝕠d kinetic impulse, then the E𝕐 kinetic displacement accelerates electrons per the -𝕠d kinetic time as in power. This is also the pulling force referred to in moving the wire here.

Energy Considerations

The environment must do work on the wire to pull it. What happens to the energy transferred to the wire by this work? Is energy conserved as the wire moves along the rail? It will be easier to answer this question if we think about power rather than work. Power is the *rate* at which work is done on the wire. You learned in Chapter 9 that the power exerted by a force pushing or pulling an object with velocity v is $P = Fv$. The power provided to the circuit by pulling on the wire is

$$P_{input} = F_{pull}v = \frac{v^2l^2B^2}{R} \qquad (30.6)$$

This is the rate at which energy is added to the circuit by the pulling force.

Power dissipation

In this model Power would be dissipated by the E𝔸/+𝕠d potential impulse, this observes the protons reacting against the motion of electrons in the wire. Here I² would be E𝕐/-𝕆D from the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy, opposing this is the ½×+e𝔸/+𝕆d ×+𝕠d rotational potential energy. When divided by R this can be regarded as the kinetic energy divided by the potential energy.

Impulse is conserved

The ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy combines -𝕆D×e𝕪 kinetic work and the E𝕐/-𝕠d kinetic impulse as E𝕐/-𝕆D, when the Power increases then E𝕐 increases and -𝕆D decreases inversely. These two impulses balance because the E𝕐/-𝕠d kinetic impulse is the inverse of the E𝔸/+𝕠d potential impulse, that comes from the +𝕆D potential and -𝕆D kinetic difference.

But the circuit also dissipates energy by transforming electric energy into the thermal energy of the wires and components, heating them up. The power dissipated by current I as it passes through resistance R is $P = I^2R$. Equation 30.4 for the induced current I gives us the power dissipated by the circuit of Figure 30.5:

$$P_{\text{dissipated}} = I^2R = \frac{v^2l^2B^2}{R} \qquad (30.7)$$

You can see that Equations 30.6 and 30.7 are identical. **The rate at which work is done on the circuit exactly balances the rate at which energy is dissipated.** Thus *energy is conserved.*

In this model changing the direction of the E𝕐/-𝕠d kinetic impulse, such as pushing the wire to the right, would reverse the +𝕆D×e𝕒 potential work being done in the loop. The +𝕆D potential difference would reverse direction, the magnet could also reverse the kinetic current by being flipped over.

Motional ℰ

In this model I/ℰ =R is a rearrangement of the equation in 1. This would be e𝕪/-𝕠d × 1/-𝕠d as F=ma. The resistance can then be e𝕒/+𝕠d as the potential current × 1/+𝕠d, here ℰ is ΔV which is in between +𝕆D and -𝕆D. It is not a force because the Δ signified a derivative. The equation would then have -𝕆D×e𝕪 kinetic work =+𝕆D×e𝕒 potential work. If -𝕆D doubles as the kinetic voltage, then +𝕆D would also double as the potential voltage.

Dissipating current

In this model the +𝕆D×e𝕒 potential work dissipates the e𝕪/-𝕠d kinetic current because +𝕆D is the potential probability, there is a randomizing direction in this work done which is like friction.

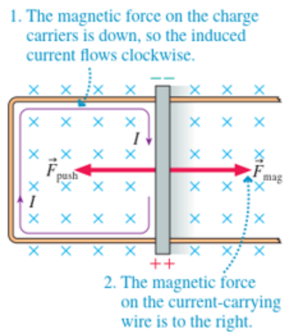
If you have to *pull* on the wire to get it to move to the right, you might think that it would spring back to the left on its own. **FIGURE 30.7** shows the same circuit with the wire moving to the left. In this case, you must *push* the wire to the left to keep it moving. The magnetic force is always opposite to the wire's direction of motion.

In both Figure 30.6, where the wire is pulled, and Figure 30.7, where it is pushed, a mechanical force is used to create a current. In other words, we have a conversion of *mechanical* energy to *electric* energy. A device that converts mechanical energy to electric energy is called a **generator.** The slide-wire circuits of Figures 30.6 and 30.7 are simple examples of a generator. We will look at more practical examples of generators later in the chapter.

We can summarize our analysis as follows:

1. Pulling or pushing the wire through the magnetic field at speed v creates a motional emf $\mathcal{E}$ in the wire and induces a current $I = \mathcal{E}/R$ in the circuit.
2. To keep the wire moving at constant speed, a pulling or pushing force must balance the magnetic force on the wire. This force does work on the circuit.
3. The work done by the pulling or pushing force exactly balances the energy dissipated by the current as it passes through the resistance of the circuit.

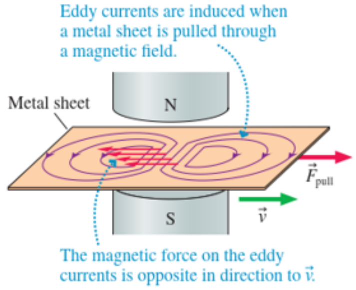
FIGURE 30.7 A pushing force is needed to move the wire to the left.



Potential eddy currents

In this model the north and south magnetic poles come from -𝕆D×e𝕪 kinetic work, the electrons are like spinning tops where one end can be north and the other south. The copper sheet does +𝕆D×e𝕒 potential work as a reaction to the E𝕐/-𝕠d kinetic impulse of moving the sheet. That forms potential eddy current from the +𝕆D potential torque, this moves electrons in loops with a -𝕆D kinetic torque. There are again three orthogonal forces as in the right-hand rule, one is the E𝕐/-𝕠d kinetic impulse from pushing the sheet, the magnet does -𝕆D×e𝕪 kinetic work and the sheet does +𝕆D×e𝕒 potential work.

FIGURE 30.9 Eddy currents.



Eddy Currents

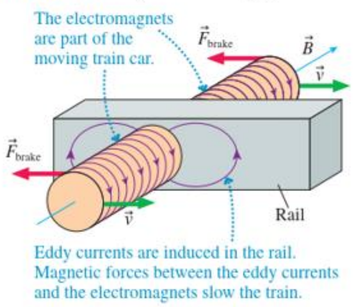
These ideas have interesting implications. Consider pulling a *sheet* of metal through a magnetic field, as shown in FIGURE 30.9a. The metal, we will assume, is not a magnetic material, so it experiences no magnetic force if it is at rest. The charge carriers in the metal experience a magnetic force as the sheet is dragged between the pole tips of the magnet. A current is induced, just as in the loop of wire, but here the currents do not have wires to define their path. As a consequence, two "whirlpools" of current begin to circulate in the metal. These spread-out current whirlpools in a solid metal are called **eddy currents.**

As the eddy current passes between the pole tips, it experiences a magnetic force to the left—a retarding force. Thus **an external force is required to pull a metal through a magnetic field.** If the pulling force ceases, the retarding magnetic force quickly causes the metal to decelerate until it stops. Similarly, a force is required to push a sheet of metal *into* a magnetic field.

Potential work in the rail

In this model the electromagnet makes +𝕆D×e𝕒 potential work in the rail, that reacts against the -𝕆D×e𝕪 kinetic work from the magnet. The e𝕧/-𝕚d inertial velocity is the train is dissipated, that reacts against the slowing by the +𝕆D×e𝕒 potential work being done. The train's inertia has a tendency to continue on with a -𝕆D inertial probability without changing.

FIGURE 30.10 Magnetic braking system.



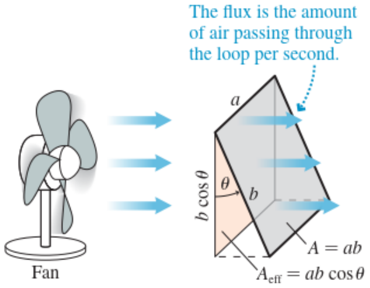
Eddy currents are often undesirable. The power dissipation of eddy currents can cause unwanted heating, and the magnetic forces on eddy currents mean that extra energy must be expended to move metals in magnetic fields. But eddy currents also have important useful applications. A good example is magnetic braking.

The moving train car has an electromagnet that straddles the rail, as shown in FIGURE 30.10. During normal travel, there is no current through the electromagnet and no magnetic field. To stop the car, a current is switched into the electromagnet. The current creates a strong magnetic field that passes *through* the rail, and the motion of the rail relative to the magnet induces eddy currents in the rail. The magnetic force between the electromagnet and the eddy currents acts as a braking force on the magnet and, thus, on the car. Magnetic braking systems are very efficient, and they have the added advantage that they heat the rail rather than the brakes.

Magnetic flux

In this model work comes from rotation and torque, a fan can do work with torque moving air forward. The fan does -𝕆D×e𝕪 kinetic work with the electricity powering it, the air then moves with -𝕀D×e𝕧 inertial work. The area of the loop is a square, as the angle changes the amount of squared work going through it changes.

FIGURE 30.11 The amount of air flowing through a loop depends on the effective area of the loop.



30.3 Magnetic Flux

Faraday found that a current is induced when the amount of magnetic field passing through a coil or a loop of wire changes. And that's exactly what happens as the slide wire moves down the rail in Figure 30.5! As the circuit expands, more magnetic field passes through. It's time to define more clearly what we mean by "the amount of field passing through a loop."

Imagine holding a rectangular loop in front of the fan shown in FIGURE 30.11. The amount of air flowing *through* the loop—the *flux*—depends on the angle of the loop. The flow is maximum if the loop is perpendicular to the flow, zero if the loop is rotated to be parallel to the flow. In general, the amount of air flowing through is proportional to the *effective area* of the loop (i.e., the area facing the fan):

$$A_{eff} = ab\cos\theta = A\cos\theta \qquad (30.8)$$

where $A = ab$ is the area of the loop and θ is the tilt angle of the loop. A loop perpendicular to the flow, with $\theta = 0°$, has $A_{eff} = A$, the full area of the loop.

Each arrow as a kinetic vector

In this model each arrow can be a eɏ kinetic vector, it measures the squared strength B of the magnetic field as -𝕆D. The side of the loop is like the hypotenuse of a Pythagorean Triangle as the angle changes, that would not be a constant Pythagorean Triangle area because the hypotenuse is a constant size. The angle θ would still be proportional to the change in the angle θ with -𝕆D×eɏ kinetic work because the relative proportions of the Pythagorean Triangle sides change in the same way.

A Pythagorean Triangle to the side of the loop

The height of the loop perpendicular to the ground would be -𝕠d here, as this decreases the length along the ground would increase inversely as eɏ. Here the size of the loop would change as the hypotenuse, this does not affect the calculations because only the -𝕠d height lets through the -𝕠d kinetic magnetic field or flux.

We can apply this idea to a magnetic field passing through a loop. FIGURE 30.12 shows a loop of area $A = ab$ in a uniform magnetic field. Think of the field vectors, seen here from behind, as if they were arrows shot into the page. The density of arrows (arrows per m^2) is proportional to the strength B of the magnetic field; a stronger field would be represented by arrows packed closer together. The number of arrows passing through a loop of wire depends on two factors:

1. The density of arrows, which is proportional to B, and
2. The effective area $A_{eff} = A\cos\theta$ of the loop.

The angle θ is the angle between the magnetic field and the axis of the loop. The maximum number of arrows passes through the loop when it is perpendicular to the magnetic field ($\theta = 0°$). No arrows pass through the loop if it is tilted 90°.

Roy electromagnetism as inverses

If the loop changes size to maintain the constant Pythagorean Triangle area, then the eɏ kinetic vector will give an accurate electric component to the -𝕠d kinetic magnetic field. That is because in Roy electromagnetism the -𝕠d kinetic magnetic field and the eɏ kinetic electric charge are inverses of each other.

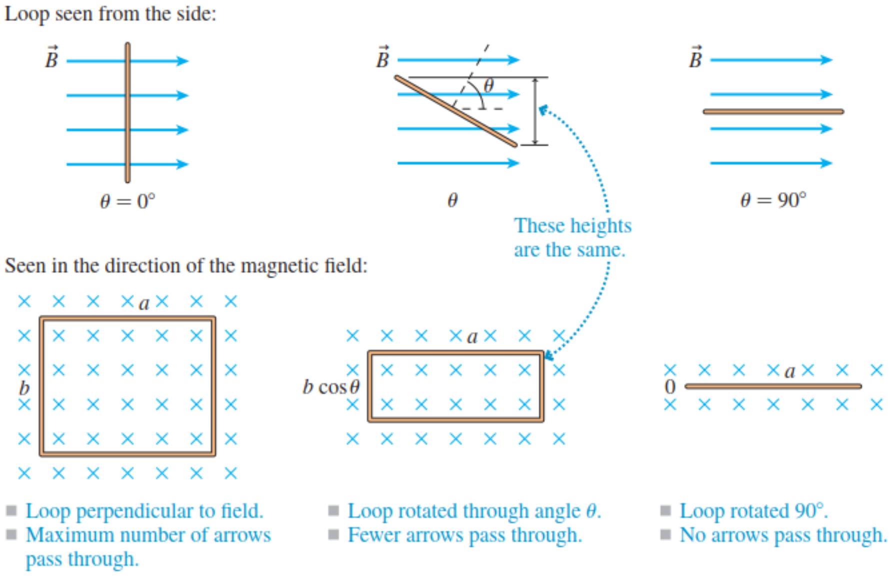


FIGURE 30.12 Magnetic field through a loop that is tilted at various angles.

The dot product and impulse

In this model the dot product would only be used with straight Pythagorean Triangle sides and impulse. The -ⓓd and ey Pythagorean Triangle has a height of -ⓓd letting through the -ⒹD magnetic flux as -ⒹD×ey kinetic work.

The cross product and work

This model gives the same answers, sinθ would use the -ⓓd vertical side letting through the flux while the loop acts as the hypotenuse. When the angle θ goes to zero there is no flux, that is the same as the vector $\vec{A}$ becoming vertical.

The cross product with spin and magnetism

In this model sinθ would be used with spin Pythagorean Triangle sides and magnetism. The Pythagorean Triangle forms a parallelogram where the hypotenuse is parallel to the other parallelogram side. The ey kinetic vector is horizontal, the other side of the parallelogram is equal to it. This was explained earlier in the text.

The parallelogram as two Pythagorean Triangles

This parallelogram is used in the cross product, the changing angle gives a spin or torque which here would be -ⓓd. In this model the parallelogram area would be constant, it can be rearranged to give two Pythagorean Triangles as a rectangle.

The hypotenuse is removed with the Pythagorean Triangles

The hypotenuse is removed from both sides so it can be set as 1. Then sinθ decreases according to -ⓓd/ζ (the hypotenuse is ζ) as with the cross product.

The window for the electromagnetic flux

The -ⓓd vertical side can be regarded as the window the loop allows the electromagnetic flux through. As the angle θ decreases this rectangle decreases in area, the width of the loop is constant so if -ⓓd halves for example then so does the window of the loop.

The magnetic flux would do -𝕆D×e𝕪 kinetic work as a square, so this would decrease as an inverse square when the angle θ decreases.

The weber as -𝕆D×e𝕪 kinetic work divided by area

The Weber would change according to this decreasing area. The Tesla here is F=ma which comes from the -𝕆D×e𝕪 kinetic work.

With this in mind, let's define the **magnetic flux** Φ_m as

$$\Phi_m = A_{eff}B = AB\cos\theta \qquad (30.9)$$

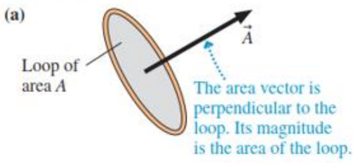
The magnetic flux measures the amount of magnetic field passing through a loop of area A if the loop is tilted at angle θ from the field. The SI unit of magnetic flux is the **weber.** From Equation 30.9 you can see that

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T m}^2$$

Equation 30.9 is reminiscent of the vector dot product: $\vec{A}\cdot\vec{B} = AB\cos\theta$. With that in mind, let's define an **area vector** $\vec{A}$ to be a vector *perpendicular* to the loop, with magnitude equal to the area A of the loop. Vector $\vec{A}$ has units of m^2. FIGURE 30.13a shows the area vector $\vec{A}$ for a circular loop of area A.

FIGURE 30.13b shows a magnetic field passing through a loop. The angle between

FIGURE 30.13 Magnetic flux can be defined in terms of an area vector $\vec{A}$.



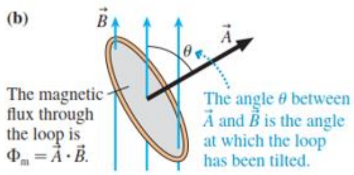
The dot product is the inverse of the cross product

The dot product is the inverse of the cross product here, that comes from the constant Pythagorean Triangle area. The answer is the same, except that the vertical window -𝕠d changes because it does -𝕆D×e𝕪 kinetic work.

vectors $\vec{A}$ and $\vec{B}$ is the same angle used in Equations 30.8 and 30.9 to define the effective area and the magnetic flux. So Equation 30.9 really is a dot product, and we can define the magnetic flux more concisely as

$$\Phi_m = \vec{A}\cdot\vec{B} \qquad (30.10)$$

Writing the flux as a dot product helps make clear how angle θ is defined: θ is the angle between the magnetic field and the axis of the loop.



Summing integral areas

In this model the integral areas of -𝕆D×e𝕪 kinetic work can be added up. A nonuniform magnetic field would come from constructive and destructive interference, such as with multiple magnets. Some integral areas of -𝕆D kinetic probability would have this interference, the same answer can come from taking the -𝕆D×e𝕪 kinetic work from each magnet and calculating the constructive and destructive interference.

An integral is not bounded by straight Pythagorean Triangle sides

In this model the integral is not an area bounded by straight Pythagorean Triangle sides. Instead, it is a single straight vector such as e𝕪 associated with a given strength of the -𝕠d kinetic magnetic field, they are inverses from the constant Pythagorean Triangle area. This can also define the integral field because as -𝕠d changes at different kinetic points e𝕪, that does different amounts of -𝕆D×e𝕪 kinetic work.

Magnetic Flux in a Nonuniform Field

Equation 30.10 for the magnetic flux assumes that the field is uniform over the area of the loop. We can calculate the flux in a nonuniform field, one where the field strength changes from one edge of the loop to the other, but we'll need to use calculus.

FIGURE 30.15 shows a loop in a nonuniform magnetic field. Imagine dividing the loop into many small pieces of area dA. The infinitesimal flux $d\Phi_m$ through one such area, where the magnetic field is $\vec{B}$, is

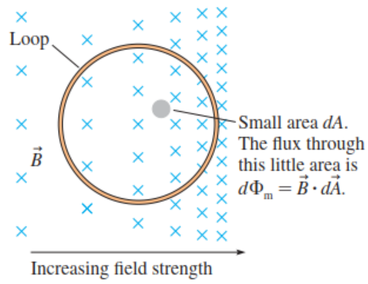
$$d\Phi_m = \vec{B} \cdot d\vec{A} \qquad (30.11)$$

The total magnetic flux through the loop is the sum of the fluxes through each of the small areas. We find that sum by integrating. Thus the total magnetic flux through the loop is

$$\Phi_m = \int_{\text{area of loop}} \vec{B} \cdot d\vec{A} \qquad (30.12)$$

Equation 30.12 is a more general definition of magnetic flux. It may look rather formidable, so we'll illustrate its use with an example.

FIGURE 30.15 A loop in a nonuniform magnetic field.



A changing magnetic flux

In this model the magnetic flux changes with -𝕆D×e𝕪 kinetic work, the -𝕆D kinetic probabilities are changing and so the electrons in the wire change their probabilities of where they would be measured. That creates a kinetic current moving the meter.

30.4 Lenz's Law

We started out by looking at a situation in which a moving wire caused a loop to expand in a magnetic field. This is one way to change the magnetic flux through the loop. But Faraday found that a current can be induced by any change in the magnetic flux, no matter how it's accomplished.

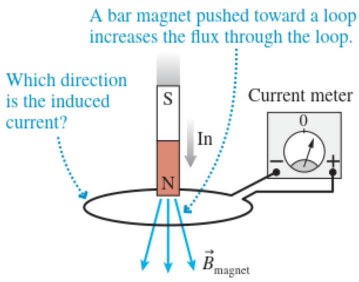
For example, a momentary current is induced in the loop of FIGURE 30.18 as the bar magnet is pushed toward the loop, increasing the flux through the loop. Pulling the magnet back out of the loop causes the current meter to deflect in the opposite direction. The conducting wires aren't moving, so this is not a motional emf. Nonetheless, the induced current is very real.

The German physicist Heinrich Lenz began to study electromagnetic induction after learning of Faraday's discovery. Three years later, in 1834, Lenz announced a rule for determining the direction of the induced current. We now call his rule **Lenz's law,** and it can be stated as follows:

> **Lenz's law** There is an induced current in a closed, conducting loop if and only if the magnetic flux through the loop is changing. The direction of the induced current is such that the induced magnetic field opposes the *change* in the flux.

Lenz's law is rather subtle, and it takes some practice to see how to apply it.

FIGURE 30.18 Pushing a bar magnet toward the loop induces a current.



A reaction from potential work

In this model the magnet does -𝕆D×e𝕪 kinetic work as it moves, also if the loop area changes the -𝕆D kinetic probability changes. The +𝕆D×e𝕒 potential work of the protons in the wire react against this which creates a +𝕆D potential probability or difference. That moves electrons towards it making the meter move.

NOTE One difficulty with Lenz's law is the term *flux*. In everyday language, the word *flux* already implies that something is changing. Think of the phrase, "The situation is in flux." Not so in physics, where *flux*, the root of the word *flow*, means "passes through." A steady magnetic field through a loop creates a steady, *un*changing magnetic flux.

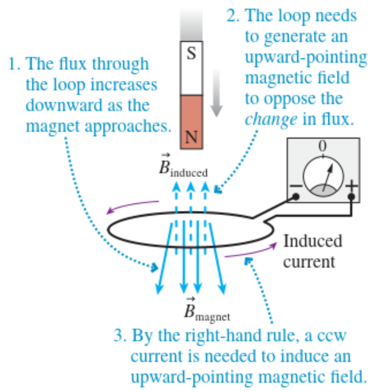
Lenz's law tells us to look for situations where the flux is *changing*. This can happen in three ways.

1. The magnetic field through the loop changes (increases or decreases),
2. The loop changes in area or angle, or
3. The loop moves into or out of a magnetic field.

Lenz's law depends on the idea that an induced current generates its own magnetic field $\vec{B}_{induced}$. This is the *induced magnetic field* of Lenz's law. You learned in Chapter 29 how to use the right-hand rule to determine the direction of this induced magnetic field.

In Figure 30.18, pushing the bar magnet toward the loop causes the magnetic flux to *increase* in the downward direction. To oppose the *change* in flux, which is what Lenz's law requires, the loop itself needs to generate the *upward*-pointing magnetic field of **FIGURE 30.19**. The induced magnetic field at the center of the loop will point upward if the current is ccw. Thus pushing the north end of a bar magnet toward the loop induces a ccw current around the loop. The induced current ceases as soon as the magnet stops moving.

FIGURE 30.19 The induced current is ccw.



An electron in an orbital

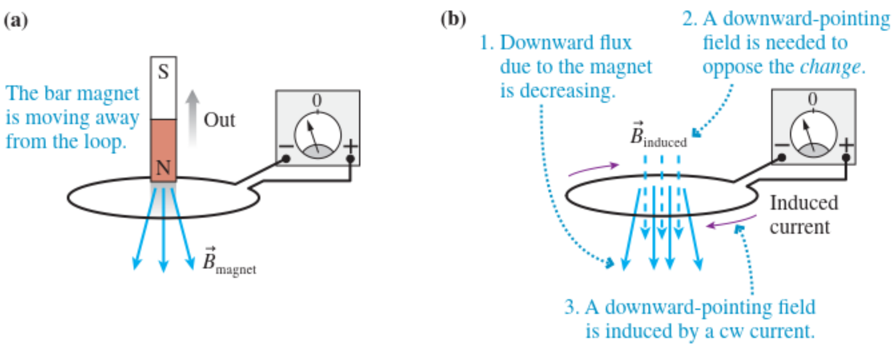
In this model the +𝕆D×e𝕒 potential work reacts against any change, this is like the proton reacting against an electron in an orbital around it. Withdrawing the magnet is like the electron moving higher in an orbital or leaving the atom, the proton does +𝕆D×e𝕒 potential work against this. When the electron has not absorbed a photon with -𝔾D×e𝕪 light work, then this reaction would move it back to a lower orbital.

Pushing and pulling on the electron

When an external magnetic field pushes an electron to a lower orbital, the proton also does +𝕆D×e𝕒 potential work reacting against this. That is because the -𝕆D×e𝕪 kinetic work of the electron, and the +𝕆D×e𝕒 potential work of the proton are inverses of each other. They tend towards a balance which in the diagram would be where the magnet is not moving. Then the -𝕆D kinetic probability, and the +𝕆D potential probability of the proton, would not be changing. There would be no work being done.

Now suppose the bar magnet is pulled back away from the loop, as shown in **FIGURE 30.20a**. There is a downward magnetic flux through the loop, but the flux *decreases* as the magnet moves away. According to Lenz's law, the induced magnetic field of the loop *opposes this decrease*. To do so, the induced field needs to point in the *downward* direction, as shown in **FIGURE 30.20b**. Thus as the magnet is withdrawn, the induced current is clockwise (cw), opposite to the induced current of Figure 30.19.

FIGURE 30.20 Pulling the magnet away induces a cw current.



A flux has no forces

In this model a flux is a flow without forces, there is no -𝕆D×e𝕪 kinetic work being done by it. There is not reaction to the protons in the wire to the flux itself, only if there is -𝕆D×e𝕪 kinetic work being done. A change in the e𝕪 kinetic position with this work is when the magnet is moved.

> **NOTE** Notice that the magnetic field of the bar magnet is pointing downward in both Figures 30.19 and 30.20. It is not the *flux* due to the magnet that the induced current opposes, but the *change* in the flux. This is a subtle but critical distinction. If the induced current opposed the flux itself, the current in both Figures 30.19 and 30.20 would be ccw to generate an upward magnetic field. But that's not what happens. When the field of the magnet points down and is increasing, the induced current opposes the increase by generating an upward field. When the field of the magnet points down but is decreasing, the induced current opposes the decrease by generating a downward field.

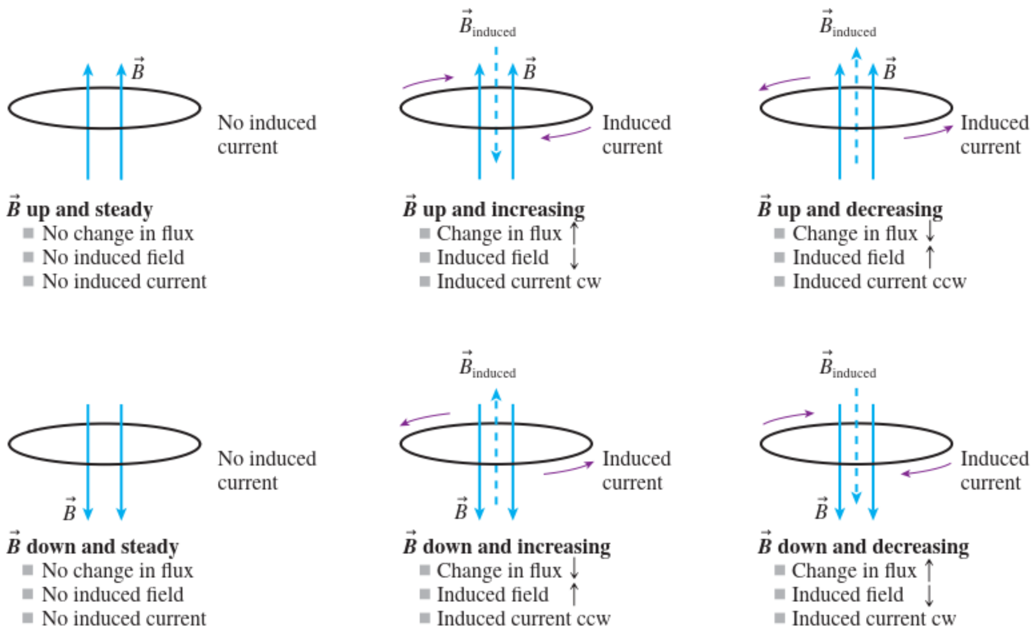
Using Lenz's Law

FIGURE 30.21 shows six basic situations. The magnetic field can point either up or down through the loop. For each, the flux can either increase, hold steady, or decrease in strength. These observations form the basis for a set of rules about using Lenz's law.

Potential work reacting

Here $\vec{B}$ is the e𝕪 kinetic vector from the magnet, when this is steady the e𝕪 kinetic position is not changing so there is no -𝕆D×e𝕪 kinetic work being done. There is no +𝕆D×e𝕒 potential work being done either. The reaction where there is -𝕆D×e𝕪 kinetic work is an inverse as +𝕆D×e𝕒 potential work.

FIGURE 30.21 The induced current for six different situations.



An induced current

In this model $I_{induced}$ is the $\mathcal{E}$ emf or ΔV divided by the resistance where here would be +𝕠d as the potential magnetic field. The induced kinetic current would be e𝕪/-𝕠d and the reactive potential current is e𝕒/+𝕠d. This current moves between the -𝕆D kinetic difference and the

+𝕆D potential difference. Here $\mathcal{E}$ would be -𝕠d as the kinetic magnetic field and +𝕠d as the potential magnetic field would be from the resistance.

Current and resistance

When there is no change in the magnet's kinetic position e𝕪 there is no -𝕆D×e𝕪 kinetic work being done. Because +𝕠d and -𝕠d are inverses then so are -𝕆D and +𝕆D. Here e𝕪/-𝕠d = e𝕒/+𝕠d so rearranging gives -𝕠d/+𝕠d = e𝕪/e𝕒, that comes from the constant Pythagorean Triangle area. This would give -𝕠d/+𝕠d = $\mathcal{E}$/R, that can also comes from -𝕆D×e𝕪 kinetic work = +𝕆D×e𝕒 potential work. Rearranging is then -𝕠d/+𝕠d = E𝕐/E𝔸 where the E𝕐/-𝕠d kinetic impulse equals the +𝕆D×e𝕒 potential work. (30.13) would be the -𝕠d kinetic clock gauge and +𝕠d potential clock gauge on which the opposing impulses are observed over time.

30.5 Faraday's Law

Charges don't start moving spontaneously. A current requires an emf to provide the energy. We started our analysis of induced currents with circuits in which a *motional emf* can be understood in terms of magnetic forces on moving charges. But we've also seen that a current can be induced by changing the magnetic field through a stationary circuit, a circuit in which there is no motion. There *must* be an emf in this circuit, even though the mechanism for this emf is not yet clear.

The emf associated with a changing magnetic flux, regardless of what causes the change, is called an **induced emf** $\mathcal{E}$. Then, if there is a complete circuit having resistance R, a current

$$I_{\text{induced}} = \frac{\mathcal{E}}{R} \qquad (30.13)$$

Faraday's law

In this model $\mathcal{E}$ would not be a magnitude because here that refers to a straight Pythagorean Triangle side vector. The magnetic flux is $d\Phi_m/dt$ which here would be the E𝕐/-𝕠d kinetic impulse changing with -𝕠d kinetic time. Because this is a force $d\Phi_m$ would be E𝕐 as the kinetic difference, how the electrons would accelerated in the wire to move the meter. The E𝕐/-𝕠d kinetic impulse is the inverse of -𝕆D×e𝕪 kinetic work so this would be the particle/wave duality. The magnet does -𝕆D×e𝕪 kinetic work, the electrons are accelerated in the circuit with a E𝕐/-𝕠d kinetic impulse.

A change over time or distance

The circuit is like that using a battery or capacitor, there would be a -𝕆D kinetic difference moving the electrons. The magnet here is moving the electrons with a E𝕐/-𝕠d kinetic impulse as particles. It would depend on whether the electrons are observed with a change over time, with the -𝕆D×e𝕪 kinetic work there is a change in the position of the magnet not over time so that is work.

Work leads to impulse

A change in the -𝕆D×e𝕪 kinetic work can then result in a E𝕐/-𝕠d kinetic impulse with the electrons. In the magnet the electrons are still confined to atoms so they have a wave nature, their spins line up with -𝕆D kinetic constructive interference. With the wire the electrons have left the atoms, they move more as particles with a E𝕐/-𝕠d kinetic impulse.

The potential impulse attracts the electrons

As the magnet does -𝕆D×e𝕪 kinetic work there is a reaction according to Lenz's law, the protons in the wire have a E𝔸/+𝕠d potential impulse which moves the electrons towards it. This is also a reaction like +𝕆D×e𝕒 potential work, when electrons are near positive ions they are moving with a E𝕐/-𝕠d kinetic impulse. The protons react against this with a E𝔸/+𝕠d potential impulse moving them towards being absorbed by the ions. The wire would react against the -𝕆D×e𝕪 kinetic work of the magnet by becoming more positive on one end, that attracts the electron particles with a E𝕐/-𝕠d kinetic impulse.

is established in the wire as a *consequence* of the induced emf. The direction of the current is given by Lenz's law. The last piece of information we need is the size of the induced emf $\mathcal{E}$.

The research of Faraday and others eventually led to the discovery of the basic law of electromagnetic induction, which we now call **Faraday's law.** It states:

> **Faraday's law** An emf $\mathcal{E}$ is induced around a closed loop if the magnetic flux through the loop changes. The magnitude of the emf is
>
> $$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right| \quad (30.14)$$
>
> and the direction of the emf is such as to drive an induced current in the direction given by Lenz's law.

Potential torque in the coil

The induced emf here would be a rate of change over time. The electrons would also react against this with an E𝕍/-𝕚d inertial impulse. When there are N coils in the wire each reacts against the bar magnet with a +𝕆D potential torque.

In other words, the induced emf is the *rate of change* of the magnetic flux through the loop.

As a corollary to Faraday's law, an N-turn coil of wire in a changing magnetic field acts like N batteries in series. The induced emf of each of the coils adds, so the induced emf of the entire coil is

$$\mathcal{E}_{coil} = N\left|\frac{d\Phi_{per\,coil}}{dt}\right| \quad \text{(Faraday's law for an } N\text{-turn coil)} \quad (30.15)$$

The work increases as the wire moves

In the diagram B⃗ is the kinetic vector from the magnetic field and B would be -𝕆D from -𝕆D×e𝕪 kinetic work. The area is not significant in this model except that as it changes so does the -𝕆D×e𝕪 kinetic work. The width of the loop does not change so there is no force from it. Here x gives the E𝕐/-𝕠d kinetic impulse of the motion to the right as e𝕪/-𝕠d.

As a first example of using Faraday's law, return to the situation of Figure 30.5, where a wire moves through a magnetic field by sliding on a U-shaped conducting rail. **FIGURE 30.26** shows the circuit again. The magnetic field $\vec{B}$ is perpendicular to the plane of the conducting loop, so $\theta = 0°$ and the magnetic flux is $\Phi = AB$, where A is the area of the loop. If the slide wire is distance x from the end, the area is $A = xl$ and the flux at that instant of time is

$$\Phi_m = AB = xlB \quad (30.16)$$

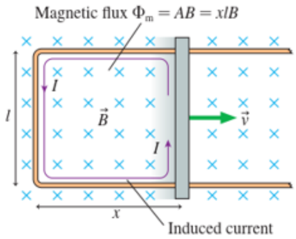
The flux through the loop increases as the wire moves. According to Faraday's law, the induced emf is

$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right| = \frac{d}{dt}(xlB) = \frac{dx}{dt}lB = vlB \quad (30.17)$$

where the wire's velocity is $v = dx/dt$. We can now use Equation 30.13 to find that the induced current is

$$I = \frac{\mathcal{E}}{R} = \frac{vlB}{R} \quad (30.18)$$

FIGURE 30.26 The magnetic flux through the loop increases as the slide wire moves.



A potential induced current

In this model the induced magnetic field comes from +𝕆D×e𝕒 potential work, that gives I as the e𝕒/+𝕠d potential current. That is v, it varies according to the width of the loop as l because more -𝕆D×e𝕪 kinetic work and +𝕆D×e𝕒 potential work is done. Here it is multiplied by B as -

𝕆D, if this was stronger with a larger or closer magnet then the velocity would increase. The resistance is the E𝔸/+𝕠d potential impulse reacting against the E𝕐/-𝕠d kinetic impulse of the wire moving.

> The flux is increasing into the loop, so the induced magnetic field opposes this increase by pointing out of the loop. This requires a ccw induced current in the loop. Faraday's law leads us to the conclusion that the loop will have a ccw induced current $I = vlB/R$. This is exactly the conclusion we reached in Section 30.2, where we analyzed the situation from the perspective of magnetic forces on moving charge carriers. Faraday's law confirms what we already knew but, at least in this case, doesn't seem to offer anything new.

Changing the interference

When the loop changes there is different +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work, the constructive and destructive interference means the electrons are more likely to be measured in different positions. The magnetic field can change with a stronger magnet, or one doing a different amount of -𝕆D×e𝕪 kinetic work by changing the e𝕪 kinetic position of it. For example it can be moved closer to the loop. In each case there is a change in the +𝕆D potential and -𝕆D kinetic difference, like with a battery or capacitor.

Changing the loop with a potential torque

In (30.19) this is adding together the changes in the loop with the changes in the magnet. The first term is the change in the area as a +𝕆D potential torque, because this is a square it grows like the squared area A does. As the loop changes size so does the torque, it is like a revolving top that increases in its radius. That changes the rotational +𝕠d×e𝕒/+𝕠d angular momentum which is a force as +𝕆D×e𝕒 potential work.

The particle/wave duality

Here the change is taken over time as a E𝔸/+𝕠d potential impulse, in this model the induced current would come from +𝕆D×e𝕒 potential work reacting against the magnet. The E𝔸/+𝕠d potential impulse is observing this as how electrons move towards this as particles, there is a particle/wave duality.

Changing the magnet

The second term is where the -𝕆D×e𝕪 kinetic work changes by using a stronger magnet or moving it closer. The change is in $\vec{B}$ which is e𝕪, that means -𝕆D as the kinetic probability also must change in this magnet. That is also described here as a E𝕐/-𝕠d kinetic impulse, the change in the magnet also has a particle/wave duality, it can be regarded as moving the electrons in the loop with a E𝕐/-𝕠d kinetic impulse.

Two work and one impulse, or two impulse and one work

In this model there are three orthogonal forces, all three cannot be work because they cannot all spin in three orthogonal directions. There also cannot be three orthogonal impulse, that would be like moving in three directions at once. There can be two of work and one of impulse, or one of work and two of impulse. Because the wire moves with a velocity it is regarded as a E𝕐/-𝕠d kinetic impulse so the other two must be +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work.

The wire can move with work or impulse

If the wire was moving with -𝕆D×e𝕪 kinetic work or -𝕀D×e𝕧 inertial work, then the magnet can be regarded as moving electrons with a E𝕐/-𝕠d kinetic impulse. The resistance to this would be a E𝔸/+𝕠d potential impulse.

What Does Faraday's Law Tell Us?

The induced current in the slide-wire circuit of Figure 30.26 can be understood as a motional emf due to magnetic forces on moving charges. We had not anticipated this kind of current in Chapter 29, but it takes no new laws of physics to understand it. The induced currents in Examples 30.8 and 30.9 are different. We cannot explain these induced currents on the basis of previous laws or principles. This is new physics.

Faraday recognized that all induced currents are associated with a changing magnetic flux. There are two fundamentally different ways to change the magnetic flux through a conducting loop:

1. The loop can expand, contract, or rotate, creating a motional emf.
2. The magnetic field can change.

We can see both of these if we write Faraday's law as

$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right| = \left|\vec{B}\cdot\frac{d\vec{A}}{dt} + \vec{A}\cdot\frac{d\vec{B}}{dt}\right| \qquad (30.19)$$

The rate of change of the magnetic flux

In this model the rate of change of the magnetic flux comes from -𝕆D×e𝕪 kinetic work and +𝕆D×e𝕒 potential work, there is a changed probability of where the electrons are likely to be measured. This can also be observed as the inverse with a E𝕐/-𝕠d kinetic impulse and E𝔸/+𝕠d potential impulse.

The first term on the right side represents a motional emf. The magnetic flux changes because the loop itself is changing. This term includes not only situations like the slide-wire circuit, where the area A changes, but also loops that rotate in a magnetic field. The physical area of a rotating loop does not change, but the area *vector* $\vec{A}$ does. The loop's motion causes magnetic forces on the charge carriers in the loop.

The second term on the right side is the new physics in Faraday's law. It says that an emf can also be created simply by changing a magnetic field, even if nothing is moving. This was the case in Examples 30.8 and 30.9. Faraday's law tells us that the induced emf is simply the rate of change of the magnetic flux through the loop, *regardless* of what causes the flux to change.

Explaining with work or impulse

In this model the current comes from -𝕆D×e𝕪 kinetic work as the magnet changes, there is also -𝕆D×e𝕪 kinetic work done in the loop as it changes size. That is reacted against by +𝕆D×e𝕒 potential work. These can explain both the change in the magnet and the area here, they can also be explained with a E𝕐/-𝕠d kinetic impulse and E𝔸/+𝕠d potential impulse.

Displacement in the wire

The changed -𝕆D×e𝕪 kinetic work of the magnet would then have a changed E𝕐/-𝕠d kinetic impulse in the wire, then the electric displacement is the observed force rather than the measured magnetic work. The changes in the magnet and the area appear to be opposites when the movement of the wire is the E𝕐/-𝕠d kinetic impulse. That is because the other two forces must be one of work and one of impulse. If the wire moves with work, then the magnet and area can both do work, or both be impulse.

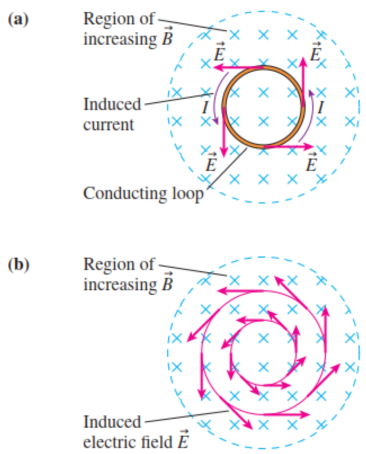
A change in position or time

When the magnet is changed, such as moving it closer to the loop, then there is a change in the e𝕪 kinetic position and -𝕆D×e𝕪 kinetic work. The change in the electrons can then be regarded as being over -𝕠d kinetic time, they move faster. The wire also moves to the right faster, so this can also be regarded as impulse. When the loop changes size this can be observed as being over +𝕠d potential time, it can also be a change in the size of l as e𝕒 in +𝕆D×e𝕒 potential work.

Coulomb and non-Coulomb

When a magnetic field changes in this model that is work, there is a -𝕆D force as a difference, a probability, or a torque. The non-Coulomb electric field can be a change in e𝕪 as the kinetic position of the magnet is changed. The Coulomb magnetic field would be the E𝕐/-𝕠d kinetic impulse, such as when a negative charge is attracted to a positive charge and its E𝔸/+𝕠d potential impulse.

FIGURE 30.30 An induced electric field creates a current in the loop.



30.6 Induced Fields

Faraday's law is a tool for calculating the strength of an induced current, but one important piece of the puzzle is still missing. What *causes* the current? That is, what *force* pushes the charges around the loop against the resistive forces of the metal? The agents that exert forces on charges are electric fields and magnetic fields. Magnetic forces are responsible for motional emfs, but magnetic forces cannot explain the current induced in a *stationary* loop by a changing magnetic field.

FIGURE 30.30a shows a conducting loop in an increasing magnetic field. According to Lenz's law, there is an induced current in the ccw direction. Something has to act on the charge carriers to make them move, so we infer that there must be an *electric* field tangent to the loop at all points. This electric field is *caused* by the changing magnetic field and is called an **induced electric field.** The induced electric field is the mechanism that creates a current inside a stationary loop when there's a changing magnetic field.

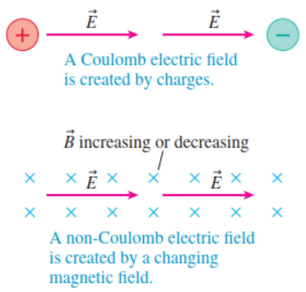
The conducting loop isn't necessary. The space in which the magnetic field is changing is filled with the pinwheel pattern of induced electric fields shown in **FIGURE 30.30b.** Charges will move if a conducting path is present, but the induced electric field is there as a direct consequence of the changing magnetic field.

But this is a rather peculiar electric field. All the electric fields we have examined until now have been created by charges. Electric field vectors pointed away from positive charges and toward negative charges. An electric field created by charges is called a **Coulomb electric field.** The induced electric field of Figure 30.30b is caused not by charges but by a changing magnetic field. It is called a **non-Coulomb electric field.**

Coulomb and non-Coulomb fields

Here a Coulomb field, as particles not a field in this model, comes from the E𝔸/+𝕠d potential impulse and E𝕐/-𝕠d kinetic impulse. The non-Coulomb field from +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work. In both cases there are straight Pythagorean Triangle sides as e𝕒 and e𝕪, when there is no measurement or observation these are part of the +𝕠d×e𝕒 and -𝕠d×e𝕪 integral fields.

FIGURE 30.31 Two ways to create an electric field.



So it appears that there are two different ways to create an electric field:

1. A Coulomb electric field is created by positive and negative charges.
2. A non-Coulomb electric field is created by a changing magnetic field.

Both exert a force $\vec{F} = q\vec{E}$ on a charge, and both create a current in a conductor. However, the origins of the fields are very different. **FIGURE 30.31** is a quick summary of the two ways to create an electric field.

We first introduced the idea of a field as a way of thinking about how two charges exert long-range forces on each other through the emptiness of space. The field may have seemed like a useful pictorial representation of charge interactions, but we had little evidence that fields are *real*, that they actually exist. Now we do. The electric field has shown up in a completely different context, independent of charges, as the explanation of the very real existence of induced currents.

The electric field is not just a pictorial representation; it is real.

Conserved reactive forces

In this model there are reactive forces from the E𝔸/+𝕠d potential impulse and +𝕆D×e𝕒 potential work, these come from the protons in the loop. They react against the motion of electrons, when the magnet does -𝕆D×e𝕪 kinetic work making a e𝕪/-𝕠d kinetic current there is +𝕆D×e𝕒 potential work and a e𝕒/+𝕠d potential current.

Moving towards or away from atoms

Even if the loop had an even amount of kinetic current there is still a motion towards a positive charge. When an electron moves with this current it leaves the vicinity of an atom against the proton's attraction with constructive interference. That proton can attract another electron. The electrons tend to spread out with repulsion from destructive interference, then get attracted into a sphere of influence around the protons in the wire.

Calculating the Induced Field

The induced electric field is peculiar in another way: It is nonconservative. Recall that a force is conservative if it does no net work on a particle moving around a closed path. "Uphills" are balanced by "downhills." We can associate a potential energy with a conservative force, hence we have gravitational potential energy for the conservative gravitational force and electric potential energy for the conservative electric force of charges (a Coulomb electric field).

The potential difference in the magnet and loop

In this model there is a +𝕆D potential difference when the protons do +𝕆D×e𝕒 potential work. The electrons are moved with -𝕆D×e𝕪 kinetic work and a -𝕆D kinetic difference. The magnet does -𝕆D×e𝕪 kinetic work only on the loop, but this is reacted against by the +𝕆D×e𝕒 potential work from the magnet's protons. When the magnet does -𝕆D×e𝕪 kinetic work on the wire, there is a reaction tending to disrupt the magnet's spin directions. This is reacted against by the protons in the magnet.

Work to separate a charge

In (30.20) the e𝕪/-𝕆D kinetic work is divided by the -𝕠d×e𝕪/-𝕠d kinetic momentum, that leaves 1/-𝕠d as the $\mathcal{E}$.

But a charge moving around a closed path in the induced electric field of Figure 30.30 is always being pushed *in the same direction* by the electric force $\vec{F} = q\vec{E}$. There's never any negative work to balance the positive work, so the net work done in going around a closed path is not zero. Because it's nonconservative, we cannot associate an electric potential with an induced electric field. Only the Coulomb field of charges has an electric potential.

However, we can associate the induced field with the emf of Faraday's law. The emf was defined as the work required per unit charge to separate the charge. That is,

$$\mathcal{E} = \frac{W}{q} \qquad (30.20)$$

Work over a distance

Here q is the -𝕠d×e𝕪/-𝕠d kinetic momentum in Coulombs, that is to the left of the integral sign. Work is a force measured over a distance $\vec{s}$ here, $\vec{E}$ can be regarded as the e𝕪 kinetic vector in -𝕆D×e𝕪 kinetic work. That would make it the same as $\vec{s}$ in units, as $\vec{E}$ changes then so would -𝕆D as the kinetic probability.

Electric fields not allowed

In (30.21) the force F would be ma as -𝕠d×e𝕪/-𝕆D, as -𝕆D×e𝕪 kinetic work this would have the force -𝕆D measured over a distance $d\vec{s}$ as e𝕪. When $q\vec{E}$ is used that would be -𝕠d×E𝕐/-𝕠d as the E𝕐/-𝕠d kinetic impulse over a distance. With the particle/wave duality the force here comes from the electric field in conventional physics. In this model there is no electric field, only a magnetic field so the units are different but the forces are the same.

In batteries, a familiar source of emf, this work is done by chemical forces. But the emf that appears in Faraday's law arises when work is done by the force of an induced electric field.

If a charge q moves through a small displacement $d\vec{s}$, the small amount of work done by the electric field is $dW = \vec{F} \cdot d\vec{s} = q\vec{E} \cdot d\vec{s}$. The emf of Faraday's law is an emf around a *closed curve* through which the magnetic flux Φ_m is changing. The work done by the induced electric field as charge q moves around a closed curve is

$$W_{\text{closed curve}} = q\oint \vec{E} \cdot d\vec{s} \tag{30.21}$$

Work divided by charge

In (30.22) the $\mathcal{E}$ is work divided by charge, that would be ey/-𝕆D divided by ey/-𝕠d to leave 1/-𝕠d or ΔV. Then E⃗ as the integral would be -𝕆D according to this model from -𝕆D×ey kinetic work.

Squaring the emf

The -𝕆D×ey kinetic work being done would have a -𝕆D kinetic difference, there is also +𝕆D×e𝕒 potential work with a +𝕆D potential difference. In between these would be $\mathcal{E}^2$, there is no actual force because the two differences balance as inverses, giving a constant current. In between +𝕆D and -𝕆D would be the same as in between +𝕠d and -𝕠d with the E𝔸/+𝕠d potential impulse and E𝕐/-𝕠d kinetic impulse.

Work and impulse as the particle/wave duality

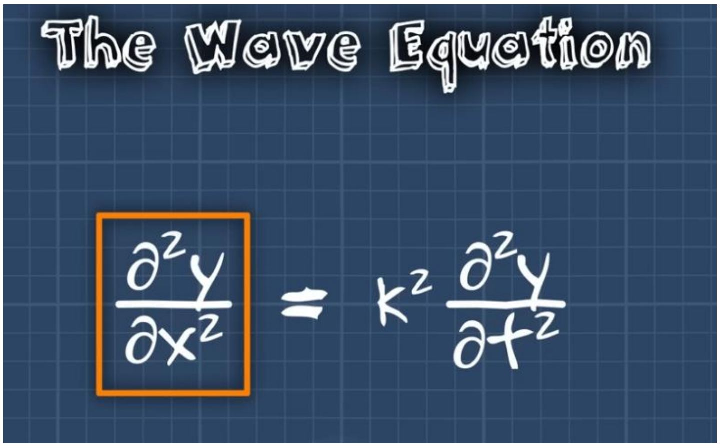
In (30.23) the left-hand side would be -𝕆D×ey kinetic work because it is measured over a distance s⃗. In conventional terminology E⃗ would be replaced by dB as -𝕆D. On the right-hand side there is dB/dt which would be the E𝕐/-𝕠d kinetic impulse, that would become E⃗/dt. This is the particle wave duality.

A loop needs torque

The bar magnet for example is not moving electrons in a circuit, they remain in its atoms. This is doing work because the force comes from magnetism not electric charge. The loop is moving electrons around it, that can be regarded as the E𝕐/-𝕠d kinetic impulse and E𝔸/+𝕠d potential impulse. In this model it would also be +𝕆D×e𝕒 potential work reacting against the -𝕆D×ey kinetic work of the magnet, this is because impulse only acts in straight lines and a loop's motion needs a torque.

The wave equation

Here (30.23) can be regarded as a wave equation, the left-hand side would be +𝕆D/e𝕒 potential work where ds⃗ is a distance. On the right-hand side there is a E𝕐/-𝕠d kinetic impulse where -𝕠d is dt as kinetic time. This can be rearranged as +𝕆D/e𝕒 =k× E𝕐/-𝕠d, → +𝕆D × -𝕠d × 1/e𝕒 × 1/E𝕐 = k. -𝕆D is the inverse of E𝕐 and ey is the inverse of e𝕒 so → +𝕆D × -𝕆D × -𝕠d × ey. Here +𝕆D and -𝕆D are inverses so they give 1 and -𝕠d×ey is the -𝕠d and ey Pythagorean Triangle which is a constant area k.



Faraday’s law a wave equation

This would make Faraday’s law a wave equation in conventional physics. Lenz’s law has a reactionary force so here the magnet would do -𝕆D×e𝕪 kinetic work. The electrons are not moving, and the magnetic force comes from the -𝕆D kinetic torque. The electrons in the loop would be accelerated with a E𝕐/-𝕠d kinetic impulse in the wave equation, reacting against this would be the E𝔸/+𝕠d potential impulse.

Potential work does not equal kinetic work

That would be written as +𝕆D×e𝕒 potential work ≠ -𝕆D×e𝕪 kinetic work. Rearranging gives +𝕆D/-𝕆D × e𝕒/e𝕪 so as +𝕆D increases then -𝕆D decreases inversely. As +𝕆D increases then e𝕒 decreases and e𝕪 increases inversely to it. This is not equal to a constant k, the -𝕆D×e𝕪 kinetic work can increase from the bar magnet and the +𝕆D×e𝕒 potential work being done would react but be overcome by it.

Lenz’s law

This gives Lenz’s law, the -𝕆D×e𝕪 kinetic work done by the magnet is always reacted against by the +𝕆D×e𝕒 potential work in the loop. The +𝕆D potential torque creates a +𝕆D potential probability where the protons in the loop react against any change. When the magnet is moved this changes the -𝕆D×e𝕪 kinetic work so the +𝕆D×e𝕒 potential work reacts inversely to this.

Current and resistance

If the +𝕆D×e𝕒 potential work was resistance this can overcome the -𝕆D×e𝕪 kinetic work in a resistor dissipating the kinetic current. This is a consequence of the wave equation in this model, it gives action/reaction pairs as inverses.

Gravity and inertia

The +𝕀D×e𝕙 gravitational work ≠ -𝕀D×e𝕧 inertial work because gravity overcomes the reactionary forces of inertia. For example an asteroid passing a planet might be slowed enough to go into orbit, the -𝕀D×e𝕧 inertial work reacts against this but it overcome by the +𝕀D×e𝕙 gravitational work. -𝕆D×e𝕪 kinetic work ≠ -𝕀D×e𝕧 inertial work because pushing a block on a frictionless surface would give an equal and opposite work force with inertia. The -𝕆D×e𝕪 kinetic work would overcome this and the block would move.

Probabilities interfering

In each case this is calculated by probabilities interfering with work and displacement with impulse. The asteroid may have a larger -𝕆D inertial probability than the +𝕆D gravitational probability it encounters, then the asteroid would pass the planet. When the +𝕆D potential probability of the resistor is less than the -𝕆D kinetic difference or probability of the current, then electrons flow through the resistor. If not then the resistor blocks the current. If the change of the -𝕆D×e𝕪 kinetic work from the magnet is weak in reversing the loop current, then the +𝕆D×e𝕒 potential work from Lenz's law in the loop can overcome that change. The current would be reduced but stay in the same direction as +𝕆D would be larger than -𝕆D.

Displacement interference

The inverse of this would be E𝔸/+𝕠d potential impulse and E𝕐/-𝕠d kinetic impulse, the electron passing through a resistor can also be regarded as particles. Reacting against this is the E𝔸/+𝕠d potential impulse of the resistor's protons, E𝕐 and E𝔸 are force vectors so they are vector subtracted. If E𝔸 is larger then the resistor blocks the current, if smaller then E𝕐 electrons pass through it.

where the integration symbol with the circle is the same as the one we used in Ampère's law to indicate an integral around a closed curve. If we use this work in Equation 30.20, we find that the emf around a closed loop is

$$\mathcal{E} = \frac{W_{\text{closed curve}}}{q} = \oint \vec{E} \cdot d\vec{s} \qquad (30.22)$$

If we restrict ourselves to situations such as Figure 30.30 where the loop is perpendicular to the magnetic field and only the field is changing, we can write Faraday's law as $\mathcal{E} = |d\Phi_m/dt| = A|dB/dt|$. Consequently

$$\oint \vec{E} \cdot d\vec{s} = A\left|\frac{dB}{dt}\right| \qquad (30.23)$$

Induced potential work

In this model there would be an induced +𝕆D×e𝕒 potential work as the -𝕆D×e𝕪 kinetic work of the magnet changes. In an electromagnet the -𝕆D×e𝕪 kinetic work would increase with an increased kinetic current. There would be a reaction against this from the protons in the solenoid coil as +𝕆D×e𝕒 potential work.

Equation 30.23 is an alternative statement of Faraday's law that relates the induced electric field to the changing magnetic field.

The solenoid in **FIGURE 30.32a** provides a good example of the connection between $\vec{E}$ and $\vec{B}$. If there were a conducting loop inside the solenoid, we could use Lenz's law to determine that the direction of the induced current would be clockwise. But Faraday's law, in the form of Equation 30.23, tells us that **an induced electric field is present whether there's a conducting loop or not.** The electric field is induced simply due to the fact that $\vec{B}$ is changing.

Potential work reacting against kinetic work

In (a) the current is increasing, this can be from -𝕆D×e𝕪 kinetic work done in the wire. For example there could be a battery connected to it that has a +𝕆D potential and -𝕆D kinetic difference. Then the coil reacts against this with +𝕆D×e𝕒 potential work according to Lenz's law. That is overcome as a potential inertia, the -𝕆D×e𝕪 kinetic work in the coil then increases.

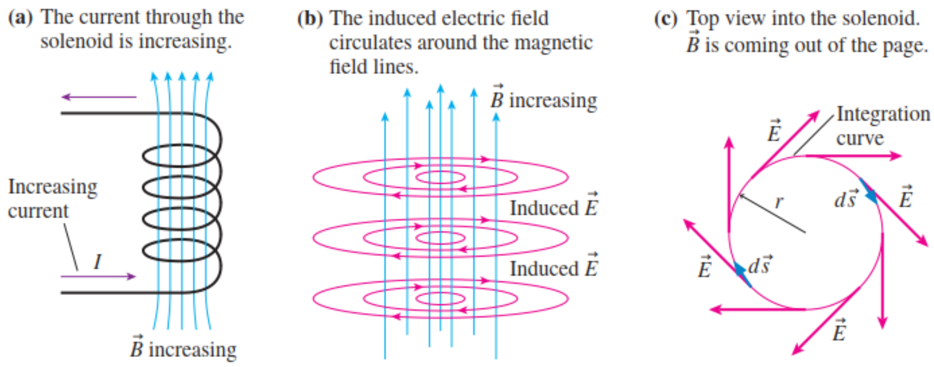
Induced potential torque

In (b) there would be +𝕆D×e𝕒 potential work reacting against the -𝕆D×e𝕪 kinetic work from the battery. That would be an induced +𝕆D potential torque not an electric field in this model. It can also be regarded as a E𝔸/+𝕠d potential impulse which is an electric force but not a field.

The potential integration curve

In (c) the +𝕆D×e𝕒 potential work is shown being induced by the -𝕆D×e𝕪 kinetic work from the battery. Here r is e𝕒 and +𝕆D is the potential torque from the protons in the coil, +𝕆D is a potential integral. They react against the electrons being moved by the increased -𝕆D×e𝕪 kinetic work. That is like protons holding onto their electrons in atoms against an external -𝕆D kinetic difference. For example, a resistor reacts against -𝕆D×e𝕪 kinetic work pushing electrons through it, and from this kinetic work stripping its atoms of its electrons.

FIGURE 30.32 The induced electric field circulates around the changing magnetic field inside a solenoid.



An electric field cannot circulate in a loop

In this model an electric displacement force cannot circulate in a loop, it can only move in a straight-line. It does not stop between charges, but the curve can contain small E𝕐 kinetic displacement vectors where electrons are moved away from protons. These would come from the Coulomb interaction, also the electrons would repel each other by colliding with this E𝕐/-𝕠d kinetic impulse.

The shape and direction of the induced electric field have to be such that it *could* drive a current around a conducting loop, if one were present, and it has to be consistent with the cylindrical symmetry of the solenoid. The only possible choice, shown in **FIGURE 30.32b**, is an electric field that circulates clockwise around the magnetic field lines.

NOTE Circular electric field lines violate the Chapter 23 rule that electric field lines have to start and stop on charges. However, that rule applied only to Coulomb fields created by source charges. An induced electric field is a non-Coulomb field created not by source charges but by a changing magnetic field. Without source charges, induced electric field lines *must* form closed loops.

The closed loop as potential work

In this model the closed loop would be +𝕆D×e𝕒 potential work where e𝕒 is the radius r, it reacts against the -𝕆D×e𝕪 kinetic work of the changing magnetic field. Here l is the circumference of a circle, +𝕆D×e𝕒 potential work in an atom would have this as a circular orbital. Then -𝕆D×e𝕪 kinetic work would be done by an electron, a change in its orbital would be reacted against by induced +𝕆D×e𝕒 potential work.

The magnetic flux area

Here the magnetic flux area is the +𝕆D potential probability or torque, that gives the probable changes the induced magnetic field can make to electrons in the coil. This induced +𝕆D×e𝕒 potential work is here equal to the changing magnetic field dB over time dt. That would be the

E𝕐/-𝕠d kinetic impulse of the current in the wire as it increased, E𝕐 would be the kinetic displacement as an electric force not magnetic. In this model the forces are reversed, the magnetic force is with respect to distance ds⃗ and the electric force is with respect to time.

To use Faraday's law, choose a *clockwise* circle of radius r as the closed curve for evaluating the integral. **FIGURE 30.32c** shows that the electric field vectors are everywhere tangent to the curve, so the line integral of $\vec{E}$ is

$$\oint \vec{E} \cdot d\vec{s} = El = 2\pi r E \tag{30.24}$$

where $l = 2\pi r$ is the length of the closed curve. This is exactly like the integrals we did for Ampère's law in Chapter 29.

The line integral as a circumference

Here the line integral is taken around the circumference in this model +𝕆D×e𝕒 potential work would come from the e𝕒 radius. An electric displacement force could not move around a loop in this model, it can be small straight force vectors in this loop. The area A would change the +𝕆D potential probability or torque as both are squares. This would be defined as +𝕆D, as the area A increased for example then +𝕆D would decrease in strength and e𝕒 as the radius would increase linearly.

Potential work proportional to kinetic impulse

This would change with respect to time as a E𝕐/-𝕠d kinetic impulse, the +𝕆D/e𝕒 potential work has +𝕆D proportional to E𝕐 and -𝕠d proportional to e𝕒. For example, in an electron orbital moving to a higher orbital decreases E𝕐 with a slower e𝕧/-𝕚d inertial velocity. The +𝕆D potential torque decreases with the inverse square rule. The higher orbital has a larger -𝕠d kinetic time, that increased linearly with the e𝕒 altitude.

If we stay inside the solenoid ($r < R$), the flux passes through area $A = \pi r^2$ and Equation 30.24 becomes

$$\oint \vec{E} \cdot d\vec{s} = 2\pi r E = A\left|\frac{dB}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right| \tag{30.25}$$

Thus the strength of the induced electric field inside the solenoid is

$$E_{\text{inside}} = \frac{r}{2}\left|\frac{dB}{dt}\right| \tag{30.26}$$

This result shows very directly that the induced electric field is created by a *changing* magnetic field. A constant $\vec{B}$, with $dB/dt = 0$, would give $E = 0$.

Kinetic work and reactive potential impulse

In (30.27) $\mathcal{E}$ is the -𝕆D×e𝕪 kinetic work done by the magnet, that is reacted against by the E𝔸/+𝕠d potential impulse. These are proportional to each other, as the magnet increases its -𝕆D×e𝕪 kinetic work then Lenz's law gives a reactive force from the protons in the coil. When the -𝕆D×e𝕪 kinetic work ceases to change then the E𝔸/+𝕠d potential impulse also stops because these are proportional.

The emf is not squared

The $\mathcal{E}$ is not squared here because there are two opposing squared forces that are inverses. For example, in an atom there is +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work, though these are two forces they counterbalance so an electron can have a constant e𝕪/-𝕠d kinetic velocity.

Occasionally it is useful to have a version of Faraday's law without the absolute value signs. The essence of Lenz's law is that the emf $\mathcal{E}$ opposes the *change* in Φ_m. Mathematically, this means that $\mathcal{E}$ must be opposite in sign to dB/dt. Consequently, we can write Faraday's law as

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt} \tag{30.27}$$

For practical applications, it's always easier to calculate just the magnitude of the emf with Faraday's law and to use Lenz's law to find the direction of the emf or the induced current. However, the mathematically rigorous version of Faraday's law in Equation 30.27 will prove to be useful when we combine it with other equations, in Chapter 31, to predict the existence of electromagnetic waves.

Maxwell's Theory of Electromagnetic Waves

In 1855, less than two years after receiving his undergraduate degree, the Scottish physicist James Clerk Maxwell presented a paper titled "On Faraday's Lines of Force." In this paper, he began to sketch out how Faraday's pictorial ideas about fields could be given a rigorous mathematical basis. Maxwell was troubled by a certain lack of

A kinetic gradient

In this model there would be -𝕆D×e𝕪 kinetic work and a E𝔸/+𝕠d potential impulse induced as a reaction to it. There would also be a E𝕐/-𝕠d kinetic impulse with induced +𝕆D×e𝕒 potential work. Changing a magnetic field would be from -𝕆D×e𝕪 kinetic work, the gradient of this force changes along a wire. This creates a -𝕆d kinetic difference and a current flow like water running down a +𝕀D gravitational gradient.

A potential gradient

A changing electric field would refer to the E𝕐/-𝕠d kinetic impulse, that induces a reactive +𝕆D×e𝕒 potential work as a potential gradient. The electrons flowing in a coil induce a +𝕆D potential torque like a gradient against the electron particles.

A potential and gravitational gradient

This can be illustrated in a Hydrogen atom, the proton does +𝕆D×e𝕒 potential work and has a +𝕆D potential gradient around it. This is like in general relativity where a planet has a +𝕀D gravitational gradient around it. That can be shown as a planet on a rubber mesh that deforms giving the gradient corresponding to relativistic effects. In this model the proton would also have this gravitational gradient from its +𝕚d gravitational mass.

A kinetic impulse and potential work

An electron in a circular orbital would move at an e𝕒 altitude on this +𝕆D potential gradient. It can be regarded as having a E𝕐/-𝕠d kinetic impulse, then that is proportional to the -𝕆D×e𝕪 kinetic work. A change in this E𝕐/-𝕠d kinetic impulse, such as from a collision with a photon with a e𝕐/-𝕘d light impulse, would be reacted against by the proton's +𝕆D×e𝕒 potential work.

Kinetic work and a potential impulse

The electron also moves as a wave with -𝕆D×e𝕪 kinetic work, there is a kinetic gradient in between a current orbital and moving upwards when absorbing a photon. Both these gradients would be quantized in steps, but the angle of the gradient would be the same. Reacting against this would be the E𝔸/+𝕠d potential impulse of the proton as a particle.

Displacement and torque

In the first case there is an electric displacement change over time as the E𝕐/-𝕠d kinetic impulse, that is reacted against by a changed magnetic field over a distance. The second case is a magnetic field changing over a distance, that is reacted against be a changed electric displacement over time.

Photons have no forces

In this model photons are e𝕪 and -𝕘d Pythagorean Triangles, when they are not being observed as the e𝕐/-𝕘d light impulse, or measured as -𝔾D×e𝕪 light work, they have no forces. They are not then inducing a change from an electric to a magnetic force or vice versa, the photon Pythagorean Triangle has an electric straight side and a magnetic spin side.

The photon is self-sustaining from its area

The photon is self-sustaining because the constant Pythagorean Triangle area is conserved, it can be absorbed into an electron in an orbital as -𝔾D×e𝕪 light work. It can also collide with an electron with a e𝕐/-𝕘d light impulse that changes its angle θ opposite -𝕘d.

Forces in a photon inducing each other

When a magnet does -𝕆D×e𝕪 kinetic work on a loop, this can induce a reactive +𝕆D×e𝕒 potential work from the protons in it. That is not the same as -𝔾D×e𝕪 light work and a e𝕐/-𝕘d light impulse inducing each other in a photon. These are active forces because the photon is emitted or absorbed from the active force changes of electrons.

Photon waves and particles are orthogonal

The photon in this model can be measured as an electromagnetic wave doing -𝔾D×e𝕪 light work, it can also be observed as an electromagnetic particle with a e𝕐/-𝕘d light impulse. These are perpendicular to each other because e𝕪 and -𝕘d are perpendicular in the e𝕪 and -𝕘d Pythagorean Triangle.

Forces would change the photon

If the photon did -𝔾D×e𝕪 light work and then had a e𝕐/-𝕘d light impulse then these would be perpendicular, but they would also change the photon. -𝔾D×e𝕪 light work would require it to be absorbed by an atom or to go through a double slit to form an interference pattern. E𝕐/-𝕘d light impulse would change its Pythagorean Triangle side ratios with a collision as a particle. To remain the same the photon has no forces in this model.

The photon has three orthogonal forces

The photon has three orthogonal forces as with the moving loop and the magnet. The e𝕐/-𝕘d light impulse and -𝔾D×e𝕪 light work are orthogonal, that leaves the third orthogonal direction the photons moves in as its inertial velocity. This can be observed as an E𝕍/-𝕚d inertial impulse when the photon collides with an electron in the Compton effect. It is can be measured when the photons is absorbed into an atom.

The permittivity and permeability constants in a photon

The photon moves with a combination of the permittivity constant ϵ which is a squared force from the E𝕐/-𝕠d kinetic impulse of an electron. It also has μ as the permittivity squared constant when -𝕆D×e𝕪 kinetic work changes with a photon emission or absorption.

Constant squared forces

In this model both are constant squared forces like gravity, the square root of each is taken to give the constant inertial velocity of c. This refers to the ratio between the two, ϵ and μ can change from this ratio to give different inertial velocities of moving electron for example. For example 1/√(ϵ×μ) has the inertial velocity of c. Here √μ is 1/-𝕠d as kinetic time and √ϵ is e𝕪 in the e𝕪/-𝕠d kinetic velocity. To change to a lower e𝕧/-𝕚d inertial velocity there would be a squared force from either ϵ or μ. These squares represent the changes between inertial velocities.

A constant force not a constant value

This is like h as Planck’s constant or -𝕠d ×e𝕐/-𝕠d. In this model that is a squared force observation of the E𝕐/-𝕠d kinetic impulse of an electron. It would be in a quantized orbital, in between these orbitals the E𝕐/-𝕠d kinetic impulse would change as a constant square E𝕐. That strength of that change would be ϵ. It is similar to gravity where there is a constant squared force, though gravity itself can be weaker at a greater e𝕙 height above a gravitational mass.

Decelerating from near c

If a rocket was moving close to c, then it can decelerate with -𝕆D×e𝕪 kinetic work or a E𝕐/-𝕠d kinetic impulse. These would be squared forces in increments of μ and ϵ respectively. An electron in changing orbitals would have different ϵ and μ squared force increments in between them. A deceleration from c in this model can only be done with this squared force. It can be quantized in between orbitals or continuously with an electron in free space.

A photon rolling wheel

In this model the photon is represented as a rolling wheel, the e𝕪 and -𝕘d Pythagorean Triangle has the -𝕘d rotational axle and the e𝕪 Pythagorean Triangle side is the spoke which rotates. That gives two orthogonal forces. The third force is where the wheel can roll, that can be observed as a e𝕐/-𝕘d light impulse or measured as -𝔾D×e𝕪 light work. This is like the three forces in the right-hand rule.

Two of work and one of impulse, or two of impulse and one of work

The wheel can then be two directions of work and one of impulse, or two of impulse and one of work, like with the loop and the magnet. In all three orthogonal directions there are no forces, the wheel turns with a constant rotational frequency so there is no torque or work. The spoke does not change in magnitude so there is no impulse. The photon moves with a constant velocity so there is no work or impulse.

The photon as a sine and cosine wave

When the photon is represented in conventional physics there are two orthogonal forces inducing each other. There is a sine wave which would be from -𝔾D×e𝕪 light work, and a cosine wave as the inverse which would be the e𝕐/-𝕘d light impulse but this is not a wave. These sine and cosine curves then come from the forces inducing each other, in order to maintain the photon as it moves. In this model the cosine wave would have a straight-line force like a spring, not as an oscillation.

symmetry. Faraday had found that a changing magnetic field creates an induced electric field, a non-Coulomb electric field not tied to charges. But what, Maxwell began to wonder, about a changing *electric* field?

To complete the symmetry, Maxwell proposed that a changing electric field creates an **induced magnetic field,** a new kind of magnetic field not tied to the existence of currents. FIGURE 30.33 shows a region of space where the *electric* field is increasing. This region of space, according to Maxwell, is filled with a pinwheel pattern of induced magnetic fields. The induced magnetic field looks like the induced electric field, with $\vec{E}$ and $\vec{B}$ interchanged, except that—for technical reasons explored in the next chapter—the induced $\vec{B}$ points the opposite way from the induced $\vec{E}$. Although there was no experimental evidence that induced magnetic fields existed, Maxwell went ahead and included them in his electromagnetic field theory. This was an inspired hunch, soon to be vindicated.

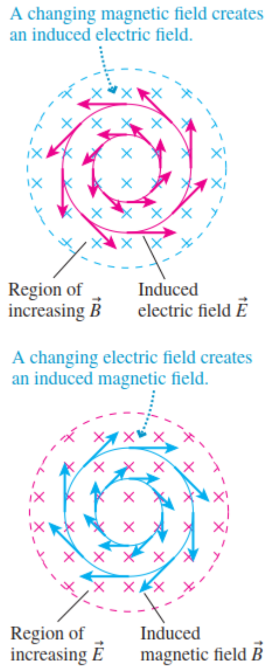
Maxwell soon realized that it might be possible to establish self-sustaining electric and magnetic fields that would be entirely independent of any charges or currents. That is, a changing electric field $\vec{E}$ creates a magnetic field $\vec{B}$, which then changes in just the right way to recreate the electric field, which then changes in just the right way to again recreate the magnetic field, and so on. The fields are continually recreated through electromagnetic induction without any reliance on charges or currents.

Maxwell was able to predict that electric and magnetic fields would be able to sustain themselves, free from charges and currents, if they took the form of an **electromagnetic wave.** The wave would have to have a very specific geometry, shown in FIGURE 30.34, in which $\vec{E}$ and $\vec{B}$ are perpendicular to each other as well as perpendicular to the direction of travel. That is, an electromagnetic wave would be a *transverse* wave.

Furthermore, Maxwell's theory predicted that the wave would travel with speed

$$v_{\text{em wave}} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

FIGURE 30.33 Maxwell hypothesized the existence of induced magnetic fields.



The sine and cosine as inverse forces

In this model E⃗ is e𝕪 -𝕆D×e𝕪 kinetic work, it would be e𝕪 as the kinetic electric charge of the photon. B⃗ would then be -𝕘d as the rotational frequency. Neither of these are squared because the photon has no forces, unless it is observed or measured. When these are represented as a sine and cosine wave, they are tracing out squared forces. The sine and cosine here are inverse forces, so in between there would be no actual force in the photon.

A centrifugal force opposed by a centripetal force

The diagram below could be regarded as an electron having a centrifugal force being held by a centripetal force in the atom. That presents its orbital as doing -𝕆D×e𝕪 kinetic work and being reacted against with +𝕆D×e𝕒 potential work. In this model the forces are not necessary because they are inverses. When an electron jumps to a higher orbital the -𝔾D×e𝕪 light work increases the -𝕆D×e𝕪 kinetic work of the electrons, reacted against by the +𝕆D×e𝕒 potential work of the proton.

Assuming all forces exist

The photon can be represented as having these forces with -𝔾D×e𝕪 light work and then the photon, electron, and proton can be regarded as measuring each other. They can also interact with a e𝕐/-𝕘d light impulse, E𝕐/-𝕠d kinetic impulse, and E𝔸/+𝕠d potential impulse where the electron is outside the atom.

A Bohmian pilot wave

This is like the Bohmian pilot wave theory where the electron is proposed to always be a particle with a pilot wave around it. Then it can be measured in a given position, also observed at the same time with the uncertainty principle hiding this. The -𝕆D×e𝕪 kinetic work of the electron would give the -𝕆D kinetic probability of where the electron is. Then the E𝕐/-𝕠d kinetic impulse would allow this existing electron to be observed.

Position and time cannot coincide

Instead of the particle and wave both existing, this model has a particle/wave duality where an iota can be observed at a time or measured at a position. The position and time cannot coincide because that would be a Pythagorean Triangle with a zero area. Instead the uncertainty comes from only being able to square one Pythagorean Triangle side and use the other to compare it to.

A rolling wheel does not change

In this model a Pythagorean Triangle such as an electron can change in several ways, it can change its time of observation which is measured as a E𝕐/-𝕠d kinetic impulse. It can change its position which is measured as -𝕆D×e𝕪 kinetic work. This is consistent with a rolling wheel, the -𝕠d kinetic time acts as an axle. Then it can be observed as an impulse with this acting like a clock gauge. The rolling wheel has a spoke as e𝕪, this can trace out a circumference of one wavelength as to how far the wheel rolls. Then the -𝕆D kinetic torque of the axle gives this work being done with the change of position.

The spoke as a radius

The e𝕪 spoke would be like a radius, so it would be the circumference wavelength divided by the square root of 2π. It is a square root because the wavelength would be observed with a E𝕐/-𝕠d kinetic impulse by for example stopping the wheel and observing a change in its vector direction.

The second Fiegenbaum constant

In this model this uses ≈ √(2π) as β the second Fiegenbaum constant. That gives a quantized value of a -𝕘d size in -𝔾D×e𝕪 light work so the photon is absorbed into an atom. The observed wavelength is then a rotation of the wheel, the change in this wavelength occurs when an electron collides with a photon for example. Then this gives a tine value of β as the changed e𝕪 electron spoke. When the electron is in an atom this is exactly √(2π) as a quantization of its orbital. Then the change in e𝕪 as the deBroglie wavelength, such as with a photon absorption, increases the e𝕒 altitude of the electron as the radius.

Orbitals with α

In this model α is also the ground state of the Hydrogen atom in Biv space-time. It is known to be ≈ 1/137 of c as the inertial velocity of the electron in the lowest orbital. In Roy electromagnetism α is $e^{-𝕠d}$ which d=1, 1/tanα is ≈ 1/e. That makes tanθ 1/137, the quantized values of the circular orbitals are for different values of d in the exponent. When squared as $e^{-𝕆D}$ they are integer values of -𝕆D×e𝕪 kinetic work.

Three Pythagorean Triangles in the proton

In Biv space-time the proton gravitational mass is 1836 times the inertial mass of the electron. Here $1836^{-7.506}$ is ≈ $e^{-𝕠d}$ with d=1. Here 7.506 is ≈ 3 times the second Feigenbaum constant β of 2.502, there are three orthogonal +𝕠d and e𝕒 Pythagorean Triangles as a proton which adds to ≈ 7.506. Here √(2π) is ≈ 2.5066 so β approaches a regular quantized value as increments of a radius. Then the circumference is where the electrons moves with a quantized -𝕆D×e𝕪 kinetic work increments of α.

Relative strengths of electromagnetism and gravity

In this model β is the limit of a tine width in chaos, that approaches a constant value like a regular quantization. This connects α in Roy electromagnetism with α in Biv space-time as a quantized amount of inertial velocity. That comes from the gravitational mass of the proton and the inertial mass of the electron. These proportions then give the relative strengths of gravity to electromagnetism here.

Balancing electron charge and mass

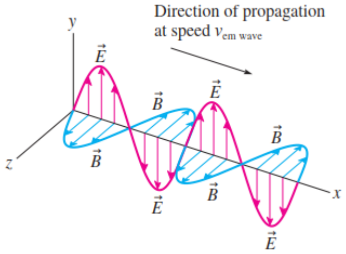
The three orthogonal +𝕠d and e𝕒 Pythagorean Triangles form the proton of a Hydrogen atom for example. Each has a quark value, a proton would be two +𝕠2/3 Pythagorean Triangle sides and a -𝕠1/3 side. The motion of these quarks is quantized according to the tine values, that connects the charge values with $e^{-𝕠d}$ as α. The quarks must have this level of motion and mass so that the Pythagorean Equation is balanced in the ground state with the electron charge and mass.

where ϵ_0 is the permittivity constant from Coulomb's law and μ_0 is the permeability constant from the law of Biot and Savart. Maxwell computed that an electromagnetic wave, if it existed, would travel with speed $v_{\text{em wave}} = 3.00 \times 10^8$ m/s.

We don't know Maxwell's immediate reaction, but it must have been both shock and excitement. His predicted speed for electromagnetic waves, a prediction that came directly from his theory, was none other than the speed of light! This agreement could be just a coincidence, but Maxwell didn't think so. Making a bold leap of imagination, Maxwell concluded that **light is an electromagnetic wave.**

It took 25 more years for Maxwell's predictions to be tested. In 1886, the German physicist Heinrich Hertz discovered how to generate and transmit radio waves. Two years later, in 1888, he was able to show that radio waves travel at the speed of light. Maxwell, unfortunately, did not live to see his triumph. He had died in 1879, at the age of 48.

FIGURE 30.34 A self-sustaining electromagnetic wave.



Work through a changing window

In this model the rotating loop has a changing window through which the -𝕆D×e𝕪 kinetic work of the magnet is done, that uses sinθ here as the angle rotates not cosθ. The force here comes from the spin Pythagorean Triangle side -𝕆D so this uses sinθ in this model. The loop is rotating with a frequency ω, with the time t that can be regarded as a squared time -𝕆D. There is a -𝕆D kinetic torque on the loop so this rotation is not smooth, the ω frequency is uncertain because of the changing acceleration and deceleration of the loop.

30.7 Induced Currents: Three Applications

There are many applications of Faraday's law and induced currents in modern technology. In this section we will look at three: generators, transformers, and metal detectors.

Generators

A generator is a device that transforms mechanical energy into electric energy. **FIGURE 30.35** on the next page shows a generator in which a coil of wire, perhaps spun by a windmill, rotates in a magnetic field. Both the field and the area of the loop are constant, but the magnetic flux through the loop changes continuously as the loop rotates. The induced current is removed from the rotating loop by *brushes* that press up against rotating *slip rings*.

The flux through the coil is

$$\Phi_m = \vec{A} \cdot \vec{B} = AB\cos\theta = AB\cos\omega t \qquad (30.28)$$

In (30.29) the $\mathcal{E}_{\text{coil}}$ would be the difference between the +𝕠d potential magnetic field of the protons in the coil and the -𝕠d kinetic magnetic field of the electrons in it. In this model the +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work is not being measured by the time but by changing e𝕒 and e𝕪 positions. As the loop moves these positions changing along with the amount of torque. That gives an AC voltage because the +𝕆D potential and -𝕆D kinetic differences are changing as the loop turns.

The change in the potential and kinetic voltage also occurs in a wire in between two terminals of a battery or two plates of a capacitor. At different positions on a wire the ratio of the +𝕆D potential difference and -𝕆D kinetic difference changes. This is the same as the changing -𝕆D kinetic difference as the loop rotates, and the reacting +𝕆D potential difference. It is also the same as when an electromagnet is switched on, the -𝕆D kinetic difference changes which makes the electrons more likely to be found elsewhere.

Exponential decay and discharge

With the battery or capacitor the +𝕆D potential and -𝕆D kinetic difference also exponentially decay as they discharge, that reduces the likelihood of where the electrons are measured and so the kinetic current will slow. As the loop turns the exponential curve comes from where the straight Pythagorean Triangle side is squared as E𝕐 compared to -𝕠d with a constant Pythagorean Triangle area. With work there is an inverse exponential as a normal curve probability, the -𝕆D Pythagorean Triangle side is squared compared to e𝕪 with a constant Pythagorean Triangle area.

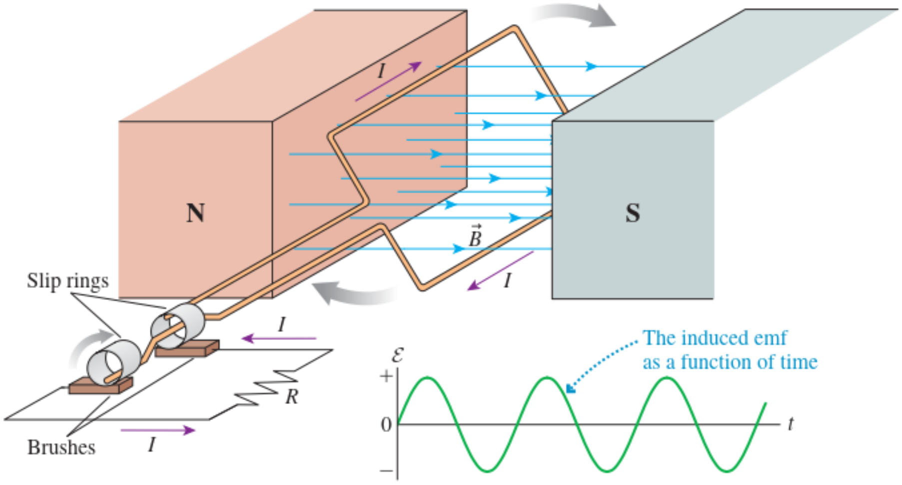
where ω is the angular frequency ($\omega = 2\pi f$) with which the coil rotates. The induced emf is given by Faraday's law,

$$\mathcal{E}_{coil} = -N\frac{d\Phi_m}{dt} = -ABN\frac{d}{dt}(\cos\omega t) = \omega ABN\sin\omega t \qquad (30.29)$$

where N is the number of turns on the coil. Here it's best to use the signed version of Faraday's law to see how $\mathcal{E}_{coil}$ alternates between positive and negative.

Because the emf alternates in sign, the current through resistor R alternates back and forth in direction. Hence the generator of Figure 30.35 is an alternating-current generator, producing what we call an *AC voltage*.

FIGURE 30.35 An alternating-current generator.



Quantized transformer loops

In a transformer each side does different amounts of -𝕆D×e𝕪 kinetic work, these are integer relations because a loop is complete as 1. The change in -𝕆D×e𝕪 kinetic work from one side to another changes the probability of where an electron is likely to be measured as the current flows. The electrons move as waves with -𝕆D×e𝕪 kinetic work, this is also like electrons in an atom changing orbitals with different amounts of -Od kinetic torque and -𝕆D×e𝕪 kinetic work.

Alternating current in a transformer

The alternating current changes the -𝕆D×e𝕪 kinetic work being done on both sides of the transformer, the probabilities of where the electrons are changes according to the ratio of the coil numbers.

Quantized gearing

It is also like gears in Biv space-time, two geared wheels can be turning so one has a -𝕀d inertial weight or torque, this rotation is changed into a different rotation rate in the second gear. This is also quantized with an integer number of teeth on both sides.

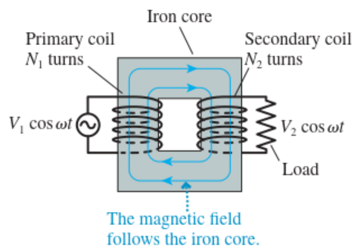
Impulse and work in gearing

It is also transformed in the sense that it now can have a faster e𝕧 velocity but a lower -𝕚d torque. This is seen in a car when shifting to a higher gear, it is harder to get up a hill because of the reduced -𝕀D inertial torque. The +𝕀D×e𝕙 gravitational work is being done against the car moving up the hill to a different e𝕙 height or position, +𝕀D is gravitational torque here compared to the gearing's -𝕀D inertial torque. It is also like a satellite's larger -𝕀D inertial torque allowing it to orbit at a greater e𝕙 height.

A lower gear has more torque but a slower velocity

This can be regained by shifting down to a lower gear with more -𝕀d inertial weight or torque but a weaker impulse. So a lower gear in a car can push people back in their seats more with this -𝕀d inertial weight, it has a weaker E𝕍/-𝕚d inertial impulse so the e𝕧/-𝕚d inertial velocity is lower before changing up a gear.

FIGURE 30.36 A transformer.



Transformers

FIGURE 30.36 shows two coils wrapped on an iron core. The left coil is called the **primary coil.** It has N_1 turns and is driven by an oscillating voltage $V_1 \cos \omega t$. The magnetic field of the primary follows the iron core and passes through the right coil, which has N_2 turns and is called the **secondary coil.** The alternating current through the primary coil causes an oscillating magnetic flux through the secondary coil and, hence, an induced emf. The induced emf of the secondary coil is delivered to the load as the oscillating voltage $V_2 \cos \omega t$.

The changing magnetic field inside the iron core is inversely proportional to the number of turns on the primary coil: $B \propto 1/N_1$. (This relation is a consequence of the coil's inductance, an idea discussed in the next section.) According to Faraday's law, the emf induced in the secondary coil is directly proportional to its number of turns:

Transforming the kinetic voltage

In this model the number of turns in each coil is quantized as a -𝕆D kinetic torque. That is reacted against by a +𝕆D potential torque by the protons in the coils and core. There cannot be a part turn so these are integers like electron orbitals. The two values of N are like the different D values in -𝕆D×e𝕪 kinetic work in between two electron orbitals. These have a different -𝕆D kinetic difference or voltage.

$\mathcal{E}_{sec} \propto N_2$. Combining these two proportionalities, the secondary voltage of an ideal transformer is related to the primary voltage by

$$V_2 = \frac{N_2}{N_1} V_1 \qquad (30.30)$$

Depending on the ratio N_2/N_1, the voltage V_2 across the load can be *transformed* to a higher or a lower voltage than V_1. Consequently, this device is called a **transformer.** Transformers are widely used in the commercial generation and transmission of electricity. A *step-up transformer,* with $N_2 \gg N_1$, boosts the voltage of a generator up to several hundred thousand volts. Delivering power with smaller currents at higher voltages reduces losses due to the resistance of the wires. High-voltage transmission lines carry electric power to urban areas, where *step-down transformers* $(N_2 \ll N_1)$ lower the voltage to 120 V.

Metal detectors

In this model the eddy currents are a +𝕆D potential torque or probability, they add to the -𝕆D kinetic torque of the electrons in the transmitter coil which reduces the -𝕆D×e𝕪 kinetic work being measured. This gives the strength of the +𝕆D×e𝕒 potential work being done in the material being detected.

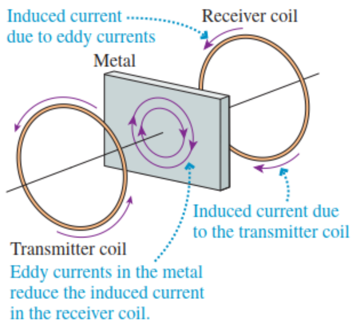
Metal Detectors

Metal detectors, such as those used in airports for security, seem fairly mysterious. How can they detect the presence of *any* metal—not just magnetic materials such as iron—but not detect plastic or other materials? Metal detectors work because of induced currents.

A metal detector, shown in **FIGURE 30.37**, consists of two coils: a *transmitter coil* and a *receiver coil.* A high-frequency alternating current in the transmitter coil generates an alternating magnetic field along the axis. This magnetic field creates a changing flux through the receiver coil and causes an alternating induced current. The transmitter and receiver are similar to a transformer.

Suppose a piece of metal is placed between the transmitter and the receiver. The alternating magnetic field through the metal induces eddy currents in a plane parallel to the transmitter and receiver coils. The receiver coil then responds to the *superposition* of the transmitter's magnetic field and the magnetic field of the eddy currents. Because the eddy currents attempt to prevent the flux from changing, in accordance with Lenz's law, the net field at the receiver *decreases* when a piece of metal is inserted between the coils. Electronic circuits detect the current decrease in the receiver coil and set off an alarm. Eddy currents can't flow in an insulator, so this device detects only metals.

FIGURE 30.37 A metal detector.



The Henry

The inductor stores a +𝕆D potential and -𝕆D kinetic torque or probability in its coil. That means electrons are more likely to be measured in different positions with it. Here Θ is the -𝕆D kinetic torque, with the +𝕆D potential torque it can also be regarded as -𝕠d and +𝕠d respectively. Then (30.31) would be -𝕠d×-𝕠d/e𝕪 or -𝕆D/e𝕪 as kinetic work. As a reaction against the current this would be +𝕠d×+𝕠d/e𝕒 from the potential current as +𝕆D×e𝕒 potential work.

The Tesla and work

This also comes from the Tesla as -𝕠d×e𝕪/-𝕆D and +𝕠d×e𝕒/+𝕆D as f=ma. That changes according to the area of a coil loop which contains the -𝕆D and +𝕆D kinetic and potential probabilities respectively. This is divided by A as amperes which is e𝕪/-𝕠d for the kinetic current and e𝕒/+𝕠d for the potential current.

The work comes from the coil geometry

In this model the potential current as e𝕒/+𝕠d would react with +𝕆D/e𝕒 potential work, that comes from the +𝕆D potential torque of the coils. The overall work comes from the coil geometry, there is also a +𝕆D potential and -𝕆D kinetic difference driving the current. The Henry is then a combination of the wire voltage and the coil geometry.

30.8 Inductors

Capacitors are useful circuit elements because they store potential energy U_C in the electric field. Similarly, a coil of wire can be a useful circuit element because it stores energy in the magnetic field. In circuits, a coil is called an **inductor** because, as you'll see, the potential difference across an inductor is an *induced* emf. An *ideal inductor* is one for which the wire forming the coil has no electric resistance. The circuit symbol for an inductor is —ᘓᘓᘓᘓ—.

We define the **inductance** L of a coil to be its flux-to-current ratio:

$$L = \frac{\Phi_m}{I} \tag{30.31}$$

Strictly speaking, this is called *self-inductance* because the flux we're considering is the magnetic flux the solenoid creates in itself when there is a current. The SI unit of inductance is the **henry,** named in honor of Joseph Henry, defined as

$$1 \text{ henry} = 1 \text{ H} \equiv 1 \text{ Wb/A} = 1 \text{ T m}^2\text{/A}$$

Practical inductances are typically millihenries (mH) or microhenries (μH).

It's not hard to find the inductance of a solenoid. In Chapter 29 we found that the magnetic field inside an ideal solenoid having N turns and length l is

$$B = \frac{\mu_0 NI}{l}$$

The inductance is reactive only

In this model the magnetic flux in an electromagnet comes from the -𝕆D kinetic torque of the current, the stronger the +𝕆D potential and -𝕆D kinetic difference then the stronger the kinetic current is. The inductance reacts against this current, so it is reactive only from the +𝕆D×e𝕒 potential work of the coil geometry. This is because the +𝕆D potential torque can only come from curved wires in a coil.

An inductor as a reservoir of electrons

The -𝕆D×e𝕪 kinetic work done in the coil is the inverse of the inductance, that is not the same as the -𝕆D×e𝕪 kinetic work being done by a battery for example with the current. The inductor can hold a number of electrons like a reservoir with this reactive +𝕆D×e𝕒 potential work.

Capacitance geometry and impulse

A capacitor can hold electrons because the E𝕐/-𝕠d kinetic impulse cannot jump in between the plates. The electrons in the negative plate do -𝕆D×e𝕪 kinetic work as it charges, that can affect the probable positions of electrons on the positive plate. This E𝕐/-𝕠d kinetic impulse depends only on the distance e𝕪 between the plates, their area increases the capacitance only through its size like many capacitors in parallel combined into one.

The magnetic flux through one turn of the coil is $\Phi_{\text{per turn}} = AB$, where A is the cross-section area of the solenoid. The total magnetic flux through all N turns is

$$\Phi_m = N\Phi_{\text{per turn}} = \frac{\mu_0 N^2 A}{l} I \tag{30.32}$$

Thus the inductance of the solenoid, using the definition of Equation 30.31, is

$$L_{\text{solenoid}} = \frac{\Phi_m}{I} = \frac{\mu_0 N^2 A}{l} \tag{30.33}$$

The inductance of a solenoid depends only on its geometry, not at all on the current. You may recall that the capacitance of two parallel plates depends only on their geometry, not at all on their potential difference.

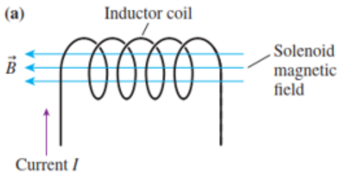
A changing current

When the kinetic current is changing, the +𝕆D×e𝕒 potential work in the inductor reacts against the probable positions of electrons. When this is not longer changing, the potential probability no longer reacts as there is no force.

The quantized Hall effect

In the Hall effect an external magnetic field orthogonal to a wire does -𝕆D×e𝕪 kinetic work on the electrons in it. That makes them separate more into steps like orbitals, each with a different -𝕆D kinetic probability. That comes from the +𝕆D potential probability as a reaction to the external magnetic field. This is like a potential gradient that does not change over time like impulse in the current.

FIGURE 30.38 Increasing the current through an inductor.



The Potential Difference Across an Inductor

An inductor is not very interesting when the current through it is steady. If the inductor is ideal, with $R = 0\ \Omega$, the potential difference due to a steady current is zero. **Inductors become important circuit elements when currents are changing.** FIGURE 30.38a shows a steady current into the left side of an inductor. The solenoid's magnetic field passes through the coils of the solenoid, establishing a flux.

In FIGURE 30.38b, the current into the solenoid is increasing. This creates an increasing flux to the left. According to Lenz's law, an induced current in the coils will oppose this increase by creating an induced magnetic field pointing to the right. This requires the induced current to be *opposite* the current into the solenoid. This induced current will carry positive charge carriers to the left until a potential difference is established across the solenoid.

You saw a similar situation in Section 30.2. The induced current in a conductor moving through a magnetic field carried positive charge carriers to the top of the wire and established a potential difference across the conductor. The induced current in the

Work changes over distance

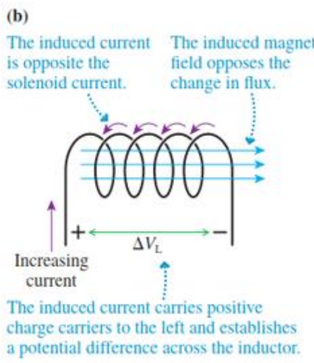
In this model the -𝕆D×e𝕪 kinetic work would be changing over a distance not over time. When the -𝕆D kinetic probability is increasing, this is already time squared. Dividing time squared by time is like seconds^2/second, distances are not in the observation or measurement. As the -𝕆D×e𝕪 kinetic work increases the electrons are more likely to be found in different positions in the wire. That is because the work forms a gradient which moves the electrons, when the current is constant or stopped then the gradient of work disappears.

A gradient of electron density

The gradient of electron density changes according to the strength of the -𝕆D×e𝕪 kinetic work. When the magnetic field is no longer changing, then the electron gradient goes back to being approximately constant. The angle of this kinetic gradient corresponds to the amount of -𝕆D×e𝕪 kinetic work being done, also reacting against this with the amount of +𝕆D×e𝕒 potential work is a potential gradient.

A gradient does not change in time

A change in time is not necessary, the angle is like θ in the -𝕠d and e𝕪 Pythagorean Triangle electrons. When this angle is larger then so is the gradient with a higher -𝕆D kinetic probability or difference.



moving wire was due to magnetic forces on the moving charges. Now, in Figure 30.38b, the induced current is due to the non-Coulomb electric field induced by the changing magnetic field. Nonetheless, the outcome is the same: a potential difference across the conductor.

We can use Faraday's law to find the potential difference. The emf induced in a coil is

$$\mathcal{E}_{\text{coil}} = N\left|\frac{d\Phi_{\text{per turn}}}{dt}\right| = \left|\frac{d\Phi_{\text{m}}}{dt}\right| \qquad (30.34)$$

where $\Phi_{\text{m}} = N\Phi_{\text{per turn}}$ is the total flux through all the coils. The inductance was defined such that $\Phi_{\text{m}} = LI$, so Equation 30.34 becomes

$$\mathcal{E}_{\text{coil}} = L\left|\frac{dI}{dt}\right| \qquad (30.35)$$

The potential gradient

In the diagram the e𝕪/-𝕠d kinetic current is decreasing, and the e𝕒/+𝕠d potential current is increasing as the inverse. The -𝕆D×e𝕪 kinetic work is decreasing as the +𝕆D×e𝕒 potential work increases, it is like two opposing gradients. The electrons are more probably measured on the lower side of the gradient, like falling downhill. They repel each other with a destructive interference, that moves them down the gradient.

Both gradients become horizontal

The potential gradient reacts against this, when the -𝕆D×e𝕪 kinetic work slows the kinetic gradient has a shallower slope. The potential gradient increases so the electrons are increasingly attracted down the potential gradient, with constructive interference. When the current doesn't change for a time with impulse, then both gradients become horizontal.

Gradients from a battery

The wire also has a kinetic and potential gradient when connected to a battery. The kinetic gradient has electrons from the negative terminal, these destructively interfere so they move downhill away from each other. The potential gradient is from the positive terminal, this is higher there so the electrons are increasingly attracted with a constructive interference.

The charge escalator as gradients

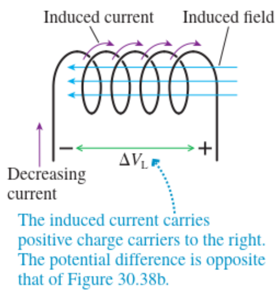
When the battery was charged the charged escalator was a kinetic gradient moving electrons from a source charge to one end of the battery. The potential gradient reacted against this as the positive terminal also became charged.

The induced emf is directly proportional to the *rate of change* of current through the coil. We'll consider the appropriate sign in a moment, but Equation 30.35 gives us the size of the potential difference that is developed across a coil as the current through the coil changes. Note that $\mathcal{E}_{\text{coil}} = 0$ for a steady, unchanging current.

FIGURE 30.39 shows the same inductor, but now the current (still *in* to the left side) is decreasing. To oppose the decrease in flux, the induced current is in the *same* direction as the input current. The induced current carries charge to the right and establishes a potential difference opposite that in Figure 30.38b.

NOTE Notice that the induced current does not oppose the current through the inductor, which is from left to right in both Figures 30.38 and 30.39. Instead, in accordance with Lenz's law, the induced current opposes the *change* in the current in the solenoid. The practical result is that it is hard to change the current through an inductor. Any effort to increase or decrease the current is met with opposition in the form of an opposing induced current. You can think of the current in an inductor as having inertia, trying to continue what it was doing without change.

FIGURE 30.39 Decreasing the current through an inductor.



A potential difference across the inductor

The inductor has a +𝕆D potential gradient against the kinetic current, this gives a +𝕆D potential and -𝕆D kinetic difference along the inductor. That is the same as the potential and kinetic difference or voltage from a battery or capacitor.

The potential gradients of the resistor

In this model a resistor does stronger +𝕆D×e𝕒 potential work, its atoms hold onto electrons more so they cannot form an electron sea like in metals. That reacts more against any kinetic current going through it, they tend to get held around the +𝕆D potential gradient around each atom. If the kinetic gradient in the wire is strong enough, then some electrons move through the resistor.

Slowing the electrons in the resistor

That makes a +𝕆D potential difference on the side of the resistor the current reaches first. The electrons reach the first part of the resistor and encounter the +𝕆D potential gradient, as they move further each atom's potential gradient adds to the electron's kinetic gradient slowing them down.

Slowing against the potential gradient

When the electrons meet this potential gradient, they slow or stop like a water flow meeting a depression. They tend to fall into the depression and then have to climb out again. Some circle the depression randomly so the forward E𝕐/-𝕠d kinetic impulse of the electrons is increasingly lost. This is -IR where the e𝕪/-𝕠d kinetic current is slowed by the +𝕆D×e𝕒 potential work of the resistor's atoms.

Impulse changes the gradient over time

In the inductor the -𝕆D×e𝕪 kinetic work being done by the current is opposed by the +𝕆D×e𝕒 potential work, the electrons are not confined mainly in atoms as with the resistor. Instead, the electron sea in a metal moves more as particles with a E𝕐/-𝕠d kinetic impulse. There would also be a E𝔸/+𝕠d potential impulse where the protons are slowing the current.

A coil slows the impulse

With a coil there is +𝕆D potential more torque on the electrons, the E𝕐/-𝕠d kinetic impulse of the current is slowed. The coil's inductance is then overcome by a larger -𝕆D kinetic difference such as with a stronger battery. The impulse of the current is changing more over time, the coil creates a +𝕆D potential gradient which changes with distance not time. Because these are inverses the coil geometry slows the straight-line geometry of the wire.

An decreasing potential gradient

The inductor has a potential gradient which is largest where the electrons enter the coil. It decreases along the coil as the -𝕆D×e𝕪 kinetic work of the electrons subtracts more and more from the +𝕆D potential gradient of the coil.

An inertial gradient

This is like in Biv space-time where water might be forced horizontally into a pipe coil by the E𝕐/-𝕠d kinetic impulse of a water pump. The coil geometry reacts against this water current with -𝕀D×e𝕧 inertial work, that slows the e𝕪/-𝕠d kinetic velocity of the water by randomizing the water direction. The straight-line current encounters a -𝕀D inertial gradient in the coil, it is at an angle to the straight water pipe entering the coil.

Lenz’s law and inertia

A straight pipe pointed horizontally into a reservoir, with no coil, is like a capacitor. An E𝕐/-𝕠d kinetic impulse forces the water into the reservoir and the E𝕍/-𝕚d inertial impulse reacts against the kinetic current as with Lenz’s law. If the E𝕐/-𝕠d kinetic impulse is slowed the inertia of the water tries to continue along the pipe for a -𝕚d inertial time. If the E𝕐/-𝕠d kinetic impulse of the water increases, the increase in the current is slowed with the reactive E𝕍/-𝕚d inertial impulse.

An equal and opposite reaction

That is an equal and opposite reaction to both the water accelerating and decelerating. Here there is an action/reaction pair with a E𝕐/-𝕠d kinetic impulse and E𝕍/-𝕚d inertial impulse. That is like the action/reaction pair of a E𝕐/-𝕠d kinetic impulse from a negative battery terminal into a capacitor. The positive terminal has a E𝔸/+𝕠d potential impulse making the other capacitor plate positive.

Inductors and capacitors

An inductor replaces the reservoir with a coil, then the action reaction pairs are -𝕆D×e𝕪 kinetic work and -𝕀D×e𝕧 inertial work, -𝕆D×e𝕪 kinetic work and +𝕆D×e𝕒 potential work. An inductor uses work and a capacitor uses impulse.

A resistor as baffles

A resistor is like a horizontal pipe with a series of baffles, these increase the reactive -𝕀D×e𝕧 inertial work the water does because it changes direction more with a -𝕀D inertial torque. In this model work has a force of probability which comes from randomness, the -𝕀D inertial probability here can point in different directions slowing the straight-line direction of the water flow.

An inertial battery

The water coil reacts with a change of the e𝕪/-𝕠d kinetic velocity of the water, whether it is increasing or slowing. If the coil is large, then it would store water pressure in it through inertia like +𝕆D×e𝕒 potential work in an inductor. If the water pressure going into the coil stops this is a change in the E𝕐/-𝕠d kinetic impulse. Then the inertia of the water continues for a -𝕚d inertial time to have it come out the other end as a reactive E𝕍/-𝕚d inertial impulse. That is like an inertial battery where the water pressure is like the charge escalator. The water pressure forms a +𝕆D kinetic gradient and the coil forms a -𝕀D inertial gradient.

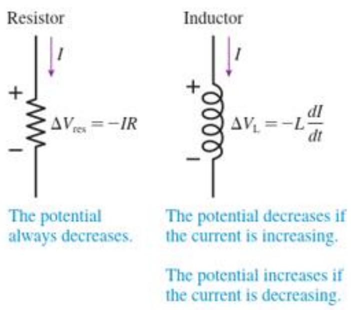
Before we can use inductors in a circuit we need to establish a rule about signs that is consistent with our earlier circuit analysis. **FIGURE 30.40** first shows current I passing through a resistor. You learned in Chapter 28 that the potential difference across a resistor is $\Delta V_{res} = -\Delta V_R = -IR$, where the minus sign indicates that the potential *decreases* in the direction of the current.

We'll use the same convention for an inductor. The potential difference across an inductor, *measured along the direction of the current*, is

$$\Delta V_L = -L\frac{dI}{dt} \qquad (30.36)$$

If the current is increasing ($dI/dt > 0$), the input side of the inductor is more positive than the output side and the potential decreases in the direction of the current ($\Delta V_L < 0$). This was the situation in Figure 30.38b. If the current is decreasing ($dI/dt < 0$), the input side is more negative and the potential increases in the direction of the current ($\Delta V_L > 0$). This was the situation in Figure 30.39.

FIGURE 30.40 The potential difference across a resistor and an inductor.



A spark from impulse

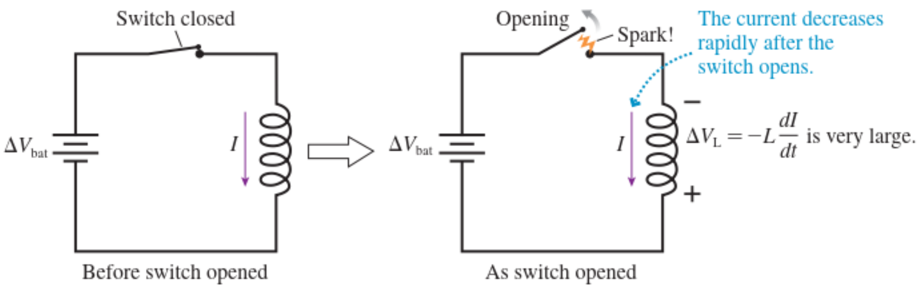
When the switch is opened there is a change in the E𝕐/-𝕠d kinetic impulse over a short -𝕠d kinetic time. That gives a larger E𝕐 kinetic displacement and a spark.

The potential difference across an inductor can be very large if the current changes very abruptly (large dI/dt). **FIGURE 30.41** shows an inductor connected across a battery. There is a large current through the inductor, limited only by the internal resistance of the battery. Suppose the switch is suddenly opened. A very large induced voltage is created across the inductor as the current rapidly drops to zero. This potential difference (plus ΔV_{bat}) appears across the gap of the switch as it is opened. A large potential difference across a small gap often creates a spark.

A spark from power

In this model the spark would come from kinetic power with the E𝕐/-𝕕d kinetic impulse. The change in the circuit over -𝕕d kinetic time also forms a -𝔻D kinetic and +𝔻D potential gradient. That allows the electrons to move across a gap as a wave of probability. If the switch remains open or closed there is no change over time, and so there is no impulse. With no impulse there is no work and no gradient of voltage.

FIGURE 30.41 Creating sparks.



Indeed, this is exactly how the spark plugs in your car work. The car's generator sends a current through the *coil,* which is a big inductor. When a switch is suddenly opened, breaking the current, the induced voltage, typically a few thousand volts, appears across the terminals of the spark plug, creating the spark that ignites the gasoline. Older cars use a *distributor* to open and close an actual switch; more recent cars have *electronic ignition* in which the mechanical switch has been replaced by a transistor.

Power as impulse

Here the current is changing with respect to time, that would be an E𝕐 kinetic displacement with the E𝕐/-𝕕d kinetic impulse. In (30.38) dU/dt would be the E𝔸/+𝕕d potential impulse of the inductor. In (30.37) this would be e𝕪/-𝕕d × Δ V or 1/-𝕕d, that would be F=ma as e𝕪/-𝔻D kinetic work. The inverse of this, with the particle/wave duality, would be the E𝕐/-𝕕d kinetic impulse. That is consistent with the change with respect to time.

Changing with respect to time or distance

In (30.38) both are changing with respect to time, so both of these would be impulse. The circuit is losing a E𝕐/-𝕕d kinetic impulse as the capacitor discharges, that is being transferred to the inductor with a E𝔸/+𝕕d potential impulse. Alternatively, the capacitor is discharging as the ΔV voltage decreases, that would be e𝕪/-𝕕d with a change as ΔV. In both case there are active forces from the capacitor discharging, then reactive forces in the inductor.

Rate of change of energy

In this model energy can change with respect to time or distance, when it change with respect to time this is impulse. An electromagnet might be run with ½×e𝕐/-𝔻d ×-𝕕d linear kinetic energy, at a greater distance the magnet can exert less kinetic energy on another magnet for example.

When the second magnet is closer it might be attracted or repelled more than if it was further away.

The inverse square law of time or distance

In this model the inverse square law is usually over a distance, for example +𝕀D×e𝕙 gravitational work decreasing as a square with a e𝕙 height above a planet. A consequence of this is with respect to time, for example a projectile when fired might fall with an increasing acceleration with respect to time. As each additional second elapses the projectile falls further than the one before. That is the inverse of meters/second^2 as meters^2/second.

Energy in Inductors and Magnetic Fields

Recall that electric power is $P_{elec} = I\Delta V$. As current passes through an inductor, for which $\Delta V_L = -L(dI/dt)$, the electric power is

$$P_{elec} = I\Delta V_L = -LI\frac{dI}{dt} \qquad (30.37)$$

P_{elec} is negative because a circuit with an increasing current is *losing* electric energy. That energy is being transferred to the inductor, which is *storing* energy U_L at the rate

$$\frac{dU_L}{dt} = +LI\frac{dI}{dt} \qquad (30.38)$$

Integrating a velocity

From this E𝕐/-𝕠d kinetic impulse and E𝔸/+𝕠d potential impulse (30.39) integrates with respect to the current as e𝕪/-𝕠d, that is like integrating with respect to e𝕧/-𝕚d velocity to go from an E𝕍/-𝕚d inertial impulse to the ½×e𝕍/-𝕀d ×-𝕚d linear inertia. This is in the typical format for kinetic energy in conventional physics. In this model an integration can only be done on the spin Pythagorean Triangle side, not on both sides.

where we've noted that power is the rate of change of energy.

We can find the total energy stored in an inductor by integrating Equation 30.38 from $I = 0$, where $U_L = 0$, to a final current I. Doing so gives

$$U_L = L\int_0^I I\,dI = \tfrac{1}{2}LI^2 \qquad (30.39)$$

The 1/2 factor in energy

Integrating in this way gives the ½ factor in energy, it is also referred to as the average between an initial and final velocity. For example when a -𝕚d inertial mass has a negligible e𝕪/-𝕚d inertial velocity, then moving to a larger inertial velocity will have their average half way between them.

A capacitor and energy

Here in electric fields a capacitor stores energy, in this model it stores the E𝕐/-𝕠d kinetic impulse and E𝔸/+𝕠d potential impulse which is part of energy. E^2 here would be E𝕐 and E𝔸, ϵ is the squared force of impulse.

An inductor and energy

In magnetic fields the inductor also stores energy, part of that equation is -𝕆D×e𝕪 kinetic work and +𝕆D×e𝕒 potential work. The squared force here is µ. Because the fractions e𝕪/-𝕠d and e𝕒/+𝕠d are allowed to be used for integration here, that gives a different answer in calculus to this model.

The potential in the inductor

In this model the +𝕆D×e𝕒 potential work gives the induction force from the protons in the wire. In (30.40) U_L is the ½×+e𝔸/+𝕆d ×+𝕠d rotational potential energy. The magnetic squared force is μ, N^2 is the number of windings in the coil and A is another square as an area. The windings can be added up into a single turn as -𝕠d, then as an area that is squared to give -𝕆D as the kinetic torque. In (30.41) B^2 becomes the magnetic force as -𝕆D.

Energy in electric and magnetic fields

Electric fields	Magnetic fields
A capacitor stores energy $U_C = \frac{1}{2}C(\Delta V)^2$	An inductor stores energy $U_L = \frac{1}{2}LI^2$
Energy density in the field is $u_E = \frac{\epsilon_0}{2}E^2$	Energy density in the field is $u_B = \frac{1}{2\mu_0}B^2$

The potential energy stored in an inductor depends on the square of the current through it. Notice the analogy with the energy $U_C = \frac{1}{2}C(\Delta V)^2$ stored in a capacitor.

In working with circuits we say that the energy is "stored in the inductor." Strictly speaking, the energy is stored in the inductor's magnetic field, analogous to how a capacitor stores energy in the electric field. We can use the inductance of a solenoid, Equation 30.33, to relate the inductor's energy to the magnetic field strength:

$$U_L = \frac{1}{2}LI^2 = \frac{\mu_0 N^2 A}{2l}I^2 = \frac{1}{2\mu_0}Al\left(\frac{\mu_0 NI}{l}\right)^2 \tag{30.40}$$

We made the last rearrangement in Equation 30.40 because $\mu_0 NI/l$ is the magnetic field inside the solenoid. Thus

$$U_L = \frac{1}{2\mu_0}AlB^2 \tag{30.41}$$

No volume for a magnetic field

In this model the volume would not be used, l gives the length of the wire which is also in -𝕆D×e𝕪 kinetic work.

But Al is the volume inside the solenoid. Dividing by Al, the magnetic field *energy density* inside the solenoid (energy per m^3) is

$$u_B = \frac{1}{2\mu_0}B^2 \tag{30.42}$$

We've derived this expression for energy density based on the properties of a solenoid, but it turns out to be the correct expression for the energy density anywhere there's a magnetic field. Compare this to the energy density of an electric field $u_E = \frac{1}{2}\epsilon_0 E^2$ that we found in Chapter 26.

Oscillation and quantization

In this model the LC circuit can oscillate with a quantized value with the inductor. The capacitor stores a E𝕐/-𝕠d kinetic impulse and E𝔸/+𝕠d potential impulse, when the switch closes at a -𝕠d kinetic and +𝕠d potential time this is making an observation of impulse. The inductor is making a measurement of the -𝕆D and +𝕆D kinetic and potential torque in it as a gradient not over time.

An oscillation dissipates with impulse

In this model an oscillation dissipates with impulse, quantization implies a normal curve with a center. It does not spread out because of the quantization, this is like the e𝕪×-𝕘d photon which cannot spread out as a wave then disappear. The oscillation of the LC circuit dissipates with an exponential decay curve from chaotic impulse.

Quantized photon exchanges

An atom would usually be stable with the -𝕆D×e𝕪 kinetic work of its electrons, a radioactive atom is where it becomes too large and some iotas become like particles. This dissipates away from the stable quantization over time with a E𝔸/+𝕠d potential impulse and E𝕐/-𝕠d kinetic impulse. When atoms are stable with quantization they can exchange e𝕪×-𝕘d photons between them, this does not dissipate the -𝔾D×e𝕪 light work being done.

Electrons dissipated chaotically

When electrons leave the atom they are dissipated chaotically, then e𝕪/-𝕘d photons can collide with them. The photons can dissipate their e𝕪/-𝕘d light velocity in these collisions while maintaining the same e𝕧/-𝕚d inertial velocity as c.

Collapsing a wave function

When a wave function is collapsed into an observation there is impulse observed over time. As this time passes there is still impulse, the particle is observed to spread out increasingly like a wave. This dissipation occurs because of the particle like state, there is no quantization and so the normal curve wave shape can become spread out. If the electron was in an atom its position would be describable by a normal curve.

Friction and impulse

When there is potential friction in a resistor, this occurs through the E𝔸/+𝕠d potential impulse of the electrons moving as particles through it. There is a dissipation of energy through chaotic motion, when the wire is a conductor the -𝕆D×e𝕪 kinetic work and +𝕆D×e𝕒 potential work dominate. Then the electrons move more like a laminar flow that changes with a distance not over time.

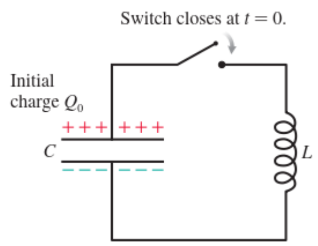
30.9 *LC* Circuits

Telecommunication—radios, televisions, cell phones—is based on electromagnetic signals that *oscillate* at a well-defined frequency. These oscillations are generated and detected by a simple circuit consisting of an inductor and a capacitor. This is called an ***LC* circuit.** In this section we will learn why an *LC* circuit oscillates and determine the oscillation frequency.

FIGURE 30.42 shows a capacitor with initial charge Q_0, an inductor, and a switch. The switch has been open for a long time, so there is no current in the circuit. Then, at $t = 0$, the switch is closed. How does the circuit respond? Let's think it through qualitatively before getting into the mathematics.

As FIGURE 30.43 shows, the inductor provides a conducting path for discharging the capacitor. However, the discharge current has to pass through the inductor, and, as we've seen, an inductor resists changes in current. Consequently, the current doesn't stop when the capacitor charge reaches zero.

FIGURE 30.42 An *LC* circuit.



The spring motion and impulse

In the diagrams the spring moves back and forth with a E𝕐/-𝕠d kinetic impulse and a E𝔸/+𝕠d potential impulse. The E𝔸/+𝕠d potential impulse comes pulling electrons further apart to a greater e𝕒 altitude in the atoms. That would be close to an oscillation here, but being quantized it would be chaotic.

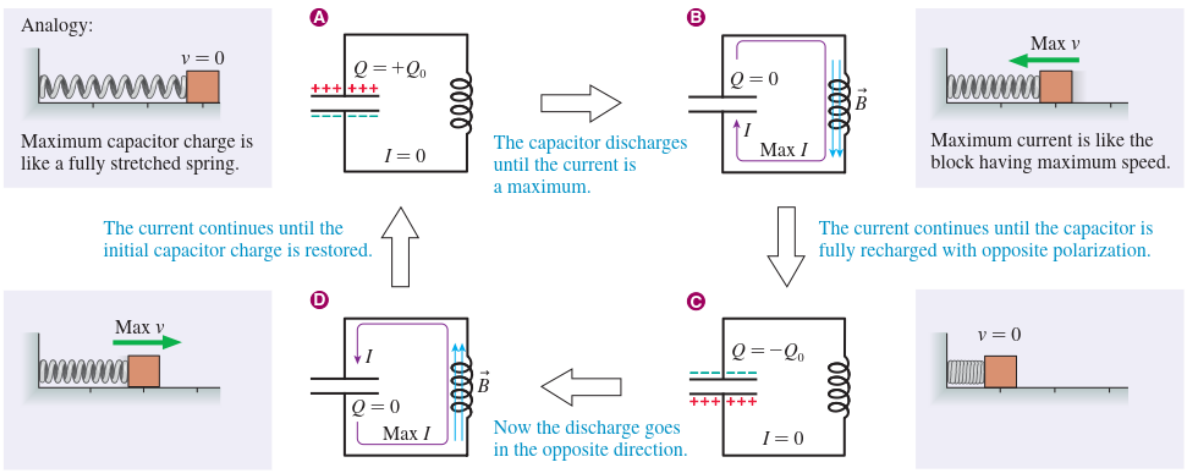
The spring motion and work

The inductor does -𝕆D×e𝕪 kinetic work and +𝕆D×e𝕒 potential work like the coil shape of the spring. There the electrons are twisted with a -𝕆D kinetic torque from their normal positions. Because these are quantized there are e𝕪×-𝕘d photons emitted and absorbed so the spring moves back to a normalized position.

The spring motion with work and impulse in Biv space-time

This is like in Biv space-time where the spring can hang vertically. Then it is pulled downwards with a Eℍ/+𝕚d gravitational impulse, reacting against this is the E𝕍/-𝕚d inertial impulse. The spring winds and unwinds with a +𝕀D gravitational and -𝕀D inertial torque in +𝕀D×e𝕙 gravitational work and -𝕀D×e𝕧 inertial work.

FIGURE 30.43 The capacitor charge oscillates much like a block attached to a spring.



The potential and inertia

In this model the inertia of the spring is reactive, in Roy electromagnetism the E𝔸/+𝕠d potential impulse and +𝕆D×e𝕒 potential work is reactive like inertia.

The kinetic and potential voltage

Here the two voltages are ΔV_C as -𝕠d where the E𝕐/-𝕠d kinetic impulse changes with -𝕠d kinetic time. The ΔV_L voltage is +𝕠d as the inductor reacts with a E𝔸/+𝕠d potential impulse. In (30.43) the two terms are equal to each other as -𝕠d+𝕠d=0 because the two impulses are equal and opposite each other.

> A block attached to a stretched spring is a useful mechanical analogy. Closing the switch to discharge the capacitor is like releasing the block. The block doesn't stop when it reaches the origin; its inertia keeps it going until the spring is fully compressed. Likewise, the current continues until it has recharged the capacitor with the opposite polarization. This process repeats over and over, charging the capacitor first one way, then the other. That is, the charge and current *oscillate*.
>
> The goal of our circuit analysis will be to find expressions showing how the capacitor charge Q and the inductor current I change with time. As always, our starting point for circuit analysis is Kirchhoff's voltage law, which says that all the potential differences around a closed loop must sum to zero. Choosing a cw direction for I, Kirchhoff's law is

$$\Delta V_C + \Delta V_L = 0 \qquad (30.43)$$

The inductor and impulse

The inductor in (30.44) changes with respect to +𝕠d potential time as impulse. In this model the force would be mainly -𝕆D×e𝕪 kinetic work and +𝕆D×e𝕒 potential work because of torque, as with a spring there is also some impulse. The charge of the capacitor is Q/C which changes with respect to -𝕠d kinetic time because it is charged with active electrons.

> The potential difference across a capacitor is $\Delta V_C = Q/C$, and we found the potential difference across an inductor in Equation 30.36. Using these, Kirchhoff's law becomes

$$\frac{Q}{C} - L\frac{dI}{dt} = 0 \qquad (30.44)$$

Work not impulse

In equation (30.45) Q is -𝕠d×e𝕪/-𝕠d kinetic momentum in kinetic Coulombs, the e𝕪 kinetic electric charge is changing with respect to -𝕠d kinetic time in a e𝕪/-𝕠d kinetic current. In this model e𝕪 is an infinitesimal as a square root, -𝕠d is an instant of time. That gives e𝕪/-𝕆D kinetic work here, it would be changing with respect to distance not time.

Equation 30.44 has two unknowns, Q and I. We can eliminate one of the unknowns by finding another relation between Q and I. Current is the rate at which charge moves, $I = dq/dt$, but the charge flowing through the inductor is charge that was *removed* from the capacitor. That is, an infinitesimal charge dq flows through the inductor when the capacitor charge changes by $dQ = -dq$. Thus the current through the inductor is related to the charge on the capacitor by

$$I = -\frac{dQ}{dt} \qquad (30.45)$$

A second derivative and work

When the e𝕪/-𝕠d kinetic current is changing with respect to -𝕠d kinetic time, that would be the E𝕐/-𝕠d kinetic impulse. As written below it would be e𝕪/-𝕠d × 1/-𝕠d as e𝕪/-𝕆D kinetic work, this is the inverse of the E𝕐/-𝕠d kinetic impulse with the particle/wave duality. When the second derivative is taken with respect to time that gives work in this model, except it comes from a second integral.

A convention in calculus

The -𝕠d and e𝕪 Pythagorean Triangle as an electron can have a first derivative always with respect to e𝕪 to give e𝕪/-𝕠d, then a second derivative with respect to e𝕪 gives E𝕐/-𝕠d. In this model the numerator is changed in differentiation because the convention is that the straight Pythagorean Triangle side is in the numerator and the spin Pythagorean Triangle side is in the denominator. In conventional physics it is for example e𝕧/-𝕚d as meters/second, with -𝕚d/e𝕧 as seconds/meter then the derivative can act on the denominator.

The kinetic and potential work as inverses

In (30.47) Q/C is ΔV or +𝕠d, the second term is the e𝕒/+𝕆D potential work from the inductor. Q here is the -𝕠d×e𝕪/-𝕠d kinetic momentum in Coulombs, this can vary according to the charge of the capacitor which is e𝕪. That makes it the -𝕠d/E𝕐 kinetic impulse because C is the capacitor charge not a voltage. The -𝕠d/E𝕐 kinetic impulse would remain on the left-hand side, the e𝕒/+𝕆D potential work can be moved to the right-hand side. They are proportional to each other where -𝕠d ∝ e𝕒 and 1/E𝕐 ∝ 1/-𝕆D.

Now I is positive when Q is decreasing, as we would expect. This is a subtle but important step in the reasoning.

Equations 30.44 and 30.45 are two equations in two unknowns. To solve them, we'll first take the time derivative of Equation 30.45:

$$\frac{dI}{dt} = \frac{d}{dt}\left(-\frac{dQ}{dt}\right) = -\frac{d^2Q}{dt^2} \qquad (30.46)$$

We can substitute this result into Equation 30.44:

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0 \qquad (30.47)$$

Inertial work and the kinetic impulse

In (30.48) there is e𝕒/+𝕆D potential work on the left-hand side, that gives the variation of the -𝕠d×e𝕪/-𝕠d kinetic momentum as it oscillates. In (30.49) the left-hand side would be e𝕧/-𝕀D inertial work as a reactive force, that is equal to the right-hand side which would be impulse. That is because with the -𝕠d/E𝕐 kinetic impulse e𝕪 in the numerator on the left is proportional to -𝕠d on the right, 1/-𝕆D on the left is proportional to 1/E𝕐 on the right. The spring moves with a combination of an active -𝕠d/E𝕐 kinetic impulse and a reactive e𝕧/-𝕀D inertial work.

Inverting work and impulse

In (30.49) the left-hand side is changing with respect to time, on the right-hand side as a distance x. This would mean the left-hand side is impulse and the right-hand side is work, the two terms become their inverses. On the left-hand side then the e𝕒/+𝕆D potential work of the spring can be written as the inverse +𝕠d/E𝔸 potential impulse, the right-hand side changes from the -𝕠d/E𝕐 kinetic impulse to the e𝕪/-𝕆D kinetic work as an inverse. The equation remains equal, the left-hand side is impulse and the right-hand side is work.

Now we have an equation for the capacitor charge Q.

Equation 30.47 is a second-order differential equation for Q. Fortunately, it is an equation we've seen before and already know how to solve. To see this, we rewrite Equation 30.47 as

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q \qquad (30.48)$$

Recall, from Chapter 15, that the equation of motion for an undamped mass on a spring is

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad (30.49)$$

Equation 30.48 is *exactly the same equation*, with x replaced by Q and k/m replaced by $1/LC$. This should be no surprise because we've already seen that a mass on a spring is a mechanical analog of the LC circuit.

A sine wave from work

In the diagrams there is a sine wave from the +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work. In this model that comes from sinθ as +𝕠d or -𝕠d over the hypotenuse of their respective Pythagorean Triangles. The cosine is not a wave here, it is like a piston moving back and forth with impulse as the work oscillates.

Impulse and displacement

The force of this piston is an E𝔸 potential and E𝕐 kinetic displacement, it is chaotic and tends to dissipate or break up the quantized oscillation of the work. In (30.51) this can be regarded as the frequency 1/+𝕠d and 1/-𝕠d. These spin Pythagorean Triangle sides come from the inductance L, inverse to this is the capacitance C as e𝕒 and e𝕪.

Torque in a wave and frequency

The force of this inductance would be 1/+𝕆D and 1/-𝕆D so the square root is taken to give the potential and kinetic time. The square root of this torque is the frequency, the torque increases and decreases as a squared force to give the sine wave.

Momentum with respect to time

The change of the charge Q as Coulombs with respect to time comes from the +𝕠d×e𝕒/+𝕠d potential momentum and -𝕠d×e𝕪/-𝕠d kinetic momentum, here the derivative is taken with respect to the spin Pythagorean Triangle side to give +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work. The +𝕠d potential mass and the -𝕠d kinetic mass in momentum is not used here, that comes from an integral not a derivative.

We know the solution to Equation 30.49. It is simple harmonic motion $x(t) = x_0 \cos \omega t$ with angular frequency $\omega = \sqrt{k/m}$. Thus the solution to Equation 30.48 must be

$$Q(t) = Q_0 \cos \omega t \qquad (30.50)$$

where Q_0 is the initial charge, at $t = 0$, and the angular frequency is

$$\omega = \sqrt{\frac{1}{LC}} \qquad (30.51)$$

The charge on the upper plate of the capacitor oscillates back and forth between $+Q_0$ and $-Q_0$ (the opposite polarization) with period $T = 2\pi/\omega$.

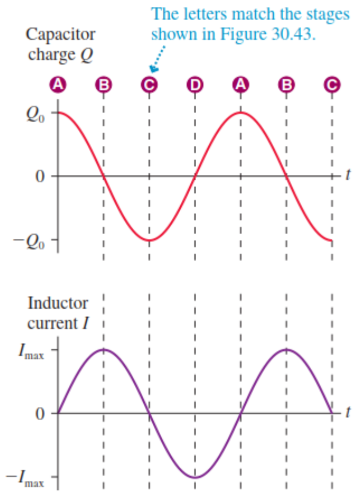
As the capacitor charge oscillates, so does the current through the inductor. Using Equation 30.45 gives the current through the inductor:

$$I = -\frac{dQ}{dt} = \omega Q_0 \sin \omega t = I_{max} \sin \omega t \qquad (30.52)$$

where $I_{max} = \omega Q_0$ is the maximum current.

An LC circuit is an *electric oscillator*, oscillating at frequency $f = \omega/2\pi$. **FIGURE 30.44** shows graphs of the capacitor charge Q and the inductor current I as functions of time. Notice that Q and I are 90° out of phase. The current is zero when the capacitor is fully charged, as expected, and the charge is zero when the current is maximum.

FIGURE 30.44 The oscillations of an LC circuit.



Oscillation frequency

Here the oscillation frequency would come from the E𝕐/-𝕠d kinetic impulse and E𝔸/+𝕠d potential impulse. This comes from the displacement of the spring, there is the spring constant k which gives the strength of the squared force as the spring oscillates. Because the square is approximately constant, that makes the frequency in the observation of the impulse also approximately constant.

A resonance

In this model a resonance comes from the +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work, this can transmit the oscillation to a second spring connected to the same surface. That synchronization comes from a constructive interference between them.

Closing the switch

When a switch has been closed for a long time, then -𝕠d would be larger, that makes the -𝕆D×e𝕪 kinetic work in the circuit stronger than the E𝕐/-𝕠d kinetic impulse. In the small amount of -𝕠d kinetic time the switch closes, there is a stronger E𝕐/-𝕠d kinetic impulse.

The inductor and potential work

The battery does -𝕆D×e𝕪 kinetic work along the circuit, the resistor reacts against this with +𝕆D×e𝕒 potential work. The inductor can act like a battery as it does +𝕆D×e𝕒 potential work against the flow of current like the positive terminal of the battery.

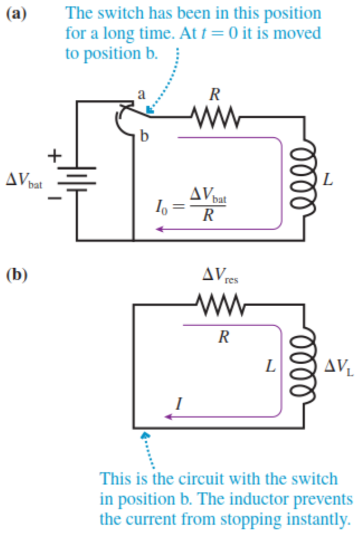
An LC circuit, like a mass on a spring, wants to respond only at its natural oscillation frequency $\omega = 1/\sqrt{LC}$. In Chapter 15 we defined a strong response at the natural frequency as a *resonance*, and resonance is the basis for all telecommunications. The input circuit in radios, televisions, and cell phones is an LC circuit driven by the signal picked up by the antenna. This signal is the superposition of hundreds of sinusoidal waves at different frequencies, one from each transmitter in the area, but the circuit responds only to the *one* signal that matches the circuit's natural frequency. That particular signal generates a large-amplitude current that can be further amplified and decoded to become the output that you hear.

30.10 *LR* Circuits

A circuit consisting of an inductor, a resistor, and (perhaps) a battery is called an ***LR* circuit.** FIGURE 30.45a is an example of an LR circuit. We'll assume that the switch has been in position a for such a long time that the current is steady and unchanging. There's no potential difference across the inductor, because $dI/dt = 0$, so it simply acts like a piece of wire. The current flowing around the circuit is determined entirely by the battery and the resistor: $I_0 = \Delta V_{bat}/R$.

What happens if, at $t = 0$, the switch is suddenly moved to position b? With the battery no longer in the circuit, you might expect the current to stop immediately. But the inductor won't let that happen. The current will continue for some period of time as the inductor's magnetic field drops to zero. In essence, the energy stored in the inductor allows it to act like a battery for a short period of time. Our goal is to determine how the current decays after the switch is moved.

FIGURE 30.45 An LR circuit.



A resistor making turbulence

Here the voltages are +𝕠d for the resistor and +𝕠d for the inductor, they react against each other and the original e𝕪/-𝕠d kinetic current. The resistor has a E𝔸/+𝕠d potential impulse as it directs the electrons in chaotic directions. That breaks up the laminar flow of the e𝕪/-𝕠d kinetic current with turbulence and a drag like friction.

Resisting the kinetic impulse

This would be the inverse of the inductor's e𝕒/+𝕆D potential work. RI would be the -𝕠d/E𝕐 kinetic impulse of the current, that is proportional to the gives e𝕒/+𝕆D ∝ -𝕠d/E𝕐. Here the e𝕒 altitude above a proton is proportional to the -𝕠d kinetic magnetic field of the electron, 1/+𝕆D as the potential probability is proportional to 1/E𝕐 as the kinetic displacement.

LR and LC circuits

The resistance here would be observed as the changes in the -𝕠d/E𝕐 kinetic impulse, this resistance can also be regarded as the reactions of the +𝕠d/E𝔸 potential impulse. Combining these two gives an LR circuit. The LC circuit had the E𝕐/-𝕠d kinetic impulse of the capacitor as the same force, here there is no resistance. Reacting against this is the same e𝕒/+𝕆D potential work as in the LR circuit. This similarity allows for two kinds of circuit.

NOTE It's important not to open switches in inductor circuits because they'll spark, as Figure 30.41 showed. The unusual switch in Figure 30.45 is designed to make the new contact just before breaking the old one.

FIGURE 30.45b shows the circuit after the switch is changed. Our starting point, once again, is Kirchhoff's voltage law. The potential differences around a closed loop must sum to zero. For this circuit, Kirchhoff's law is

$$\Delta V_{res} + \Delta V_L = 0 \qquad (30.53)$$

The potential differences in the direction of the current are $\Delta V_{res} = -IR$ for the resistor and $\Delta V_L = -L(dI/dt)$ for the inductor. Substituting these into Equation 30.53 gives

$$-RI - L\frac{dI}{dt} = 0 \qquad (30.54)$$

Logarithmic decay in work and impulse

In (30.57) the logarithm is a decay in the current, that comes from the E𝕐/-𝕠d kinetic impulse reducing in the resistor. The +𝕆D×e𝕒 potential work of the inductor also decreases exponentially as it is directly proportional to the impulse. The change in the current I is with respect to time here which can be +𝕆D×e𝕒 potential work, it can also be regarded as the E𝕐/-𝕠d kinetic impulse with respect to time.

Exponential decay and inverse exponential decay

In this model an exponential comes from impulse as a squared exponent such as $e^{E\mathbb{Y}}$ where E𝕐 is a real number because e𝕪 is a square root. The normal curve comes from a negative squared exponent here as $e^{+\mathbb{O}D}$, because these are proportional to each other they both decay logarithmically. As the +𝕆D×e𝕒 potential work decays this is a probability function, so the electrons are less likely to be changed by the inductor over this gradient.

Quantum tunneling and logarithmic decay

This proportion is also found in quantum tunneling which decreases exponentially as the e𝕧 length increases with a barrier's thickness linearly. The -𝕀D inertial probability of an electron being found at the other side of the barrier decreases as a square. When observed on the other side as a particle with a E𝕐/-𝕠d kinetic impulse, then this is an exponential decay proportional to the probabilistic decay of the normal curve.

L/R as a constant decreases logarithmically

L as +𝕆D×e𝕒 potential work is divided by the E𝕐/-𝕠d kinetic impulse as R, each decreases logarithmically maintaining the same proportional ratio. The E𝕐/-𝕠d kinetic impulse has E𝕐 decreasing as a square while the -𝕠d kinetic time increases linearly. Reacting against this, +𝕆D×e𝕒 potential work has +𝕆D as the potential torque or probability in the coil decreasing as a square as the e𝕒 altitude increases with a potential gradient.

A time and distance constant

As the E𝕐/-𝕠d kinetic impulse decreases exponentially, the protons in the coil lower their e𝕒 altitude at which they can have this +𝕆D potential torque or probability affect the electrons in the circuit. The gradient then decreases exponentially while remaining proportional to the time constant of the E𝕐/-𝕠d kinetic impulse decreasing. There would then be a distance constant as the induction gradient shrinks.

An uncertain proportion

The proportion is approximately constant with uncertainty, the time constant with the E𝕐/-𝕠d kinetic impulse changes chaotically. The distance constant with +𝕆D×e𝕒 potential work changes probabilistically. The chaos comes from β which is ≈ √(2π), the probability comes from the normal curve where the exponent values are divided by √(2π) to give an integral area of 1 under the normal curve.

$$f\left(x,\mu,\sigma^2\right) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

We're going to need to integrate to find the current I as a function of time. Before doing so, we rearrange Equation 30.54 to get all the current terms on one side of the equation and all the time terms on the other:

$$\frac{dI}{I} = -\frac{R}{L}dt = -\frac{dt}{(L/R)} \qquad (30.55)$$

We know that the current at $t = 0$, when the switch was moved, was I_0. We want to integrate from these starting conditions to current I at the unspecified time t. That is,

$$\int_{I_0}^{I} \frac{dI}{I} = -\frac{1}{(L/R)}\int_0^t dt \qquad (30.56)$$

Changing the Euler equation

In (30.58) the E𝕐/-𝕠d kinetic impulse is decreasing, that would be $e^{E𝕐\text{-}𝕠d}$. That can be regarded as a Pythagorean Triangle with the right angle at the origin not as in the Euler equation. The math would be the same, but it is drawn differently.

A hyperbolic Euler equation

Also the hypotenuse is no longer constant as the radius of the Euler circle, instead the -𝕠d and e𝕪 Pythagorean Triangle here is tangent to a hyperbola that is decreasing logarithmically. That relates to the area under a hyperbola giving logarithms, for example where e𝕪 is 1 then a vertical line up to the hyperbola gives the area e under it.

A circular Euler equation

+𝕆D×e𝕒 potential work is in circular geometry, the +𝕆D potential torque is in the coil. That would give $e^{e𝕒+𝕆D}$, the hypotenuse is no longer constant so the +𝕠d and e𝕒 Pythagorean Triangle remains constant. The circle would be drawn where e𝕒 is the radius not the hypotenuse, at right angles to this at the origin or a tangent to the circle is +𝕠d as the potential spin. This is squared as an integral area like a force in +𝕆D×e𝕒 potential work.

The angles remain the same, the hypotenuse changes

In each case the Euler equation remains the same, the main aspect is the angle θ in the -𝕠d and e𝕪 Pythagorean Triangle and +𝕠d and e𝕒 Pythagorean Triangle. When isinθ and cosθ are used in the Euler equation the angle remains the same if the hypotenuse is no longer constant. The Pythagorean Triangle area instead is the constant in constant area trigonometry.

Both are common integrals, giving

$$\ln I\Big|_{I_0}^{I} = \ln I - \ln I_0 = \ln\left(\frac{I}{I_0}\right) = -\frac{t}{(L/R)} \qquad (30.57)$$

We can solve for the current I by taking the exponential of both sides, then multiplying by I_0. Doing so gives I, the current as a function of time:

$$I = I_0 e^{-t/(L/R)} \qquad (30.58)$$

Notice that $I = I_0$ at $t = 0$, as expected.

The argument of the exponential function must be dimensionless, so L/R must have dimensions of time. If we define the **time constant** τ of the LR circuit to be

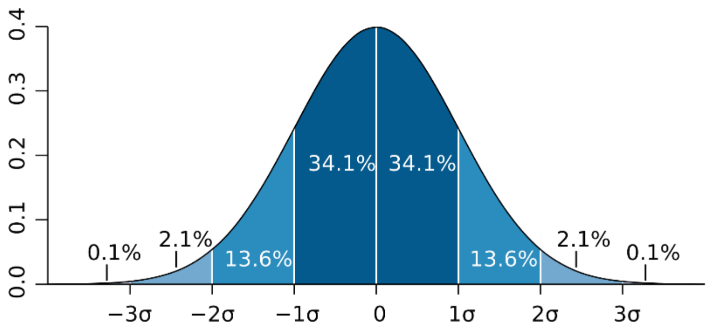
$$\tau = \frac{L}{R} \tag{30.59}$$

then we can write Equation 30.58 as

$$I = I_0 e^{-t/\tau} \tag{30.60}$$

A standard deviation in an inverse exponential

Here the time constant is where the current decreases to e^{-1}, with +𝕆D×e𝕒 potential work this can be where it decreases to the same fraction under the normal curve or as a standard deviation of 34.1% being used for both.



Exponentials and squares

The second standard deviation is 13.6% which is similar to e^{-2} here. Instead of the impulse exponent being a cube here it would be 6 because e𝕪 is a square root, it can then increase as a square exponentially.

The time constant is the time at which the current has decreased to e^{-1} (about 37%) of its initial value. We can see this by computing the current at the time $t = \tau$:

$$I(\text{at } t = \tau) = I_0 e^{-\tau/\tau} = e^{-1} I_0 = 0.37 I_0 \tag{30.61}$$

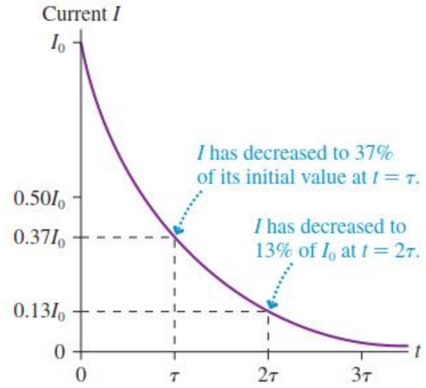
Thus the time constant for an LR circuit functions in exactly the same way as the time constant for the RC circuit we analyzed in Chapter 29. At time $t = 2\tau$, the current has decreased to $e^{-2} I_0$, or about 13% of its initial value.

The current is graphed in **FIGURE 30.46**. You can see that the current decays exponentially. The *shape* of the graph is always the same, regardless of the specific value of the time constant τ.

Integral area under the exponential curve.

Here the integral area under the exponential curve is related to the integral area under the inverse exponential as the normal curve, they would converge more as the times and distances increase. The difference between the two is another constant in this model, like Γ as the difference between the inverted integers and the hyperbolic area.

FIGURE 30.46 The current decay in an *LR* circuit.



α and e

In this model $e^{-𝕠𝕕}$ is e^{-1} as ≈ 1/tanα, that gives ≈ the quantized levels of +𝕆D×e𝕒 potential work. With an exponential decay as e^{-1} that also gives the E𝕐/-𝕠𝕕 kinetic impulse which is proportional to it. Here e^{-1} as the exponent increases as a square gives the normal curve and work. α is proportional to δ as the first Feigenbaum number with cascades.

Electromagnetic Fields and Waves

Work is measured over a distance

In this model Britney is moving with a e𝕧/-𝕠𝕕 inertial velocity, Alec measures her doing -𝕀D×e𝕧 inertial work. Her positive charge is doing +𝕆D×e𝕒 potential work and so the +𝕆D potential probability is the squared +𝕠𝕕 potential magnetic field. According to Britney she is at rest, so she cannot measure +𝕆D×e𝕒 potential work from her charge, there is no +𝕠𝕕 potential magnetic field for her.

An external magnetic field is measured over a distance

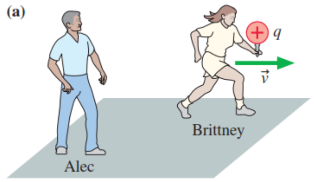
When Alec creates a magnetic field, such as with a bar magnet doing -𝕆D×e𝕪 kinetic work, then the positive charge is moving through it over a distance. There is -𝕆D×e𝕪 kinetic work done on the charge so depending on the direction it would be rotated. For Britney the -𝕆D×e𝕪 kinetic work is still not changing over a distance in her inertial and kinetic reference frames. She would not measure -𝕆D×e𝕪 kinetic work being done on her charge.

31.1 *E* or *B*? It Depends on Your Perspective

Our story thus far has been that charges create electric fields and that moving charges, or currents, create magnetic fields. But consider FIGURE 31.1a, where Brittney, carrying charge q, runs past Alec with velocity $\vec{v}$. Alec sees a moving charge, and he knows that this charge creates a magnetic field. But from Brittney's perspective, the charge is at rest. Stationary charges don't create magnetic fields, so Brittney claims that the magnetic field is zero. Is there, or is there not, a magnetic field?

Or what about the situation in FIGURE 31.1b? Now Brittney is carrying the charge through a magnetic field that Alec has created. Alec sees a charge moving in a magnetic field, so he knows there's a force $\vec{F} = q\vec{v} \times \vec{B}$ on the charge. But for Brittney the charge is still at rest. Stationary charges don't experience magnetic forces, so Brittney claims that $\vec{F} = \vec{0}$.

FIGURE 31.1 Brittney carries a charge past Alec.



Charge q moves with velocity $\vec{v}$ relative to Alec.

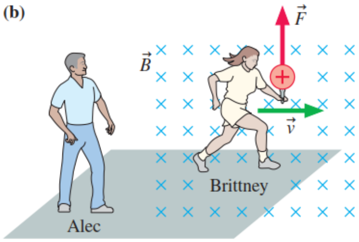
Vectors and spin

In this model $\vec{E}$ would be e𝕪 as the kinetic electric charge, $\vec{B}$ would be -𝕠𝕕 as the kinetic magnetic field. In conventional physics these two can become confused with each other. With this model the e𝕪 $\vec{E}$ is a straight-line vector only, -𝕠𝕕 as $\vec{B}$ is not a vector. It is spin, that can be a clock turning in this case to observe kinetic time or squared as a magnetic field with a kinetic torque. As $\vec{B}$ is not an actual vector then impulse is only observed over time with a rotation on a clock gauge. Here the difference in the reference frame is a change in distance which is work.

Now, we may be a bit uncertain about magnetic fields, but surely there can be no disagreement over forces. After all, forces cause observable and measurable effects, so Alec and Brittney should be able to agree on whether or not the charge experiences a force. Further, if Brittney runs with constant velocity, then both Alec and Brittney are in *inertial reference frames*. You learned in Chapter 4 that these are the reference frames in which Newton's laws are valid, so we can't say that there's anything abnormal or unusual about Alec's and Brittney's observations.

This paradox has arisen because magnetic fields and forces depend on velocity, but we haven't looked at the issue of velocity *with respect to what* or velocity *as measured by whom*. The resolution of this paradox will lead us to the conclusion that $\vec{E}$ and $\vec{B}$ are not, as we've been assuming, separate and independent entities. They are closely intertwined.

(b)



Charge q moves through a magnetic field established by Alec.

Brittney's reference frame

Brittney's potential reference frame would have an e𝕒 altitude and +𝕠d potential time, the positive charge can be moved by attracting it with an -𝕠d and e𝕪 Pythagorean Triangle kinetic reference frame. It can also have the positive charge move with a repulsion by other protons. Here Brittney may have been moved with the active force in a e𝕪 and -𝕠d reference frame, that would attract her positive charge. She could also be moving downhill with gravity, then there would be a gravitational reference frame of e𝕙 and +𝕚d. If her positive charge was repelled by another proton, that would also have an e𝕒 altitude and +𝕚d potential time reference frame.

Alec's reference frame

Alec could also be observed and measured from all four of these reference frames. To move from one reference frame to another needs a force, for example with the gravitational reference frame Alec may be on a platform and Brittney is walking down a slope. To join Britney he would have +𝕀D×e𝕙 gravitational work or a Eℍ/+𝕚d gravitational impulse move him to her. In an inertial reference frame, he would have to use an active force such as -𝕆D×e𝕪 kinetic work, or a E𝕐/-𝕠d kinetic impulse, to change his inertia to match hers.

Changing a reference frame with work and impulse

The changes in the reference frames require an observation of impulse or measurement of work, this force also changes whether $\vec{E}$ as e𝕪 or $\vec{B}$ as -𝕠d is observed or measured. In Alec's own reference frame there are no forces, except for his atoms with their work an impulse also changing with their own individual reference frame. Britney would not measure +𝕆D×e𝕒 potential work being done by her charge, it would not be moving a distance with respect to her. She also could not observe a E𝔸/+𝕠d potential impulse with her own charge because it does not change over potential time.

Reference frames are relative

A reference frame is relative as another reference frame requires a force to reach it. That means a change of reference frame needs an observable impulse or measurable work compared to one's own reference frame. Here C would have its own four reference frames in comparison to Alec and Brittney.

Integral and derivative reference frames

Alec has a magnetic field only, so his reference frame is an integral area with e𝕪 as the straight Pythagorean Triangle side and -𝕠d as the spin Pythagorean Triangle side. Britney's reference frame is not an area, it is a fraction or derivative. This comes from e𝕒 divided by +𝕠d. Alec's reference frame cannot be a derivative, that would mean his bar magnet would have to be charged. Britney's positive charge is not referred to as a magnetic field here, so it cannot be an integral.

An expanding universe and reference frames

In the expanding universe hypothesis, there is a limit of observation and measurement from one side to the opposite side. Someone in a galaxy from soon after the big bang could not send a signal to another galaxy that accelerated in the opposite direction and side. Each of these is observed and measured to be traveling over 3.5 times the speed of light, that is accounted for in conventional physics by postulating an expansion of space. Each would be outside the other's light cone, so with a limit of c the other galaxy could not be observed or measured.

Biv space-time contraction

In this model there is no Biv space-time expansion, instead the +𝕚d and e𝕙 Pythagorean Triangle as gravity points backwards in +𝕚d gravitational time towards the hypothesized big bang. That has the +𝕚d gravitational time slowing, and e𝕙 height contraction so the CMB appears to be like an event horizon of a black hole. This would be a ground state below which photons could not be measured or observed, they would be so redshifted they would not move electrons enough to be detected.

An event horizon like a ground state

In this model gravity has an event horizon like the ground state of the +𝕠d and e𝕒 Pythagorean Triangle protons. This is at a lower e𝕙 height just like the CMB as a limit is further away than the ionization boundary of the proton for electrons. As the e𝕙 height contraction occurs there is a boundary beyond which photons cannot be observed and measured, this is the CMB according to this model. Above an event horizon there is a photosphere analogous to the CMB.

Limits of the inertial velocity

These limits of the +𝕚d and e𝕙 Pythagorean Triangle as gravity, and the -𝕚d and e𝕧 Pythagorean Triangle as inertia, are different from the limits of the +𝕠d and e𝕒 Pythagorean Triangle protons and -𝕠d and e𝕪 Pythagorean Triangle electrons. Distant galaxies can appear to be traveling at e𝕧/-𝕚d inertial velocities greater than c, this approaches the limit of the inertial velocity like the event horizon exceeds c. A black hole would have a limit at c as a photosphere, in this model under that would be the event horizon with a e𝕧/-𝕚d inertial velocity greater than c.

A limit of inertial velocity and gravitational speed

There are two kinds of observation and measurement in these limits. The +𝕚d and e𝕙 Pythagorean Triangle as gravity has a limit appearing as the big bang and CMB, this is their e𝕙/+𝕚d gravitational speed. The galaxies have a limit of -𝕚d and e𝕧 Pythagorean Triangle inertia, this has a limit of the e𝕧/-𝕚d inertial velocity. That inertia is proportional to e𝕪×-𝕘d photons so light from these galaxies appears to be redshifted by their inertial velocity. The gravitational speed appears differently to the inertial velocity, that is the CMB boundary.

Going towards a black hole

In this model a rocket falling into a black hole does this with a e𝕧/-𝕚d inertial velocity. As the -𝕚d and e𝕧 Pythagorean Triangle inertia is proportional to the electron's -𝕠d and e𝕪 Pythagorean Triangle, the rocket emits light as its e𝕧 length contracts and its -𝕚d inertial time slows. Conversely as galaxies are further away, they appear to be moving with a faster e𝕧/-𝕚d inertial velocity, this can exceed c both when falling into the black hole and moving away from an observer and measurer.

A changing inertial reference frame with respect to c

When the +𝕚d and e𝕙 Pythagorean Triangle as gravity has a greater e𝕙 height approaching the CMB, then e𝕙 contracts into the CMB wall. Also the +𝕚d gravitational time is slowed there, e𝕙 contracts and +𝕚d slows more past this. When galaxies are far enough away then their e𝕧 length dilates and their -𝕚d inertial time speeds up. This makes them appear to move faster than c, the opposite of falling into a black hole making them appear to be moving more slowly. From their own inertial reference frames, they have no e𝕧 length or -𝕚d inertial time change.

Eternal galaxies

In this model the galaxies are not younger because they are more distant, their light is not from a common time when they were formed after the big bang. Instead, these galaxies are much older, that implies the heavy elements in stars must be recycled back into Hydrogen and Helium. One proposal is from around the black holes in the galaxies tearing apart higher elements. A galaxy would then last forever, or reform into other galaxies such as with collisions.

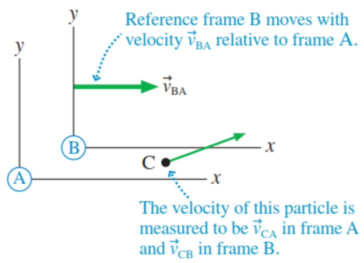
Reference Frames

We introduced reference frames and relative motion in Chapter 4. To remind you, **FIGURE 31.2** shows two reference frames labeled A and B. You can think of these as the reference frames in which Alec and Brittney, respectively, are at rest. Frame B moves with velocity $\vec{v}_{BA}$ with respect to frame A. That is, an observer (Alec) at rest in A sees the origin of B (Brittney) go past with velocity $\vec{v}_{BA}$. Of course, Brittney would say that Alec has velocity $\vec{v}_{AB} = -\vec{v}_{BA}$ relative to her reference frame. We will stipulate that both reference frames are inertial reference frames, so $\vec{v}_{BA}$ is constant.

Figure 31.2 also shows a particle C. Experimenters in frame A measure the motion of the particle and find that its velocity *relative to frame A* is $\vec{v}_{CA}$. At the same instant, experimenters in B find that the particle's velocity *relative to frame B* is $\vec{v}_{CB}$. In Chapter 4, we found that $\vec{v}_{CA}$ and $\vec{v}_{CB}$ are related by

$$\vec{v}_{CA} = \vec{v}_{CB} + \vec{v}_{BA} \qquad (31.1)$$

FIGURE 31.2 Reference frames A and B.



Impulse from one reference frame to another

In equation (31.1) there is a change from one reference frame to another with impulse. That is because these are derivatives with respect to spin Pythagorean Triangle sides, with time as 1/dt. They can also transform with work between them, then there would be integral changes with respect to a distance as straight Pythagorean Triangle sides. In conventional physics these work changes would also be written as derivatives, in this model they are integrals because they represent an area or field. The answers would be the same, there is a change in distance in moving between reference frames instead of time.

Accelerating from one reference frame to another

In this model there can be a E𝕐/-𝕠d kinetic impulse in changing between reference frames, this is with respect to time or dt as in (31.2). With the E𝕍/-𝕚d inertial impulse for example this would be in meters²/second, the force comes from the squared straight Pythagorean Triangle side. With e𝕧/-𝕀D inertial work the time component is squared as -𝕆D to give meters/second². That is the format in F=ma from Newton.

Equation 31.1, the *Galilean transformation of velocity*, tells us that the velocity of the particle relative to reference frame A is its velocity relative to frame B plus (vector addition!) the velocity of frame B relative to frame A.

Suppose the particle in Figure 31.2 is accelerating. How does its acceleration $\vec{a}_{CA}$, as measured in frame A, compare to the acceleration $\vec{a}_{CB}$ measured in frame B? We can answer this question by taking the time derivative of Equation 31.1:

$$\frac{d\vec{v}_{CA}}{dt} = \frac{d\vec{v}_{CB}}{dt} + \frac{d\vec{v}_{BA}}{dt}$$

The derivatives of $\vec{v}_{CA}$ and $\vec{v}_{CB}$ are the particle's accelerations $\vec{a}_{CA}$ and $\vec{a}_{CB}$ in frames A and B, respectively. But $\vec{v}_{BA}$ is a *constant* velocity, so $d\vec{v}_{BA}/dt = \vec{0}$. Thus the Galilean transformation of acceleration is simply

$$\vec{a}_{CA} = \vec{a}_{CB} \qquad (31.2)$$

Two kinds of forces

In this model there are two kinds of forces, work and impulse. When a magnetic field accelerates a charge this can only be -𝕆D×e𝕪 kinetic work, the spin of the magnet's electrons makes the charge change its direction. The charge is not regarded as a particle by Alec, instead it moves with -𝕆D×e𝕪 kinetic work as a wave. For Alec to observe Britney's positive charge as a particle, he would need to observe it. That could not be done with a magnet, he could only measure its e𝕒/+𝕆D potential work. From Britney's reference frame she is accelerating with a E𝔸/+𝕠d potential impulse with her positive charge.

Brittney and Alec may measure different positions and velocities for a particle, but they *agree* on its acceleration. And if they agree on its acceleration, they must, by using Newton's second law, agree on the force acting on the particle. That is, **experimenters in all inertial reference frames agree about the force acting on a particle.**

Moving in free space

In this model Alec can be regarded as moving backwards compared to Brittney. If this was in free space, then the magnet pushing on Brittney's charge would move Alec in an arc clockwise for example while pushing her counterclockwise in an arc. This is like a cyclotron making a positive charge turn, it cannot make the positive charge accelerate in a straight-line.

Two forces

There are two kinds of forces here, the positive charge moves with impulse because it comes from the straight Pythagorean Triangle side. The magnet moves with work as it comes from the spin Pythagorean Triangle side. The positive charge's velocity and impulse would not change from the magnet, this would move in an arc at the same velocity. Brittney would not feel an acceleration from Alec, she would measure a centrifugal force from the positive charge turning.

Straight and rotational reference frames

In space an electric charge as impulse and a magnetic field as work cannot swap by changing one straight reference frame for another. The change would still be observed as electric charges only. If a rotational reference frame is changed to a second rotational reference frame, such as by changing the rotational frequency with a torque, then this can only measure the effects of one magnetic field on another.

A bar or electromagnet

Instead of Alec having a bar magnet, it could instead be an electromagnet. Then Alec has both a E𝕐/-𝕠d kinetic impulse of electrons in the magnetic coil and -𝕆D×e𝕪 kinetic work as a magnetic field inside the coil. He measures the magnet's effects as work on a charge only by making Britney's positive charge turn in an arc. He could also regard this arc as coming from the E𝕐/-𝕠d kinetic impulse of the electromagnet's electrons coming from the battery. Alec observes Brittney moving with her positive electric charge in an arc from the effects of his electromagnet, that can be regarded as doing work on her charge or changing its impulse.

A rotational to a straight-line reference frame

Looking from above, Britney's positive charge moves in an arc from Alec's electromagnet. This is in a rotational reference frame, the force can only be described as torque. Changing this reference frame by 90° it becomes a straight-line reference frame, Alec observes Britney's positive charge seeming to decelerate. That would be as it turns more towards him. This deceleration can only be impulse. This would appear to be his electromagnet exerting an electrical displacement on her positive charge.

Moving parallel to the electromagnet

Britney moves parallel to the electromagnet, she measures the magnet's -𝕆D×e𝕪 kinetic work as being zero as it is not accelerating her charge. She could also observe the electrons in the electromagnetic coil moving roughly orthogonally to her, the protons in the coil are reacting against the electrons moving.

No change in both the rotational and straight-line reference frames

Because the electrons are moving orthogonally, they cannot be moving her parallel to the magnet. The electrons also move in all directions orthogonal to her motion, so they should not be deflecting her charge's motion. A rotational reference frame from above shows no -𝕆D×e𝕪 kinetic work being done, changing this by 90° to a straight-line reference frame there is also no E𝕐/-𝕠d kinetic impulse affecting her positive charge.

At an angle to the electromagnet

If she was at an angle to the magnet in a straight-line reference frame, then her positive charge would be acted on by the electrons in the coil with the E𝕐/-𝕠d kinetic impulse. That would tend to turn her positive charge because the electrons are attracted to her proton asymmetrically as they move around the curve of the coil. The protons in the coil would also change her direction with positive charges repelling asymmetrically.

An electric coil or a magnet

Changing the reference frame by 90° makes it rotational, that would be from above the electromagnet. Now as her positive charge moves it no long appears as a straight-line motion accelerating. It appears as an arc where the electromagnet exerts a torque on it. That can only be from -𝕆D×e𝕪 kinetic work, from this rotational reference frame the electromagnet has a magnetic field. From the straight-line reference frame there was no magnetic field in the coils, only an electric charge.

A gravitational reference frame

Her motion is the same, in the straight-line reference frame there is a straight-line force turning her charge in an arc. This would be like a projectile fired from a cannon, it moves in an arc trajectory as gravity pulls it straight down with a Eℍ/+𝕚d gravitational impulse. In the rotational reference frame, there is a torque turning her positive charge. That would be like gravity turning the projectile with a +𝕀D gravitational torque into an arc path.

A loop can have a torque

In a bar magnet there is no battery and coil, instead the electrons do -𝕆D×e𝕪 kinetic work as individual fermions in its atoms. The -𝕆D kinetic probability or torque of the electrons is the same as the -𝕆D kinetic difference of the battery and circuit. The wire of electromagnet's circuit is a loop, that allows for the battery's +𝕆D potential difference and the -𝕆D kinetic difference to be a torque. That potential and kinetic torque moves electrons around the loop from the rotational reference frame above it.

A straight wire has no torque

Changing this to a straight-line reference frame the loop appears as a straight wire from the side. Now the electrons appear to accelerate and decelerate with a E𝕐/-𝕠d kinetic impulse only, the battery repels them with a E𝕐/-𝕠d kinetic impulse from the negative terminal and attracts them with a E𝔸/+𝕠d potential impulse from the positive terminal. The loop cannot be observed from the side so there can be no measurable torque or voltage, just a current and impulse.

A bar magnet has no current flow because there is no loop

The bar magnet has the same +𝕆D potential torque and -𝕆D kinetic torque as in the battery and loop. The +𝕆D potential difference is in each nuclei, the -𝕆D kinetic difference is in each fermion producing the magnetization. There is no current flow as the electrons do not move in the bar magnet's metal like an electron sea with impulse, unless the bar magnet is hooked up with wires in a circuit.

A circuit has a magnetic field

There is no loop of wire so there is no voltage. The +𝕆D potential difference and -𝕆D kinetic difference produce a magnetic field like the battery and electromagnetic loop did, this is a pure magnetic attraction and repulsion. The loop as a circuit also has a magnetic field from this potential and kinetic torque, as Faraday discovered.

Moving a magnet in a circuit

When a magnet is moved in a coil, connected in a circuit, that can make an ammeter needle move. This is from a rotational reference frame above the circuit loop so it comes from a -𝕆D kinetic torque. That is from the bar magnet when it moves over a distance with work, it appears to produce a +𝕆D potential and -𝕆D kinetic difference or voltage in the circuit.

No acceleration in a rotational reference frame

The bar magnet cannot appear to make voltage with acceleration here. With a rotational reference frame there is no straight-line motion, so impulse cannot be observed. There is only a change in position as points and a change in torque as voltage.

Turning to a straight-line reference frame

Turning 90° to a straight-line reference frame, the loop is viewed from the side. There can be no torque and no work, only the acceleration and deceleration of the current in a straight-line. That acceleration is from a E𝕐/-𝕠d kinetic impulse as it changes over time. The bar magnet appears to produce electricity as its position changes with an acceleration and deceleration over time.

No voltage in a straight-line reference frame

Changing the reference frame then also changes a voltage into an acceleration of the current with power. The straight-line reference frame cannot measure voltage as it can only observe a straight-line edge of the circuit. Without a loop there can be no measurement of voltage.

A magnetic loop

If the bar magnet was twisted around into a loop, there would no longer be a north and south pole. The magnetic field would be canceled out and it could not attract another magnet. There are no change of positions on a straight-line ruler on which the magnet could do -𝕆D×e𝕪 kinetic work. An elliptical loop could be partially magnetized like having a hole along a magnet's core.

Observing electrical changes in the magnet over time

In the straight-line reference frame, Britney could observe a small E𝕐/-𝕠d kinetic impulse from the bar magnet as her positive charge disturbed each atom. The bar magnet is electrically neutral so this would not give her an impulse. With an electromagnet it would cause some electrons to move in the circuit and coil as they are attracted to her positive charge.

Impulse from the battery

This would also be happening because of the E𝔸/+𝕠d potential impulse and E𝕐/-𝕠d kinetic impulse from the battery. If a circuit switch was opened as a change with respect to time, then she would no longer observe a force from the battery. Changing 90° to a rotational reference frame, opening the switch removes the +𝕆D potential and -𝕆D kinetic difference or voltage from the battery. There can be no torque in the circuit as there is no closed loop.

No time in a rotational reference frame

The torque in the loop can be measured in a rotational reference frame, but changes cannot be observed over time. This is because a clock gauge only works with rotation, it would be comparing spin in a circuit to the spin of the clock hands. There would be no work measured because there is no straight-line ruler in the rotational reference frame. All torque forces would appear to be the same as there would be no straight-line distance to compare them with.

No work in a straight-line reference frame

Work cannot be measured in a straight-line reference frame, it would be comparing an acceleration over a distance only as the rotational clock could not be used. Then all different accelerations over the same distance would appear to be the same force.

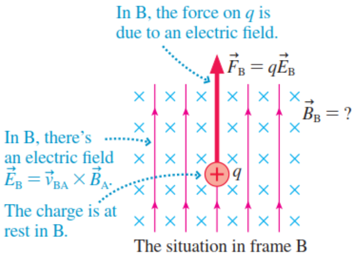
A magnet does constructive interference to protons with both poles

In this model, fermion electrons in a bar magnet have the same orientation. Each pole would tend to turn the proton's magnetic orientation with its +𝕠d potential magnetic field. This would not accelerate the proton with either pole or turn its straight-line velocity into an arc. It would change the direction of the proton's magnetic poles or make them precess.

Larmor precession

In this model an external magnetic field can make electrons in an atom precess. This is from a rotational reference frame as -𝕆D×e𝕪 kinetic work from the magnet. This is called Larmor precession, work cannot move the electron in a straight-line direction, the magnetic field can only change its spin. This leads to a precession as the third orthogonal direction available after a E𝕐/-𝕠d kinetic impulse and the quantized electron spin. A larger precession can lead to the electron flipping over from the -𝕆D kinetic torque, then flipping over again to return to the original state.

FIGURE 31.4 In frame B, the charge experiences an electric force.



As Brittney runs past Alec, she finds that at least part of Alec's magnetic field has become an electric field! **Whether a field is seen as "electric" or "magnetic" depends on the motion of the reference frame relative to the sources of the field.**

FIGURE 31.4 shows the situation from Brittney's perspective. There is a force on charge q, the same force that Alec measured in Figure 31.3, but Brittney attributes this force to an electric field rather than a magnetic field. (Brittney needs a moving charge to measure magnetic forces, so we'll need a different experiment to see whether or not there's a magnetic field in frame B.)

More generally, suppose that an experimenter in reference frame A creates both an electric field $\vec{E}_A$ and a magnetic field $\vec{B}_A$. A charge moving in A with velocity $\vec{v}_{CA}$ experiences the force $\vec{F}_A = q(\vec{E}_A + \vec{v}_{CA} \times \vec{B}_A)$ shown in **FIGURE 31.5a**. The charge is at rest in a reference frame B that moves with velocity $\vec{v}_{BA} = \vec{v}_{CA}$ so the force in B can be due only to an electric field: $\vec{F}_B = q\vec{E}_B$. Equating the forces, because experimenters in all inertial reference frames agree about forces, we find that

$$\vec{E}_B = \vec{E}_A + \vec{v}_{BA} \times \vec{B}_A \qquad (31.4)$$

Biv space-time reference frames

This also happens in Biv space-time with reference frames. Alec and Britney can be moving in free space. Alec is rotating with -𝕀D×e𝕧 inertial work in an orbit, his -𝕆D inertial torque allows him to move in a circle around a planet. The spin is like with electrons spinning in magnetism. This could also occur without gravity, then his rocket might move him in a circle.

Britney in a straight-line reference frame

Britney is moving with an E𝕍/-𝕚d inertial impulse as an equal and opposite reaction from burning rocket fuel, she is moving directly towards the center of the planet orthogonal to Alec's orbit. That is like a Coulomb electrical force as a square. She sees Alec moving in a circle doing -𝕀D×e𝕧 inertial work, the force is a -𝕀D inertial torque.

Changing her reference frame

Britney then changes her inertial reference frame 90° so she is edge on to Alec's orbit. Now she observes Alec seemingly moving from side to side like a spring oscillating. His motion now seems to be like a Coulomb electrical force. On the left side of his orbit the spring seems to be compressed, on the right side the spring seems to be expanded. He appears to move with an E𝕍/-𝕚d inertial impulse with a period according to -𝕚d inertial time.

Electric charge/magnetism duality

This is like the electric charge and magnetism duality, the positive electric charge can only move with a E𝔸/+𝕠d potential impulse and the magnetism can only do +𝕆D×e𝕒 potential work. When the reference frame is changed, the potential clock gauge of the E𝔸/+𝕠d potential impulse is changed to the ruler e𝕒 with +𝕆D×e𝕒 potential work. This happens because the +𝕠d and e𝕒 Pythagorean Triangle is now viewed orthogonally, the force also changed orthogonally from E𝔸 to +𝕆D.

Changing a reference frame requires a force

The change from one reference frame to another requires a force, in moving 90° the observation of the positive charge's E𝔸/+𝕠d potential impulse changed to +𝕆D×e𝕒 potential work. If not, then the change of force would not be conserved. Here Britney moved from being orthogonal to Alec's circular orbit to edge on, that required a rotation or torque so she did -𝕀D×e𝕧 inertial work for this.

Rotating her reference frame

The amount of -𝕀D×e𝕧 inertial work she did is equivalent to the amount needed to changing her measurement of Alec's -𝕀D×e𝕧 inertial work to her observing this as an E𝕍/-𝕚d inertial impulse. Initially she was in a rotational inertial reference frame because she could measure Alec's -

𝕀D×e𝕧 inertial work. When she changed 90° she reached a straight-line reference frame because now she could observe his E𝕍/-𝕚d inertial impulse.

No unique position like a particle

She could not observe Alec's E𝕍/-𝕚d inertial impulse from the rotational reference frame, she could only measure his -𝕆D inertial torque. His orbit did not appear as a particle, his rocket could have been at any e𝕧 position on his orbit and the -𝕀D×e𝕧 inertial work would have been the same. His position was then being measured as doing the same -𝕀D×e𝕧 inertial work anywhere on the orbit. No mattery what e𝕧 position Alec had on the circle, this could not be defined as unique just as with a wave.

Velocity and speed

His rocket moved like a wave with a speed not a velocity. In this model a velocity is straight-line motion, a speed is curved motion. Here Britney's inertial velocity would be e𝕧/-𝕚d, Alec's inertial speed can be regarded as an integral -𝕚d×e𝕧. The -𝕚d×e𝕧/-𝕚d inertial momentum here is a combination of velocity and speed, e𝕧/-𝕚d is a derivative and -𝕚d×e𝕧 is an integral.

Momentum

Alec moves with an inertial momentum where his -𝕚d inertial mass gives the orbit its shape. Britney moves with an inertial momentum where her 1/-𝕚d inertial time gives her velocity. Her -𝕚d inertial mass is not relevant because her inertial velocity is constant. His -𝕚d inertial time is not relevant because his orbit is not changing over time as a constant circle. To calculate his period of rotation, as a 1/-𝕚d inertial frequency would require a e𝕧/-𝕚d inertial speed. But then this would not be constant edge on to his orbit.

Alec's straight-line inertial reference frame

Alec looks at Britney's rocket coming towards him edge on to his orbit, as he goes around the planet, her ship seems to oscillate side to side like a spring. This is because he watches her ship, he would need to turn his head side to side to keep his eyes on it. Alec would be observing her E𝕍/-𝕚d inertial impulse with an acceleration and deceleration. This is like an electric charge, with the straight-line Coulomb force.

Alec's rotational inertial reference frame

If Alec was in an equatorial orbit then he could change it 90° with a -𝕀D inertial torque to a polar orbit. In viewing Britney's rocket it no longer appears to be moving from side to side, it is now orthogonal to his orbit whereas before it was edge on to his orbit. Her rocket now appears to be spinning as a -𝕀D inertial torque doing -𝕀D×e𝕧 inertial work. This is like a change from an electric charge to magnetism.

Charge as acceleration

In this model a charge is like acceleration. The Coulomb force accelerates two charges towards or away from each other. This is different from magnetism and mass, each only gives a curved path, for example a rocket approaching a planet tends to curve around it or go into orbit. Electromagnetism is created with a coiled wire.

Acceleration is not torque

The acceleration is not a torque, the E𝕐/-𝕠d kinetic impulse from an electron is proportional to an E𝕍/-𝕚d inertial impulse such as rocket fuel propelling a rocket. A E𝔸/+𝕠d potential impulse attracts an electron to accelerate towards it, this is like a Eℍ/+𝕚d gravitational impulse attracting Britney's rocket.

Electrons not colliding

In this model a E𝕳/+𝕚d gravitational impulse directly moving towards a gravitational mass turns into +𝕀D×e𝕙 gravitational work. This is like two electrons moving towards each other with a E𝕐/-𝕠d kinetic impulse, when the e𝕪 distance between them is small enough the -𝕆D×e𝕪 kinetic work from the orthogonal rotational reference frame gives them a -𝕆D kinetic torque.

Quarks not colliding

In a proton the +𝕠2/3 up quarks would move towards each other with a E𝕳/+𝕚d gravitational impulse, with a smaller e𝕙 height between them this would change to +𝕀D×e𝕙 gravitational work so they would not collide.

Straight special and curved general relativity

There are then two reference frames, one is only in a straight-line and one is only curved. These are also analogous to straight-line special relativity and curved general relativity. Special relativity only occurs in straight lines as a consequence of the higher e𝕧/-𝕚d inertial velocity of a rocket. General relativity only occurs in curved space as a consequence of the higher e𝕙/+𝕚d gravitational speed of a rocket near an event horizon.

Moving towards an event horizon

When a rocket moves towards an event horizon with its E𝕳/+𝕚d gravitational impulse there is a slowing of +𝕚d gravitational time. The rocket can then appear to be frozen in position. When the e𝕙 height becomes small then the +𝕀D×e𝕙 gravitational work of the event horizon is stronger like with the electrons. This stops the rocket moving closer as the +𝕀D gravitational torque is stronger. It would then go into orbit around the event horizon like with photons in the photosphere. A rotating event horizon would have the rocket appear to be moving in this orbit.

The barn paradox

In special relativity there is a paradox, a 20-meter-long pole can seem to fit into a 10-meter-long barn. It can be regarded as accelerating with a rocket on the back of it.

The time to traverse the barn

Sitting at the back of the barn, Alec would observe the pole moving towards him with a -𝕚d inertial time slowing, he cannot see a e𝕧 length contraction because it is moving directly towards him. This is a straight-line motion, so Alec concludes this is the E𝕍/-𝕚d inertial impulse of the pole. He had calculated the time it would take for the pole to enter the barn and reach the back wall. He concluded this would need half the time needed for all of the pole to get inside the barn.

A straight-line reference frame

When he observes the time the pole takes to reach the back it seems to take twice as long as he had calculated. He concludes that is enough time for the pole to fit inside the barn before hitting the back wall. He can only calculate the time, he cannot see the e𝕧 length contraction because it is coming directly towards him.

A rotational reference frame

Rotating the reference frame by 90° gives a side on view, now the pole is moving from left to right for Britney as it enters the barn. The pole is e𝕧 length contracted because now this can be observed, she cannot accurately observe the time the pole takes because the length has changed. The pole looks as if it was moving more towards her, the e𝕧 length contraction from this change in perspective allows it to fit inside the barn.

Gravitational reference frames

In terms of gravity there is also two reference frames, Alec can be moving in a low orbit around the planet with a constant e𝕙/+𝕚d gravitational speed. That means he has a fixed e𝕙 height above the ground and a fixed +𝕚d period of rotation. The planet does +𝕀D×e𝕙 gravitational work on him in moving him around with a +𝕀D gravitational torque.

Freefall and weightlessness

Britney's rocket is stopped on the ground after she landed, she only observes a Eℍ/+𝕚d gravitational impulse straight down. If she jumped off the rocket's ladder to the ground then her free fall would be a straight-line impulse. When Alec is directly above her he is rotated 90° to her. He measures weightlessness in his rocket, there seems to be no force pulling him downwards. Britney observes free fall when she jumps, she is not weightless because she is being accelerated towards the ground.

A field of torque

The gravity would seem to be curved space as a geodesic in general relativity, it could also be regarded as a field of +𝕀D gravitational torque holding Alec in orbit. This is also a +𝕀D gravitational probability that Alec would be somewhere on this circle, the e𝕙 height is the same so it would not change the strength of the torque and probability. His position is not definable from the +𝕀D×e𝕙 gravitational work, any position on the circle would have the same torque and probability so this is a wave. Alec's +𝕀D×e𝕙 gravitational work cannot be measured with a gravitational clock gauge because he seems to be suspended in the air relative to his e𝕙 height above the ground.

Linear time with a spring and displacement

Britney would observe this downward force like a spring expanding, her Eℍ height displacement defines her starting and final positions like a particle. The displacement force can only be observed in a gravitational clock gauge, otherwise different accelerations could not be differentiated from each other. Because they can be differentiated, they are derivatives, and so this is the Eℍ/+𝕚d gravitational impulse.

Weightlessness becomes weight

Alec can join Britney by firing his rocket until he lands, then he is stationary like Britney and her rocket. Now there is a force downwards he could not feel before. If he jumps off his ladder he feels weightless like in orbit, but now it comes from gravitationally accelerating downwards with a Eℍ/+𝕚d gravitational impulse.

Weight to weightlessness

If instead Britney fires her rocket and goes into orbit, she joins Alec's reference frame. Then she becomes weightless in orbit next to him, her downward force is gone even though the planet's gravity has not changed. She formerly experienced the equivalent of the straight-line electrical force, now there is a force like magnetism that only works in circles. Her free fall experienced in jumping off the rocket ladder has gone, now she does not fall when in orbit.

Einstein's elevator

Einstein said that gravity and acceleration were equivalent, a person in an elevator could not tell them apart. In this model the inertial acceleration of the elevator would come from the E𝕍/-𝕚d inertial impulse. Here gravity is pulling the elevator's occupant straight sown, that would be the Eℍ/+𝕚d gravitational impulse.

Space station inertial impulse

A large space station could have rings with different radii, it rotates to give an artificial gravity with -𝕀D×e𝕧 inertial work. Alec could enter the center of the space station feeling weightless as it rotates around the planet doing -𝕀D×e𝕧 inertial work. In moving in a radial or spoke-like tube towards the outer ring he observes an increasing E𝕍/-𝕚d inertial impulse moving straight downwards. He is in a straight-line reference frame as his motion is in a straight-line down the spoke. This is a squared acceleration like the Coulomb electrical force.

Space station inertial torque

Britney is outside the space station at 90° to Alec, she measures him spinning around from her rotational reference frame. As he moves to a different e𝕧 position down the spoke his -𝕀D inertial torque increases. She concludes his -𝕀D inertial probability increasingly makes it more likely he moves towards the end of the spoke as artificial gravity. That is not an inertial acceleration to her, the -𝕀D inertial torque is making him move in an exponential spiral. This torque is like magnetism to her, such as in a generator or electromagnetic coil.

Potential freefall for the electron

The proton and potential also have two reference frames. An electron can be in a higher orbital around the proton, from the proton's straight-line reference frame the electron emits a e𝕪×-𝕘d photon. It seems to move downwards with the proton's E𝔸/+𝕠d potential impulse. Turning 90° to a rotational reference frame looking down on the proton, and orthogonal to the electron's orbital, the electron appears to experience a +𝕆D potential torque from the proton. This makes it drop to a lower orbital at a lower e𝕒 altitude, the +𝕆D potential torque is stronger there.

Potential weight for the electron

If the electron was in free space it can be attracted towards the proton with a E𝔸/+𝕠d potential impulse, then there seems to be a straight-line force on it whereas in the orbital there is an electron cloud with no straight-line impulse. The electron would be observed as being a particle, from the proton's straight-line reference frame, as it was displacing from an initial to a final position over a time.

An electron spring

An electron edge on in a circular orbital could also appear as a particle like a spring oscillating back and forward. Turning 90° it would appear as a circle, the forces holding it in this orbital would appear as a wave of -𝕆D kinetic torque, or a +𝕆D potential torque from the proton. The electron typically is measured as a -𝕆D probability wave, an energetic photon can knock out of of an orbital as if it is a particle. From the straight-line reference frame of the photon the electron is collided with, that moves out of the atom with a E𝕐/-𝕠d kinetic impulse.

A photon's reference frame

In this model a photon can be regarded as a rolling wheel. From above the e𝕪 light spoke, it seems to oscillate backwards and forwards like a spring with a e𝕐/-𝕘d light impulse. If this collides with an electron in free space, that can change the size of the photon's and the electron's e𝕪 spoke. Turning 90° to a rotational reference frame, the photon appears as a wheel turning with a -𝔾D light torque. The photon does -𝔾D×e𝕪 light work when being absorbed by an atom, that increases the -𝕆D kinetic torque of an electron in an orbital.

A transverse or longitudinal wave

In this model an ocean wave from the side has a -𝕀D inertial torque, the water spins with a vortex under it with -𝕀D×e𝕧 inertial work. This is not from the water particles, instead it is the motion of the water as a whole wave. Changing 90° to a longitudinal reference frame, a buoy on the water would appear to move backwards and forwards like a longitudinal motion as an E𝕍/-𝕚d inertial impulse. The water molecules would also move backwards and forwards as they accelerated then reversed direction.

A magnet only turns charges

Alec and Britney changed their reference frames, from a straight-line reference frame to a rotational reference frame, or vice versa. The magnet can only turn charges. It cannot turn a positive charge moving parallel to it because its torque is acting on all sides of the charge. It is in effect trying to turn the positive charge in opposite ways, these cancel out with destructive interference. When her positive charge's path is no longer parallel to the magnet, then the magnet's destructive interference is not canceled out. The positive charge is then more -𝕆D kinetically probable to change its path.

Curving her path to across the magnet

To do this Britney had to curve her straight-line reference frame to rotational, from a parallel path to at an angle across the magnet. Then the -𝕆D×e𝕪 kinetic work would make her positive charge spin in circles like with a cyclotron. According to Britney she sees Alec with the magnet orthogonal to her. The magnet makes her path curve, but she only sees the magnet getting close just as if it was a negative charge attracting her. From the magnet's rotational reference frame, Britney would seem to oscillate from side to side with a E𝔸/+𝕠d potential impulse.

Work cannot accelerate in a straight line

In this model -𝕆D×e𝕪 kinetic work from the magnet can only turn the charge's path, it cannot accelerate it in a straight-line. That is why a stationary charge is not moved by a magnet, and why when it moves parallel to the magnet it is not accelerated. The straight-line motion would be a E𝕐/-𝕠d kinetic impulse from the magnet, this can only come from a positive or negative charge in the magnet itself.

A rotational reference frame

To make a straight-line motion appear curved, that would require a rotational reference frame. This could be created by Britney moving at an angle to Alec, this can be measured at 90° from above the magnet. Then her positive charge would be turned with a torque from work.

Changing the angle of the reference frame

To change from an electrical impulse to a magnetic work from torque, the reference frame needs to be turned 90°. That is because the +𝕠d and e𝕒 Pythagorean Triangle protons, and -𝕠d and e𝕪 Pythagorean Triangle electrons, in the magnet have work at 90° to impulse in a Pythagorean Triangle. When this angle changes, such as with Britney moving more across the magnet, then there is -𝕆D×e𝕪 kinetic work done on her charge which would tend to make it move in circles like with a cyclotron.

Work and impulse from the electromagnet

In Figure 31.5 the positive charge would have -𝕆D×e𝕪 kinetic work done in it in both cases, the magnet's electric charge would separately have an impulse that moves the positive charge here. There would be a rotational reference frame with the magnet, and a straight-line reference frame from the electric charge.

Two electrons with work and impulse

When two electrons approach each other in this model, they can almost collide with a E𝕐/-𝕠d kinetic impulse as particles. When they get close to each other the distance is small, then the -𝕠d kinetic magnetic field of the electrons is much larger. The forces then become -𝕆D×e𝕪 kinetic work on each other, the electrons destructively interfere. They would be at a small angle at least from uncertainty, the -𝕆D kinetic torque would make them turn in an arc.

Quantization between electrons colliding

As they approach each other the electrons have quantized increments like orbitals, they emit e𝕪×-𝕘d photons in between each other which is proportional to the -𝕆D kinetic improbability of them being close together. The -𝔾D×e𝕪 light work between the electrons cannot be removed by a direct collision between the electrons. If these photons were emitted, then the electrons would lose impulse in an inelastic collision. That can happen with two large negatively charged masses, photons can be emitted as heat which reduces their inertial velocities after the collision.

Approaching c has a higher impulse

If they were collided at near c then the E𝕐/-𝕠d kinetic impulse would be larger than the -𝕆D×e𝕪 kinetic work, this is from c as e𝕧/-𝕚d and e𝕪/-𝕚d. Here e𝕧 and e𝕪 approach their maximum, but -𝕚d and -𝕠d also have a minimum. The electrons would move like particles, in a collider particles can show more particle internal structure when high inertial velocities are used.

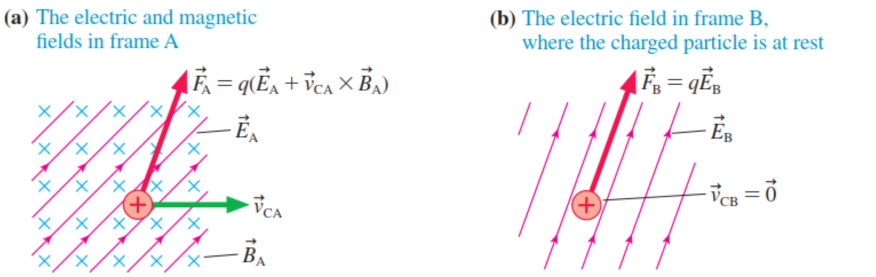
Structure inside particles

At lower velocities iotas act more like waves and are repelled with interference so there is less internal structure observed. Electrons colliding at high inertial velocities with protons would make both more like particles, so a quark structure can be observed.

Collisions at higher inertial velocities

The electrons would do some -𝕆D×e𝕪 kinetic work as they collided, they would get closer together because the -𝕠d and e𝕪 Pythagorean Triangle electrons would have the angles θ changing as they decelerated. When -𝕠d is large enough then the -𝕆D kinetic probability destructively interferes with a repulsion, this is also a E𝕐/-𝕠d kinetic impulse collision when observed as a change over -𝕠d kinetic time. Collisions like this can create additional iotas, these come from the higher E𝕐/-𝕠d kinetic impulse of the electrons.

FIGURE 31.5 A charge in reference frame A experiences electric and magnetic forces. The charge experiences the same force in frame B, but it is due only to an electric field.



Using a scale or a gauge between reference frames

When there is a transformation from an electrical force to a magnetic force, or vice versa, there is also a change in the scale used. The electrical force uses impulse which is observed over time, the positive charge would be observed to move with a velocity. When there is a magnetic force then work is done, the positive charge turns with a measurable change over a distance.

Equation 31.4 transforms the electric and magnetic fields measured in reference frame A into the electric field measured in a frame B that moves relative to A with velocity $\vec{v}_{BA}$. **FIGURE 31.5b** shows the outcome. Although we used a charge as a probe to find Equation 31.4, the equation is strictly about fields in different reference frames; it makes no mention of charges.

Changing the angle between the reference frames

In (31.5) the positive charge in not moving in Alec's reference frame A. This is the electric charge so in this model he would be observing the charge, not measuring an electric field. In the rotational reference frame Brittney, she measures the turning the magnet relative to her as its -𝕆D×e𝕪 kinetic work. Her direction of motion is changing her angle to Alec, so this acts as a rotation. This is dividing by r^2 so it is a second derivative of impulse.

To find a transformation equation for the magnetic field, **FIGURE 31.7a** shows charge q at rest in reference frame A. Alec measures the fields of a stationary point charge:

$$\vec{E}_A = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad \vec{B}_A = \vec{0}$$

What are the fields at this point in space as measured by Brittney in frame B? We can use Equation 31.4 to find $\vec{E}_B$. Because $\vec{B}_A = \vec{0}$, the electric field in frame B is

$$\vec{E}_B = \vec{E}_A = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad (31.5)$$

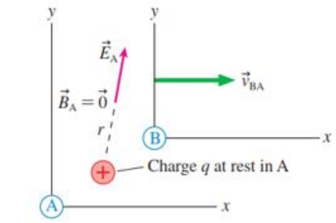
In other words, **Coulomb's law is still valid in a frame in which the point charge is moving.**

But Brittney also measures a magnetic field $\vec{B}_B$, because, as seen in **FIGURE 31.7b**, charge q is moving in reference frame B. The magnetic field of a moving point charge is given by the Biot-Savart law:

$$\vec{B}_B = \frac{\mu_0}{4\pi}\frac{q}{r^2}\vec{v}_{CB}\times\hat{r} = -\frac{\mu_0}{4\pi}\frac{q}{r^2}\vec{v}_{BA}\times\hat{r} \qquad (31.6)$$

FIGURE 31.7 A charge at rest in frame A is moving in frame B.

(a) In frame A, the static charge creates an electric field but no magnetic field.



(b) In frame B, the moving charge creates both an electric and a magnetic field.

A fraction only changes a magnitude

In (31.7) µ would be the squared magnetic force, it is multiplied by the vector v⃗ which would be an integral. A torque cannot be created by a division or derivative, a fraction only makes something larger or smaller like a vector changing its magnitude. With multiplication an area can be created, for example µ here is a squared value giving B⃗.

Parallel lines cannot enclose an area

For this to give a force there must be a motion that is not parallel, parallel lines cannot enclose an area. When that is for example at 45° it creates an area between them that allows for a magnetic integral field. That is why, in this model, the -𝕆D×e𝕪 kinetic work of the magnet can only be measured when the positive charge moves at an angle to it.

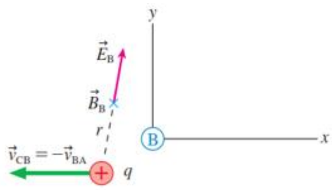
where we used the fact that the charge's velocity in frame B is $\vec{v}_{CB} = -\vec{v}_{BA}$.

It will be useful to rewrite Equation 31.6 as

$$\vec{B}_B = -\frac{\mu_0}{4\pi}\frac{q}{r^2}\vec{v}_{BA}\times\hat{r} = -\epsilon_0\mu_0\vec{v}_{BA}\times\left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}\right)$$

The expression in parentheses is simply $\vec{E}_A$, the electric field in frame A, so we have

$$\vec{B}_B = -\epsilon_0\mu_0\vec{v}_{BA}\times\vec{E}_A \qquad (31.7)$$



The Biot Savart law and rotation

Here the Coulomb force would be a E𝔸/+𝕠d potential impulse of the positive charge's motion. That changes into a rotational reference frame, if the charge was moving parallel to the magnet it would remain in a straight-line reference frame. By rotating this becomes -𝕆D×e𝕪 kinetic work which uses the Biot Savart law. In (31.6) this uses a vector r̂ to give the direction of a charge, that can be moving at an angle which gives a rotational reference frame.

Thus we find the remarkable idea that **the Biot-Savart law for the magnetic field of a moving point charge is nothing other than the Coulomb electric field of a stationary point charge transformed into a moving reference frame.**

We will assert without proof that if the experimenters in frame A create a magnetic field $\vec{B}_A$ in addition to the electric field $\vec{E}_A$, then the magnetic field $\vec{B}_B$ is

$$\vec{B}_B = \vec{B}_A - \epsilon_0 \mu_0 \vec{v}_{BA} \times \vec{E}_A \qquad (31.8)$$

This is a general transformation matching Equation 31.4 for the electric field $\vec{E}_B$.

ϵ and μ as distance and time

In this model ϵ and μ combine together to give s^2/-𝕚d×e𝕧/-𝕀D, then ϵ can be regarded as s^2 or E𝕍 as a length squared. That makes $1/m^2$ as $1/seconds^2$ which would be μ. When the square root is taken these are two squared constant forces, that gives c. From e $e=mc^2$ that gives e=m×μ×ϵ. These squared forces give work and impulse in this model. Taken as ϵ×μ=E𝕍/-𝕀D multiplying this by the mass m as the -𝕚d inertial mass this is the ½×e𝕍/-𝕀d ×-𝕚d linear inertia.

Newtons and inverted Newtons

In this model energy is not used except as an approximation, ϵ as E𝕍 comes from the E𝕍/-𝕚d inertial impulse and μ as 1/-𝕀D comes from -𝕆D×e𝕪 kinetic work. A Tesla reduces to Newtons which is f=ma, that is -𝕚d×e𝕧/-𝕆D as -𝕀D×e𝕧 inertial work. With ϵ the units are divided by Newtons 1/(-𝕠d ×e𝕪/-𝕆D) as the force. The inverse of Newtons would be impulse, that becomes e𝕪×E𝕐/-𝕠d. Now the -𝕚d inertial mass has become e𝕪 as temperature or an electric charge from Coulombs. The E𝕐 kinetic displacement is a square as ϵ, from Newtons 1/-𝕆D is μ.

Notice something interesting. The constant μ_0 has units of T m/A; those of ϵ_0 are $C^2/N\,m^2$. By definition, 1 T = 1 N/A m and 1 A = 1 C/s. Consequently, the units of $\epsilon_0\mu_0$ turn out to be s^2/m^2. In other words, the quantity $1/\sqrt{\epsilon_0\mu_0}$, with units of m/s, is a speed. But what speed? The constants are well known from measurements of static electric and magnetic fields, so we can compute

$$\frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{1}{\sqrt{(8.85 \times 10^{-12}\ C^2/N\,m^2)(1.26 \times 10^{-6}\ T\,m/A)}} = 3.00 \times 10^8\ m/s$$

Not using $1/c^2$

In this model 1/1/(√ϵ×√μ) would not be used as ϵ×μ, the E⃗ electric charge's straight-line reference frame would be impulse as a derivative fraction. The B⃗ magnetic rotational reference frame would be work as a multiplied integral.

Of all the possible values you might get from evaluating $1/\sqrt{\epsilon_0\mu_0}$, what are the chances it would turn out to be *c*, the speed of light? It is not a random coincidence. In Section 31.5 we'll show that electric and magnetic fields can exist as a *traveling wave*, and that the wave speed is predicted by the theory to be none other than

$$v_{em} = c = \frac{1}{\sqrt{\epsilon_0\mu_0}} \qquad (31.9)$$

For now, we'll go ahead and write $\epsilon_0\mu_0 = 1/c^2$. With this, our **Galilean field transformation equations** are

$$\vec{E}_B = \vec{E}_A + \vec{v}_{BA} \times \vec{B}_A$$
$$\vec{B}_B = \vec{B}_A - \frac{1}{c^2}\vec{v}_{BA} \times \vec{E}_A \qquad (31.10)$$

Reference frames are not at rest

In this model two reference frame cannot be entirely at rest with each other. Each corresponds to one of four main reference frames, as the potential, kinetic, gravitational and inertial. Because each has a time component, for example the gravitational reference frame has +𝕚d spin, moving to a second gravitational reference frame has a different gravitational time.

Reference frames cannot share an axis

Two +𝕚d and e𝕙 Pythagorean Triangles cannot share a Pythagorean Triangle side, they also could not have the same e𝕙 height. Moving from one to another requires +𝕀D×e𝕙 gravitational work or a Eℍ/+𝕚d gravitational impulse.

A macro body is not a single reference frame

There would not be a single reference frame for a planet, as an example. Instead that is made up of many +𝕚d and e𝕙 Pythagorean Triangles where each is also a gravitational reference frame. Each Pythagorean Triangle has two different aspects, $\vec{E}$ is like the e𝕙 height and $\vec{B}$ is like +𝕚d as the gravitational field.

where $\vec{v}_{BA}$ is the velocity of reference frame B relative to frame A and where, to reiterate, the fields are measured *at the same point in space* by experimenters *at rest* in each reference frame.

NOTE We'll see shortly that these equations are valid only if $v_{BA} \ll c$.

We can no longer believe that electric and magnetic fields have a separate, independent existence. Changing from one reference frame to another mixes and rearranges the fields. Different experimenters watching an event will agree on the outcome, such as the deflection of a charged particle, but they will ascribe it to different combinations of fields. Our conclusion is that **there is a single electromagnetic field that presents different faces, in terms of $\vec{E}$ and $\vec{B}$, to different viewers.**

Negative charges with special relativity

In this model negative charges would correspond with special relativity, positive charges would correspond to general relativity. In the equations below, $\vec{E}_A$ has 1/ϵ, this corresponds to the E𝕐/-𝕠d kinetic impulse for the negative charge. $\vec{B}_A$ has μ in the numerator, that gives -𝕆D×e𝕪 kinetic work so the two together give the $e=mc^2$ energy equation. When ĵ and k̂ are orthogonal they correspond to the different -𝕠d and e𝕪 Pythagorean Triangle sides.

Removing ϵ or μ

In transforming from one to another an equation may have 1/ϵ and μ as an approximation. A transformation from $\vec{B}_A$ to $\vec{B}_B$ is from one rotational reference frame to another. That can be with a torque, a probability, or a difference as voltage. In (31.11) 1/(ϵ×μ) is $1/c^2$, the subtracted part of the equation has 1/ϵ so removing this leaves μ for that part. This leaves two rotational reference frames each with the squared constant μ. Removing one of these changes the equation away from energy to work or impulse.

Combining ϵ and μ

When there is a transformation from a straight-line to a rotational reference frame, or vice versa, then both 1/ϵ and μ can be combined in the same equation. The fraction of each can depend approximately on the angle between the reference frames. When this is 90° then for example 1/ϵ might be removed so that there is no E𝕐/-𝕠d kinetic impulse as a square. Alternatively μ might be removed so there is no longer -𝕆D×e𝕪 kinetic work.

Changing from impulse to work as a squared factor

The distance between the two charges is r^2, that would come from the E𝕐/-𝕠d kinetic impulse. Instead, there might be -𝕆D as the squared time in between the reference frames, that would convert it to an approximate shared rotational reference frame.

Almost Relativity

FIGURE 31.9a shows two positive charges moving side by side through frame A with velocity $\vec{v}_{CA}$. Charge q_1 creates an electric field and a magnetic field at the position of charge q_2. These are

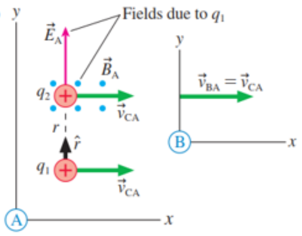
$$\vec{E}_A = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{j} \quad \text{and} \quad \vec{B}_A = \frac{\mu_0}{4\pi}\frac{q_1 v_{CA}}{r^2}\hat{k}$$

where r is the distance between the charges, and we've used $\hat{r} = \hat{j}$ and $\vec{v} \times \hat{r} = v\hat{k}$.

How are the fields seen in frame B, which moves with $\vec{v}_{BA} = \vec{v}_{CA}$ and in which the charges are at rest? From the field transformation equations,

$$\vec{B}_B = \vec{B}_A - \frac{1}{c^2}\vec{v}_{BA} \times \vec{E}_A = \frac{\mu_0}{4\pi}\frac{q_1 v_{CA}}{r^2}\hat{k} - \frac{1}{c^2}\left(v_{CA}\hat{\imath} \times \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{j}\right)$$
$$= \frac{\mu_0}{4\pi}\frac{q_1 v_{CA}}{r^2}\left(1 - \frac{1}{\epsilon_0\mu_0 c^2}\right)\hat{k} \qquad (31.11)$$

FIGURE 31.9 Two charges moving parallel to each other.



Fields seen in frame A

At rest

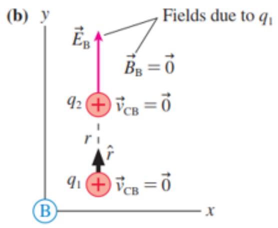
At rest can refer to no change in position or time. When the positions is not changing there is no work, when the time is not changing there is no impulse. Each measurement and observation is limited by the uncertainty principle. Here E⃗$_B$ has only 1/ϵ so it is the E𝕐/-𝕠d kinetic impulse from the Coulomb force.

where we used $\hat{\imath} \times \hat{j} = \hat{k}$. But $\epsilon_0\mu_0 = 1/c^2$, so the term in parentheses is zero and thus $\vec{B}_B = \vec{0}$. This result was expected because q_1 is at rest in frame B and shouldn't create a magnetic field.

The transformation of the electric field is similar:

$$\vec{E}_B = \vec{E}_A + \vec{v}_{BA} \times \vec{B}_A = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{j} + v_{BA}\hat{\imath} \times \frac{\mu_0}{4\pi}\frac{q_1 v_{CA}}{r^2}\hat{k}$$
$$= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}(1 - \epsilon_0\mu_0 v_{BA}^2)\hat{j} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\left(1 - \frac{v_{BA}^2}{c^2}\right)\hat{j} \qquad (31.12)$$

where we used $\hat{\imath} \times \hat{k} = -\hat{j}$, $\vec{v}_{CA} = \vec{v}_{BA}$, and $\epsilon_0\mu_0 = 1/c^2$. FIGURE 31.9b shows the charges and fields in frame B.



Fields seen in frame B

But now we have a problem. In frame B where the two charges are at rest and separated by distance r, the electric field due to charge q_1 should be simply

$$\vec{E}_B = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{j}$$

The Pythagorean Triangles and relativity

In this model the Pythagorean Triangles conform to general and special relativity, when the E𝕍/-𝕚d inertial impulse increases there is a -𝕚d inertial time slowing. When -𝕀D×e𝕧 inertial work increases there is a e𝕧 length contraction. When the Eℍ/+𝕚d gravitational impulse increases there is a +𝕚d gravitational time slowing, such as near an event horizon. When +𝕀D×e𝕙 gravitational work increases there is a e𝕙 height contraction.

The field transformation equations have given a "wrong" result for the electric field $\vec{E}_B$.

It turns out that the field transformations of Equations 31.10, which are based on Galilean relativity, aren't quite right. We would need Einstein's relativity—a topic that we'll take up in Chapter 36—to give the correct transformations. However, the Galilean field transformations in Equations 31.10 are equivalent to the relativistically correct transformations when $v \ll c$, in which case $v^2/c^2 \ll 1$. You can see that the two expressions for $\vec{E}_B$ do, in fact, agree if v_{BA}^2/c^2 can be neglected.

Thus our use of the field transformation equations has an additional rule: Set v^2/c^2 to zero. This is an acceptable rule for speeds $v < 10^7$ m/s. Even with this limitation, our investigation has provided us with a deeper understanding of electric and magnetic fields.

The loop moves in a straight-line reference frame

In the diagram the loop moves with a straight-line reference frame. That is with an E𝕍/-𝕚d inertial impulse, against this is a E𝔸/+𝕠d potential impulse where the protons in the loop react against the kinetic current. Observing the magnet under the loop, this has no magnetic field because that is only in a rotational reference frame. It would move opposite the loop with an E𝕍/-𝕚d inertial impulse.

The magnet and loop have a rotational reference frame

In the rotational reference frame of the magnet, the loop is also measured as a rotational reference frame. The motion of the loop cannot be measured as this is a straight-line reference frame or velocity. The magnet's rotational reference frame can measure rotating electrons in the loop as a +𝕆D potential and -𝕆D kinetic voltage or difference. It cannot observe electrons as particles, only as probabilities and the torque in the loop.

The change in the magnetic fields cannot be measured

The change in the magnetic fields cannot be measured, that needs a straight-line reference frame with respect to time. This change can happen with impulse, if the magnet is an electromagnet, it can be switch on. Then there is a e𝕪/-𝕠d kinetic current because there was a before the closed switch and after. That gives a straight-line reference frame with a change in the E𝕐 kinetic displacement of the electrons.

Observing the electrons in the straight wire

In the rotational reference frame of the loop, the magnet has a -𝕆D magnetic field from the electron spins. It cannot observe the magnet moving backwards under the loop as it moves to the right. In the straight-line reference frame of the loop, the electrons can be observed to move along a straight-line segment of it.

A current is a derivative fraction

These move with a e𝕪/-𝕠d kinetic current, the -𝕆D kinetic difference is not measurable in this reference frame. This can only happen with impulse because e𝕪/-𝕠d is a fraction and derivative. With work there is only -𝕠d×e𝕪 which as an integral does not change with time.

The loop and magnet have a torque

The loop and magnet have a torque in them, the -𝕆D kinetic torque comes from the magnet and this is reacted against by a +𝕆D potential torque. This could also occur if the magnet was an electromagnet being switched on. That changes the -𝕆D kinetic probabilities in the magnet, also the +𝕆D potential probabilities in the loop. The change over time as the electromagnet is switched on, that cannot be measured in the rotational reference frame.

Faraday's law

This is then Faraday's law, switching on an electromagnet, or closing a switch, and getting a current in a circuit. When the switch is on for a while the straight-line reference frame of the switch is unchanging, it is increasingly a rotational reference frame with a change. That longer time as -𝕠d means the -𝕆D×e𝕪 kinetic work in the circuit is stronger and the derivative e𝕪/-𝕠d kinetic current stops.

Bar magnet in a coil

The same can happen with a bar magnet moving in a coil, there is -𝕆D×e𝕪 kinetic work done in the coil with a rotational reference frame. This cannot measure the motion of the bar magnet back and forth. If the magnet stops moving the straight-line reference frame is not changing, then the -𝕆D×e𝕪 kinetic work in the coil also stops because the -𝕆D kinetic probabilities become constant there.

A battery on a circuit

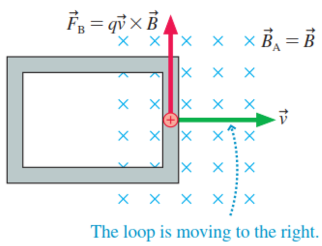
When there is a battery or capacitor hooked up to a circuit, but with a switch closed, there is only a rotational reference frame around it. The +𝕆D potential and -𝕆D kinetic difference in the circuit remains in the attached battery or capacitor. This is because they are unchanging, they represent a torque in the circuit.

Closing the switch

When the switch is closed these differences convert into a straight-line reference frame, as the e𝕪/-𝕠d kinetic current moves through the circuit with a E𝕐/-𝕠d kinetic impulse it decreases. When the battery or capacitor are depleted, then there are no changes, the +𝕆D potential and the -𝕆D kinetic difference or torque is constant.

FIGURE 31.10 A motional emf as seen in two different reference frames.

(a) Laboratory frame A



Faraday's Law Revisited

The transformation of electric and magnetic fields gives us new insight into Faraday's law. **FIGURE 31.10a** shows a reference frame A in which a conducting loop is moving with velocity $\vec{v}$ into a magnetic field. You learned in Chapter 30 that the magnetic field exerts a magnetic force $\vec{F}_B = q\vec{v} \times \vec{B} = (qvB, \text{upward})$ on the charges in the leading edge of the wire, creating an emf $\mathcal{E} = vLB$ and an induced current in the loop. We called this a *motional emf.*

How do things appear to an experimenter who is in frame B that moves with the loop at velocity $\vec{v}_{BA} = \vec{v}$ and for whom the loop is at rest? We have learned the important lesson that experimenters in different inertial reference frames agree about the outcome of any experiment; hence an experimenter in frame B agrees that there is an induced current in the loop. But the charges are at rest in frame B so there cannot be any magnetic force on them. How is the emf established in frame B?

We can use the field transformations to determine that the fields in frame B are

A change of E$^{\rightarrow}$

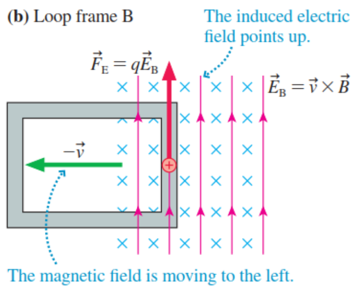
In (31.13) there is an initial state, that would be the rotational reference frame shown here as E$^{\rightarrow}$. That can mean the distances over which the work is measured. Then there is a change of E𝕐/-𝕠d kinetic impulse, that comes from v here as a derivative velocity. This is a straight-line reference frame, that can come from B$^{\rightarrow}$ which is moving a magnet to a different position. That changes the -𝕆D kinetic probabilities of where electrons are. After this motion there is again a rotational reference frame.

Special and general relativity

The second equation would be from the rotational reference frame, this cannot measure a change in time. It also cannot refer to $1/c^2$ as that would be a squared velocity in a straight-line reference frame. It does use c^2 but as an integral, in this model special relativity is in a straight-line reference frame and general relativity is in a rotational reference frame.

An electric field as impulse

The loop's reference frame is ambiguous here, the electrons can be observed moving in the straight line segments of it. That is referred to here as an induced electric field, in this model it would be a E𝕐/-𝕠d kinetic impulse of the moving electrons as particles not a field. A magnetic field cannot change in this model, that is because measuring it cannot occur over time but of distance. The electric field here is an electric displacement as impulse.



$$\vec{E}_B = \vec{E}_A + \vec{v}\times\vec{B}_A = \vec{v}\times\vec{B}$$
$$\vec{B}_B = \vec{B}_A - \frac{1}{c^2}\vec{v}\times\vec{E}_A = \vec{B} \tag{31.13}$$

where we used the fact that $\vec{E}_A = \vec{0}$ in frame A.

An experimenter in the loop's frame sees not only a magnetic field but also the electric field $\vec{E}_B$ shown in **FIGURE 31.10b**. The magnetic field exerts no force on the charges, because they're at rest in this frame, but the electric field does. The force on charge q is $\vec{F}_E = q\vec{E}_B = q\vec{v}\times\vec{B} = (qvB, \text{upward})$. This is the same force as was measured in the laboratory frame, so it will cause the same emf and the same current. The outcome is identical, as we knew it had to be, but the experimenter in B attributes the emf to an electric field whereas the experimenter in A attributes it to a magnetic field.

Field $\vec{E}_B$ is, in fact, the *induced electric field* of Faraday's law. Faraday's law, fundamentally, is a statement that **a changing magnetic field creates an electric field.** But only in frame B, the frame of the loop, is the magnetic field changing. Thus the induced electric field is seen in the loop's frame but not in the laboratory frame.

The potential Gaussian surface

The Gaussian surface around a proton would do +𝕆D×e𝕒 potential work, the electric field lines would be the e𝕒 altitude the work is measured with. This is not actually doing work unless there is a change in relation to other iotas, such as an electron in an orbital. The +𝕆D×e𝕒 potential work is not changing over time, only over a distance as an inverse square law. To induce an electric current there would be a E𝔸/+𝕠d potential impulse, that changes over +𝕠d potential time.

A moving flux as impulse

Inside the Gaussian surface there is a rotational reference frame. When electrons enter this, such as in electron orbitals, then they are also in rotational reference frames as waves. A flux can only move through this surface with a straight-line reference frame, then it can change over time with impulse.

The line integral

In (31.14) the line integral would be from this +𝕆D×e𝕒 potential work, the e𝕪 kinetic electric charge would be the boundary of the +𝕆D potential probability or torque. This would be where the electron was in its orbital, that gives +𝕆D×e𝕪 as the line integral. Contained inside the line integral is a rotational reference frame which is not changing.

Through the line integral

Here the force is 1/ϵ which is E𝔸 from the E𝔸/+𝕠d potential impulse. This would be from the straight-line reference frame at 90° to the rotational reference frame. This has a squared displacement force as 1/ϵ. When this refers to flux moving through the line integral, then it is changing over time. This is conserved because in a given time the same number of electrons would be observed going into the line integral as would be coming out.

The net change in a rotational reference frame

The net change over the line integral is zero, from the rotational reference frame at 90° the electrons are only measured as probabilities that do not change. The field lines can be loops in the rotational reference frame, leaving and entering so there is not change over time with impulse.

The charge Q

With a -𝕠d×e𝕪/-𝕠d kinetic momentum of the electrons in Coulombs, this is a combination of a -𝕠d×e𝕪 rotational reference frame and at 90° a straight-line e𝕪/-𝕠d reference frame. This allows for momentum to not change like a rotational reference frame, such as with a rolling wheel. It can also change by rolling in a straight-line reference frame. It then represents two reference frames with no forces at 90° to each other.

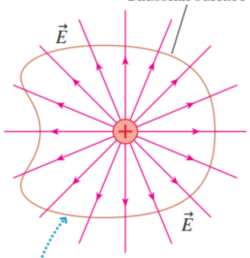
Three reference frames with the wavelength

To that can be added a third straight-line reference frame, a spot on the rolling wheel can be the end of the e𝕪 spoke as it rotates. This can extend between two crests of the sine wave being drawn as the wavelength. This is unchanging in the rotational reference frame as the frequency of the rolling wheel axle is not changing. It is changing in the straight-line reference frame as the wheel draws new wavelengths as it moves over time.

deBroglie wavelengths

An electron in an orbital can have deBroglie wavelengths in this straight-line reference frame, that means the electron has the same wavelength at different times. The electron can have a -𝕠d×e𝕪/-𝕠d kinetic momentum where it rotates as a rolling wheel with the rotational reference frame. The -𝕠d kinetic axle is at 90° to the wavelength. It can also move with a straight-line reference frame around the orbital, that can be observed with h as -𝕠d ×e𝕐/-𝕠d with a discrete spectrum.

FIGURE 31.11 A Gaussian surface enclosing a charge.



31.2 The Field Laws Thus Far

Let's remind ourselves where we are in terms of discovering laws about the electromagnetic field. Gauss's law, which you studied in Chapter 24, states a very general property of the electric field. It says that charges create electric fields in such a way that the electric flux of the field is the same through *any* closed surface surrounding the charges. **FIGURE 31.11** illustrates this idea by showing the field lines passing through a Gaussian surface enclosing a charge.

The mathematical statement of Gauss's law for the electric field says that for any *closed* surface enclosing total charge Q_{in}, the net electric flux through the surface is

$$(\Phi_e)_{\text{closed surface}} = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0} \tag{31.14}$$

The circle on the integral sign indicates that the integration is over a closed surface. Gauss's law is the first of what will turn out to be four *field equations*.

There's an analogous equation for magnetic fields, an equation we implied in Chapter 29—where we noted that isolated north or south poles do not exist—but didn't explicitly write it down. **FIGURE 31.12** shows a Gaussian surface around a magnetic dipole. Magnetic field lines form continuous curves, without starting or stopping, so every field line leaving the surface at some point must reenter it at another. Consequently, the net magnetic flux over a *closed* surface is zero.

Magnetic fields in rotational reference frames

Here magnetic fields are loops so they are in rotational reference frames. Starting and stopping refers to time, that is in a straight-line reference frame at 90° on an electric charge. There are no north and south poles in this model, an -𝕠d and e𝕪 Pythagorean Triangle electron has only one spin Pythagorean Triangle side. This can be viewed from above or underneath as a north and south pole. If a top could only spin clockwise, then its spin from above would seem always different from its spin from underneath.

A change in between two rotational reference frames

In (31.15) the area A does not change as this is an integral field. In (31.16) Faraday's law is a change with respect to time which would be in a straight-line reference frame. In this model a switch would change in a straight-line reference frame. That would be in between two rotational reference frames as magnetic fields. Here $d\Theta_m$ would be a change in the rotational reference frame integral, from one magnetic field to another.

We've shown only one surface and one magnetic field, but this conclusion turns out to be a general property of magnetic fields. Because every north pole is accompanied by a south pole, we can't enclose a "net pole" within a surface. Thus Gauss's law for magnetic fields is

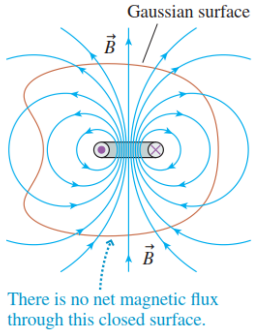
$$(\Phi_m)_{\text{closed surface}} = \oint \vec{B} \cdot d\vec{A} = 0 \quad (31.15)$$

Equation 31.14 is the mathematical statement that Coulomb electric field lines start and stop on charges. Equation 31.15 is the mathematical statement that magnetic field lines form closed loops; they don't start or stop (i.e., there are no isolated magnetic poles). These two versions of Gauss's law are important statements about what types of fields can and cannot exist. They will become two of Maxwell's equations.

The third field law we've established is Faraday's law:

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt} \quad (31.16)$$

FIGURE 31.12 There is no net flux through a Gaussian surface around a magnetic dipole.



Sign conventions

The sign convention here is positive when the +𝕠d and e𝕒 Pythagorean Triangle protons are greater in number. They are negative when the -𝕠d and e𝕪 Pythagorean Triangle electrons are greater. When a magnetic field changes it does so only over a distance, the -𝕆D kinetic probability density would be larger closer to a magnet.

Measuring reality only as fields

Much of the world can be explained only in terms of +𝕀D×e𝕙 gravitational work and -𝕀D×e𝕧 inertial work in Biv space-time. Then there is only a present with no past and future.

Quantum field theory

Where there are objects, they can be regarded as more dense fields like in quantum field theory. These are quantized so that the objects are solid, they resist other objects going through these quantum states. This ignores motion which would be in straight line reference frames only. A quantized level is like a plane or wall, a plane wave is also like an integral area.

Flat surfaces and gradients

Under +𝕀D×e𝕙 gravitational work a liquid would have a flat surface like an integral. The atoms would have -𝕆D probability densities of electrons, without motion they would have stable molecular bonds between the atoms. Gases would be stable in equilibrium. The +𝕀D×e𝕙 gravitational work of a planet would be like a +𝕀D gravitational gradient, this becomes denser as the e𝕙 height decreases around the planet to form a geodesic.

No need for a past and future

When there are changes, these can be interpreted without needing a past and future. Then the present changes the positions of where the Pythagorean Triangles probably are. There is no time, as the present does not allow for the passage of time to be observed on a clock gauge. Instead, the probabilities change when measured at points or positions.

Velocity over time or distance

A velocity can be regarded as occurring over a distance or of time, it can be meters/second or seconds/meter. Light years are the time light takes to travel a distance, that defines it as e𝕧/-𝕚d inertial velocity. But it could also be defined as the distance light travels in a year, then discard the idea of a year. This might be called a light reay as the opposite of a year. Then cosmology can be measured in light reays everywhere, they would contract around event horizons. In special relativity only the e𝕧 length contraction would be measured not the -𝕚d inertial time slowing.

Riding a bike at different velocities

If someone rides a bike from a first to a second position, then this can be measured only as a distance. If they road faster then they would accelerate more quickly initially. That can be fully described as a stronger -𝕆D kinetic torque on their wheels and then when stopping a reversed torque. That makes different accelerations distinguishable, that is because the work is always the inverse of the impulse. It can be more cumbersome, but with some uncertainty the description of the universe would be similar.

Motion as movie frames

Representing motion as a series of integrals, in rotational reference frames, there can be a series of movie frames. These are like quantization in that they show no motion in a frame, some might be blurred indicating uncertainty or changing probability densities. A journey on a bike would be measured as these stopped rotational movie frames. By removing continuous motion then changes over time can be measured as quantized intervals of distance.

A strobe showing rotational reference frames

For example if people were looking at a disco with a strobe light they would see only stationary events. The atoms composing these events would be in different positions like the bike's motion. Each would have some uncertainty, and a probability density, as the motion in between the rotational frames occurred.

Quantized levels like movie frames

This is like in quantum mechanics where e𝕪×-𝕘d photons are emitted and absorbed only in quantized levels, there is no continuous change in between them and so work can describe them. The -𝔾D×e𝕪 light work is emitted and absorbed with -𝔾D light probabilities, the rotational movie frames are not showing objects but probability densities. It is not known exactly where these atoms where, the space in between the frames allows many probable paths they could have followed.

Least action and least time

These are like the path integrals, each path has a probability of occurring. Many of these pair up on either side of the path of least action, then they cancel out destructively to leave a single path. Here the least action is like the shortest path while impulse would observe the least time between objects moving.

Path integrals

However one of these destructively interfered paths could still have occurred with a given probability. These were regarded by Feynman as being probable paths between double slits, as more and more of them were added between a light source and a screen. In describing Roy electromagnetism only with rotational reference frames, electrons move in between orbitals which are like these movie frames. What they do in between them is only measured with probabilities and interference.

Emitting and absorbing photons as work

E𝕪×-𝕘d photons are emitted and absorbed between these atoms, which are themselves only described as concentrations of probability densities like in quantum field theory. The photons move between these atoms as a distance in reays, no time elapses for them in their rotational reference frames. In measuring a photon, it appears stationary as a vortex or spinning wheel. Its position can be measured on a Gaussian curve.

Fitting together movie frames

This is why, according to this model, photons do not change over long distances. There is no time to change when they are measured by their emission and absorption. Measuring the universe in this way requires putting movie frames in a row, it appears to take up more space than reality does. In this model this is because the 90° straight-line reference frame is removed. Then there are only integral fields, the passage of time is represented by straight lines of these movie frames.

In between the movie frames is impulse

To add the straight-line reference frame the spin Pythagorean Triangle sides must be observed on a clock gauge. Then the rotational reference frame moves without forces as the passage of time. That happens in between the movie frames so some events become continuous, this allows for e𝕪/-𝕘d photons to have a continuous spectrum.

Small distances and rotational reference frames

In this model small distances are mainly measured by work, there is little impulse. The universe described by work only, as only a rotational reference frame, is how these small distances operate. This leads to unusual phenomena with electric charges and magnetic field, while in the macro world they change over time over small distances they are mainly field densities.

Small distances require rotational reference frames

This is why atoms have electrons around them in orbitals, because they are small. In Biv space-time stars have planets around them because these distances are small compared to the distance to the CMB.

Turning fields into particles to observe them

In quantum mechanics field densities and quantization dominates, they need to be perturbed with observations to turn them temporarily into particles to be observed. In general relativity, gravity is best described by a geodesic of curved lines. That gives no straight-line reference frame.

Problematic terms and reference frames

With the differences between the rotational and straight-line reference frames, some terms become problematic. Using the word "because" implies a cause-and-effect relationship over time. That happens only over time in a deterministic way with no probability.

Logic is deterministic

Logic is derived from this cause and effect, quantum logic is where probabilities can measure the same events but in a rotational reference frame. To measure events in work the concepts of before and after, therefore, must occur, etc should be avoided. If they become necessary that indicates a straight-line reference frame is needed, and a change of 90° to the other Pythagorean Triangle sides.

Newtonian physics

To represent the universe only in straight-line reference frames, this gives the clockwork universe of Newtonian physics. There are no fields, particles can be observed at the same instant of time while the positions are uncertain as a displacement. This is because there is no quantization, particles can collide, a point between them is common to both at the same time. In the rotational reference frame at 90° these are separated by a quantized distance, the time is uncertain as a probability.

Gravity was not observable

Only having straight-line reference frames led to problems even in Newton's time. Planets moved in circular and elliptical orbits, there seemed to be something between them and a star that was not observable. It was thought gravity was an instantaneous force, but how it acted at a distance was not understood. Light seemed to be like particles according to Newton, later Young showed that they could diffract like waves.

The infrared catastrophe

As electrons collide, they can emit a continuous spectrum of e𝕪/-𝕘d photons, this led to the ultraviolet catastrophe with blackbody radiation and quantization. Conversely describing a blackbody only with rotational reference move frames could be regarded as an infrared catastrophe. The quantized levels would seem to disappear and the spectrum would become continuous.

A changing electric field

Because of this a changing electric field is problematic according to this model. It spans two kinds of reference frames, it says that electric fields in rotational reference frames can change over time. But in this model only particles can change over time, and only in straight-line reference frames at 90°.

> where the line integral of $\vec{E}$ is around the closed curve that bounds the surface through which the magnetic flux Φ_m is calculated. Equation 31.16 is the mathematical statement that an electric field can also be created by a changing magnetic field. The correct use of Faraday's law requires a convention for determining when fluxes are positive and negative. The sign convention will be given in the next section, where we discuss the fourth and last field equation—an analogous equation for magnetic fields.

Changing integrals and voltage

In the diagram there is a +𝕆D potential probability or difference around the wires, also a -𝕆D inertial probability or difference to give a current. That is in the rotational reference frame, it can be regarded as integral area like in the diagram. These change in between the terminals of a battery or capacitor, closer to the negative terminal or plate there is a higher -𝕆D kinetic difference. Closer to the positive terminal or plate there is a higher +𝕆D potential difference.

Impulse and current

The line integrals are rotational reference frames, at 90° to that is the straight-line reference frame of the current. This would have a E𝔸/+𝕠d potential impulse towards the positive terminal of a battery, and a E𝕐/-𝕠d kinetic impulse towards the negative terminal of a battery.

Movie frames of voltages

The line integral in (31.17) has μ as the squared constant, that means the current is moved with +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work from these rotational reference frames. They can be regarded like the movie frames, each has a different +𝕆D potential and -𝕆D kinetic probability of where the electrons are measured in it.

Displacement in between rotational movie frames

A displacement can occur from one rotational movie reference frame to the next with the E𝔸/+𝕠d potential impulse and E𝕐/-𝕠d kinetic impulse. This model is consistent with using a battery or capacitor, also with turning on an electromagnet or moving a bar magnet in a coil. Turning on and off a switch can also change these rotational reference movie frames of voltages, in between at 90° there is a change in the displacement of the electrons.

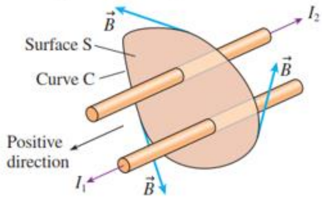
31.3 The Displacement Current

We introduced Ampère's law in Chapter 29 as an alternative to the Biot-Savart law for calculating the magnetic field of a current. Whenever total current $I_{through}$ passes through an area bounded by a closed curve, the line integral of the magnetic field around the curve is

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{through} \qquad (31.17)$$

FIGURE 31.13 illustrates the geometry of Ampère's law. The sign of each current can be determined by using Tactics Box 31.1. In this case, $I_{through} = I_1 - I_2$.

FIGURE 31.13 Ampère's law relates the line integral of $\vec{B}$ around curve C to the current passing through surface S.



The wire turn as a rotational reference frame

In the diagram the wire turns, that gives a rotational reference frame. The closed surface on one side would have a higher -𝕆D kinetic probability or distance than the other, also a higher -𝕆D kinetic torque with the turn.

The wire turn as a rotational reference frame

The current is in a straight-line reference frame only, in the middle of the turn it has changed by 90° so there is a rotational reference frame component. That is because there can be three orthogonal reference frames through a point or instant. The turn then gives an additional rotational reference frame, this allows for a magnetic field to turn the current around with a torque.

The Archimedean screw

An analogy for voltage would be an Archimedean screw where the torque of its rotation is the voltage moving the water like electrons with work. There is a rotational reference frame moving the water forward with torque, at 90° the water moves with a straight line reference frame as a velocity or current.

Movie frame turning with the wire

In the closed curve the net distance is canceled out as e𝕒, so the +𝕆D×e𝕒 potential work done in this segment is zero. Each part of the curved volume can be regarded as a rotational movie reference frame, here they would curve as the wire turns instead of all being in straight lines. Each movie frame is at 90° to the straight-line reference frame the current movies in.

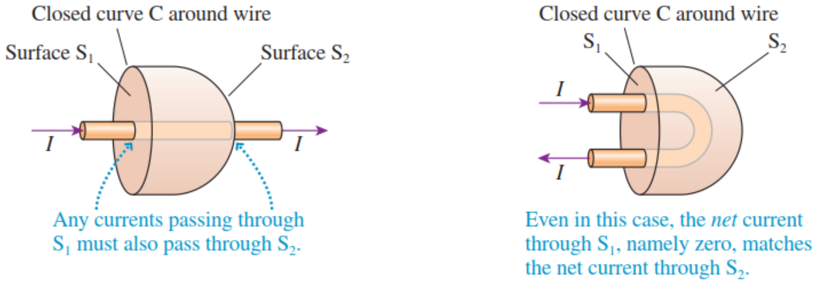
An additional torque

There is no need for an electric field changing with time, according to this model. The movie frames measure an additional torque at 90° to them, the third orthogonal direction. That would also be work, like a generator turning in a circle. A rotary pump works in a similar way, it turns water around in a circle in between an inlet and an outlet pipe.

Something Is Missing

Nothing restricts the bounded surface of Ampère's law to being flat. It's not hard to see that any current passing through surface S_1 in **FIGURE 31.14** must also pass through the curved surface S_2. To interpret Ampère's law properly, we have to say that the current $I_{through}$ is the net current passing through *any* surface S that is bounded by curve C.

FIGURE 31.14 The *net* current passing through the flat surface S_1 also passes through the curved surface S_2.



The space between the plates as work

In this model the charging occurs over time in a straight-line reference frame. At 90° to this from above there is a rotational reference frame. This covers the space between the plates. When the impulse changes over time in the straight reference frame, the +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work in the rotational reference frame changes its probability densities over positions. That occurs in between the plates as well as there is an integral field between them.

Work in between protons and electrons in an atom

It is like a Hydrogen atom, there is +𝕆D×e𝕒 potential work from the proton and -𝕆D×e𝕪 kinetic work from the electron as fields that do not change. This is also a gap that the probability densities can occupy as integrals.

Movie frames and charging

When the capacitor is charging in the straight-line reference frame, the rotational reference frame changes as a series of quantized movie like frames. In between these at 90° the straight-line reference frame charges continuously. The squared quantized values of +𝕆D and -𝕆D have a dual identity, they are a squared spin Pythagorean Triangle side but can also be a squared straight Pythagorean Triangle side.

Squares as both straight-line and rotational

That allows for a continuous change through the quantized levels with a E𝔸/+𝕠d potential impulse and E𝕐/-𝕠d kinetic impulse. When it passes through the quantized values it can only do so as a displacement, so -𝕆D and E𝕐 can have E=D, +𝕆D and E𝔸 can have E=D as well.

The squares remain separate

However the quantized values in the rotational reference frame cannot be measured in the straight-line reference frame as a displacement, and vice versa.

Sign addition and vector addition

The difference is in the signs, +𝕆D and -𝕆D can be added as plus and minus. E𝔸 and E𝕐 can be vector added, if they are opposed then the answer is the same as an absolute value. Also when they are at an angle with vector addition, this can be equivalent to adding the constructive or destructive interference between the probabilities as an inverse of this.

But this leads to an interesting puzzle. **FIGURE 31.15a** shows a capacitor being charged. Current *I*, from the left, brings positive charge to the left capacitor plate. The same current carries charges away from the right capacitor plate, leaving the right plate negatively charged. This is a perfectly ordinary current in a conducting wire, and you can use the right-hand rule to verify that its magnetic field is as shown.

No volume around a wire

In this model there is no volume around the wire surrounding the wire and a plate. That would combine a straight-line reference frame and an integral at 90° to it. This can be done as a cell in the Pythagorean Triangle where a derivative and integral are multiplied together. That gives permutations and combinations, also coefficients and powers in formulae.

Integral movie frames between the plates

Instead, there is a straight-line reference frame in the wire, this terminates at the end of the plate. At 90° to this is the negative plate for example. This is an integral, in between this and the positive plate there would be integrals like movie frames that are each at 90° to the wire. In this space there is no wire, so there is no straight-line reference frame there unless there is a spark across the plates. The plates are part of a loop, that gives a +𝕆D potential and -𝕆D kinetic torque to the protons and electrons to move onto the plates.

Changing probabilities as the plates charge

When the plates are charging the E𝔸/+𝕠d potential impulse and E𝕐/-𝕠d kinetic impulse increases the number of -𝕠d and e𝕪 Pythagorean Triangle electrons on the negative plate. This leaves the positive plate with more +𝕠d and e𝕒 Pythagorean Triangle protons. As this happens there are changing +𝕆D potential and -𝕆D kinetic probabilities in the integrals at 90° to the wire.

Probabilities in the gap

The -𝕆D kinetic probability of measuring an -𝕠d and e𝕪 Pythagorean Triangle electron between the plates is approximately zero. This is because the E𝔸/+𝕠d potential impulse and E𝕐/-𝕠d kinetic impulse would require an electron to move there with a current. Instead, there is only a rotational reference frame, the probabilities are ≈ zero but there is still some chance of electrons tunneling through the gap.

Curve C is a closed curve encircling the wire on the left. The current passes through surface S_1, a flat surface across C, and we could use Ampère's law to find that the magnetic field is that of a straight wire. But what happens if we try to use surface S_2 to determine $I_{through}$? Ampère's law says that we can consider *any* surface bounded by curve C, and surface S_2 certainly qualifies. But *no* current passes through S_2. Charges are brought to the left plate of the capacitor and charges are removed from the right plate, but *no* charge moves across the gap between the plates. Surface S_1 has $I_{through} = I$, but surface S_2 has $I_{through} = 0$. Another dilemma!

Looking along the wire

In the rotational reference frame from above, there is a magnetic flux in between the plates. Measuring along the wire the rotational reference frame also appears at 90°, it is like a plate at different points along the wire. This terminates at the plates themselves, looking along the wire it appears as if there is a final rotational reference frame at the positive and negative plates.

In between the plates

In between the plates there can be more rotational reference frames as integrals at 90°, from looking down the wire. The -𝕆D kinetic probability of measuring an electron in the gap drops to ≈ zero as before. Because the electrons are only measured here as probability densities, not particles there is still some probability they will appear in the gap or tunnel across. That means there is a constructive interference in between the plates, the -𝕆D kinetic voltage on the negative plate appears to attract the +𝕆D potential voltage on the positive plate.

Voltage at a distance

This is like voltages in a battery making electrons move along a wire, even though the battery can be at a distance. It is also like electrostatic forces, such as with a van de Graff generator, causes someone's hair to stand up at a distance. The same happens with charging a glass rod with wool, then using it to pick up pieces of paper.

The voltage changes differently to the electron collisions

The rotational reference frames from the battery, charging the capacitor, still create a voltage even though the wire's electrons as particles are not colliding with the electrons in the negative terminal. That can also be accounted for by collisions in between electrons in the straight-line reference frame. However, the voltage moves the electrons faster than the collisions can occur.

An elongated loop

This can be shown with an elongated loop, a battery is hooked up to a loop that extends kilometers on both sides of it. From above the rotational reference frame is measured to be a large ellipse, the major axis is much larger than the minor axis. Opposite the battery there can be a lamp centimeters away. When the loop is connected to the battery the lamp lights before the electron collisions can get to the lamp in their straight-line reference frame.

A changing quantized gradient along the wire

The rotational reference frame from above would measure +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work moving across to the lamp. This occurs in quantized levels along the wires on either side. These appear as a +𝕆D potential and -𝕆D kinetic gradient that changes in these movie frames.

Moving across the gap between the battery and lamp

The work done moves at a limit of c as e𝕪×-𝕘d photons, they move straight over to the map in a straight-line reference frame with a e𝕧/-𝕚d inertial velocity of c. from above this appears as e𝕪×-𝕘d photons which are integrals, if they collide with the wire around the lamp then there would be a e𝕐/-𝕘d light impulse. More usually they would be absorbed into the wire around the map with -𝔾D×e𝕪 light work.

The plates attract each other

The plates would tend to come together if allowed as the capacitor charges. This is like in an atom where the constructive interference between the proton nucleus and electron leads to an attraction. There is no orbital here and so the two plates would attract each other. In an atom there is a -𝕆D kinetic torque where the electron orbits at a constant e𝕒 altitude. The elliptical wire would also tend to narrow the distance between the battery and lamp with constructive interference.

It would appear that Ampère's law is either wrong or incomplete. Maxwell was the first to recognize the seriousness of this problem. He noted that there may be no current passing through S_2, but, as **FIGURE 31.15b** shows, there is an electric flux Φ_e through S_2 due to the electric field inside the capacitor. Furthermore, this flux is *changing* with time as the capacitor charges and the electric field strength grows. Faraday had discovered the significance of a changing magnetic flux, but no one had considered a changing electric flux.

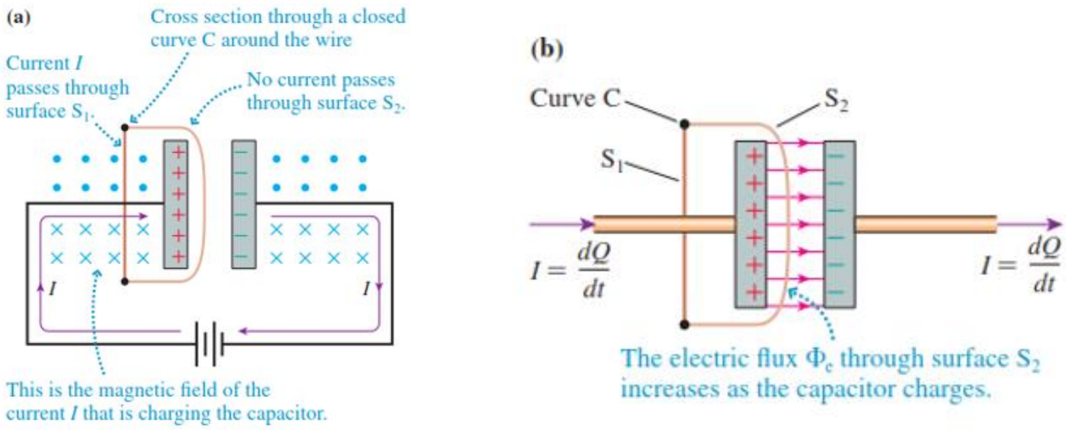
The current I passes through S_1, so Ampère's law applied to S_1 gives

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} = \mu_0 I$$

Torque and interference

In the diagram there is a rotational reference frame from above. The magnetic field points in and out of the page as a distance. Each would have a -𝕆D kinetic torque from the electrons acting as electromagnets. Looking along the wire there is a second rotational reference frame, the integral appears like a -𝕆D kinetic torque around the negative plate. Looking down the positive wire there would be an opposing +𝕆D potential torque around the plate. The two plates interfere constructively and so they are more likely to move towards each other.

FIGURE 31.15 There is no current through surface S_2 as the capacitor charges, but there is a changing electric flux.



Electric and magnetic flux

In the rotational reference frame Φ=EA becomes -𝕆D for the area A of the negative plate, this is multiplied by e𝕪 as a distance E. The electric flux then becomes the magnetic flux with work. The plate size doesn't change the -𝕆D×e𝕪 kinetic work here when there are a fixed number of electrons. They are spread out more but do the same -𝕆D×e𝕪 kinetic work over a distance e𝕪. This is because the -𝕆D kinetic probabilities add up with a constructive interference in the direction of e𝕪. They interfere destructively with each other, that makes them move apart with a repulsion.

ϵ and μ in different reference frames

In (31.18) the force is 1/ϵ in the straight-line reference frame, from above at 90° it is 1/μ in the rotational reference frame. Here 1/ϵ equals E𝕐 from the E𝕐/-𝕠d kinetic impulse, 1/μ equals -𝕆D from -𝕆D×e𝕪 kinetic work.

Q as two reference frames

The charge Q is the -𝕠d×e𝕪/-𝕠d kinetic momentum, that combines the straight-line reference frame with e𝕪/-𝕠d and the rotational reference frame with -𝕠d×e𝕪.

We believe this result because it gives the correct magnetic field for a current-carrying wire. Now the line integral depends only on the magnetic field at points on curve C, so its value won't change if we choose a different surface S to evaluate the current. The problem is with the right side of Ampère's law, which would incorrectly give zero if applied to surface S_2. We need to modify the right side of Ampère's law to recognize that an electric flux rather than a current passes through S_2.

The electric flux between two capacitor plates of surface area A is

$$\Phi_e = EA$$

The capacitor's electric field is $E = Q/\epsilon_0 A$; hence the flux is actually independent of the plate size:

$$\Phi_e = \frac{Q}{\epsilon_0 A}A = \frac{Q}{\epsilon_0} \tag{31.18}$$

The derivative and integral of the momentum

In (31.19) the derivative is with respect to time as 1/dt, that is in a straight-line reference frame so the squared force is 1/ϵ. With dQ/dt as a derivative, that is the -𝕠d×e𝕪/-𝕠d kinetic momentum which removes -𝕠d in the numerator to leave e𝕪/-𝕠d as the kinetic current. Dividing this by ϵ makes it E𝕐/-𝕚d as the kinetic impulse. Conversely taking the integral of the -𝕠d×e𝕪/-𝕠d kinetic momentum removes -𝕠d from the denominator to leave -𝕠d×e𝕪. Dividing this by μ gives e𝕪/-𝕆D as kinetic work, it does not change the value if -𝕆D is moved from the numerator to the denominator.

The *rate* at which the electric flux is changing is

$$\frac{d\Phi_e}{dt} = \frac{1}{\epsilon_0}\frac{dQ}{dt} = \frac{I}{\epsilon_0} \tag{31.19}$$

where we used $I = dQ/dt$. The flux is changing with time at a rate directly proportional to the charging current I.

Equation 31.19 suggests that the quantity $\epsilon_0(d\Phi_e/dt)$ is in some sense "equivalent" to current I. Maxwell called the quantity

$$I_{disp} = \epsilon_0 \frac{d\Phi_e}{dt} \tag{31.20}$$

Electrons as a compressible fluid

In this model electrons can act like a compressible fluid. They repel each other with a destructive interference, this moves them apart. With an atomic gas the collisions occur with an E𝕍/-𝕚d inertial impulse, when the atoms get close together the electrons in them repel each other with a -𝕆D destructive interference. Proportionally the atoms repel each other with a -𝕀D destructive interference as well.

No velocity with repulsion

This happens in a rotational reference frame, the e𝕪/-𝕠d kinetic velocity of the electrons and the gas are not measured here. Instead there is the -𝕆D kinetic and -𝕀D inertial destructive interference as a repulsion. Then the electrons and gas atoms can move in a straight-line reference frame as a current like a fluid.

Two reference frames combined

In (31.21) there are two reference frames combined, this is because the magnetic flux is added to Maxwell's electric flux. In this model that would have μ as the squared magnetic force in a rotational reference frame. That would be at 90° to the current and also at 90° to the displacement in between the capacitor plates and when the wire turns 180°.

Changing the force by 90°

Instead, the displacement force is changed to be ϵ, the electric flux Φ then is the E𝕐/-𝕠d kinetic impulse in the straight-line reference frame at 90°. The two forces combine so that the displacement force in between the plates moves at c.

the **displacement current.** He had started with a fluid-like model of electric and magnetic fields, and the displacement current was analogous to the displacement of a fluid. The fluid model has since been abandoned, but the name lives on despite the fact that nothing is actually being displaced.

Maxwell hypothesized that the displacement current was the "missing" piece of Ampère's law, so he modified Ampère's law to read

$$\oint \vec{B} \cdot d\vec{s} = \mu_0(I_{\text{through}} + I_{\text{disp}}) = \mu_0\left(I_{\text{through}} + \epsilon_0 \frac{d\Phi_e}{dt}\right) \qquad (31.21)$$

Separate reference frames

The two reference frames are used to give the gap between the capacitor plates. In this model the two reference frames remain separated, the gap is composed of integral fields of a rotational reference frame. These are like movie frames quantized at 90° to the current. When the current is not changing then this rotational reference frame gives a constant constructive interference between the plates.

Quantized movie frames

When the current is changing over time this gives rotational reference frames along the wire at different positions. These are different like quantized move frames. In between the plates these movie frames give changing probability densities. When the current changes several times the movie frames would appear as the different probabilities with e𝕒 and e𝕪 linear vectors between them giving the sequence in which they would occur. In the straight-line reference frame the current would be changing over time.

Equation 31.21 is now known as the Ampère-Maxwell law. When applied to Figure 31.15b, the Ampère-Maxwell law gives

$$S_1: \quad \oint \vec{B} \cdot d\vec{s} = \mu_0\left(I_{\text{through}} + \epsilon_0 \frac{d\Phi_e}{dt}\right) = \mu_0(I + 0) = \mu_0 I$$

$$S_2: \quad \oint \vec{B} \cdot d\vec{s} = \mu_0\left(I_{\text{through}} + \epsilon_0 \frac{d\Phi_e}{dt}\right) = \mu_0(0 + I) = \mu_0 I$$

where, for surface S_2, we used Equation 31.19 for $d\Phi_e/dt$. Surfaces S_1 and S_2 now both give the same result for the line integral of $\vec{B} \cdot d\vec{s}$ around the closed curve C.

Connecting the current in the gap

Here the displacement current as e𝕪/-𝕠d would be -𝕠d×e𝕪 as a kinetic field. It is continuous in the sense that changing 90° in the gap allows for the current to connect through it. This is not a flow of charge as a e𝕪/-𝕠d kinetic velocity, instead the change to a rotational reference frame occurs in the gap.

NOTE The displacement current I_{disp} between the capacitor plates is numerically equal to the current I in the wires to and from the capacitor, so in some sense it allows "current" to be continuous all the way through the capacitor. Nonetheless, the displacement current is *not* a flow of charge. The displacement current is equivalent to a real current in that it creates the same magnetic field, but it does so with a changing electric flux rather than a flow of charge.

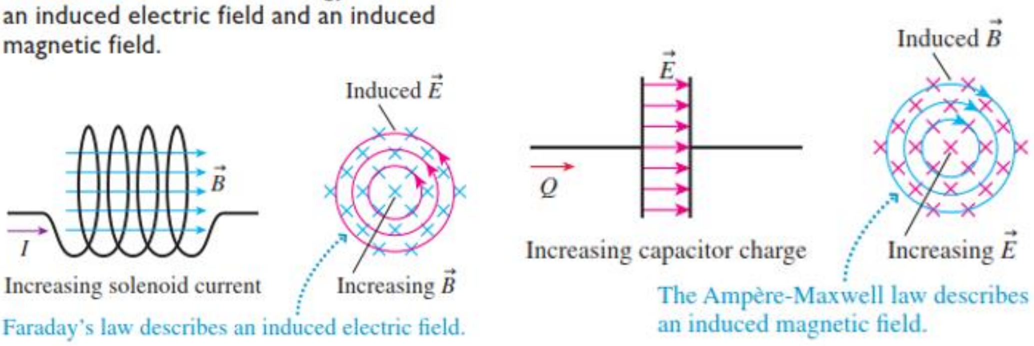
As if there were an electric flux

In the left diagrams B⃗ is like e𝕪 in the straight-line reference frame. At 90° to this there is the rotational reference frame referred to as E⃗ here. In the right diagrams there is a straight-line reference frame between the plates, this changes as if there was an electric flux. If there were electrons in this gap they would be accelerates with a E𝕐/-𝕠d kinetic impulse in this frame, not as an electric field.

Changing probabilities and acceleration

At 90° to this there are movie like frames of integral probabilities, this are additions of the +𝕆D potential and -𝕆D kinetic differences which change across the gap with μ. That change in the straight-line reference frame is equivalent to electrons there being accelerated with a square ϵ force.

FIGURE 31.16 The close analogy between an induced electric field and an induced magnetic field.



A single rotational reference frame

In this model a changing magnetic field occurs as successive movie like frames in the rotational reference frame. This induced magnetic field is the same rotational reference frame, as the changed electric field.

The Induced Magnetic Field

Ordinary Coulomb electric fields are created by charges, but a second way to create an electric field is by having a changing magnetic field. That's Faraday's law. Ordinary magnetic fields are created by currents, but now we see that a second way to create a magnetic field is by having a changing electric field. Just as the electric field created by a changing $\vec{B}$ is called an induced electric field, the magnetic field created by a changing $\vec{E}$ is called an *induced magnetic field*.

Electrons as particles not a field

In this model there is a E𝕐/-𝕠d kinetic impulse through the coil when a battery for example is connected to it. This is in the straight-line reference frame, the e𝕪/-𝕠d kinetic current increases after the connection. There is no electric field, the electrons are observed as particles. They can collide with each other to move the current, much like with an ocean current.

The electron sea

Because there is an electron sea outside of the atoms, this moves primarily in the straight-line reference frame. The electrons remaining in the atoms are primarily in a rotational reference frame. The electron sea is moved by the +𝕆D potential and -𝕆D kinetic difference in the rotational reference frame by the battery. This is like the ocean current being moved by tides as a change in the +𝕀D gravitational gradient from the -𝕀D×e𝕧 inertial work of the moon's motion.

No spin direction

There is no particular direction to the spin in this model, the rotation can be measured in either direction by changing the position from under it to over it. The opposed spin can come from +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work.

Lanz's law and the spin direction

When the battery is connected, the induction in the coil reacts against the change according to Lenz's law. This would be +𝕆D×e𝕒 potential work against the E𝕐/-𝕠d kinetic impulse of the current. This is in a coil, so the rotational reference frame has a positive or negative sign. In the wire the current does not have a plus or minus sign, the electrons collide with each other as vector addition and subtraction.

No electric field in the gap

In the capacitor there is no electric field, according to this model, so there is no opposing sign to +𝕆D×e𝕒 potential work in the coil. The electrons move into the negative plate with vector addition, in the straight-line reference frame. The distance across the plates is a straight-line reference frame as well, unlike the rotational reference frame of the coil.

A rotational reference frame from above and below

From above there is an integral field around the wire and through the plate gap, this is also a rotational reference frame. This gives the -𝕆D kinetic probabilities of where the electrons would be measured, it can be rotated around to the sides and underneath to cover all the measuring positions.

The inverse square law

These probabilities decrease as squares according to μ at larger positions away from the wire and plates. All these rotational reference frames are at 90° to the straight-line reference frame along the wire and in between the plates. The -𝕆D kinetic probability also decreases as a square with a larger gap, this is the same as with quantum tunneling. If the plates were very close, then there could be more electrons tunneling as in some transistors.

Quantum tunneling and the inverse square law

Looking down the wire the rotational reference frame is measured at 90°, there is an integral area of changing work at different positions along it. When this gets to the plates there is no more measurement of a magnetic field, there is no more current. In the plate gap there are more rotational reference frame integral areas. These gives the -𝕆D kinetic probabilities of measuring electrons there, which is low but they are still at 90° to the current in the wire. The -𝕆D kinetic probability again decreases as a square through the gap, this allows for some electrons to quantum tunnel through it as waves.

Destructive interference and repulsion

When the current stops the plates are full with no changes over time, then there are only the rotational reference frames. These give the -𝕆D kinetic probabilities of where the electrons are. With the destructive interference between them, these are spread out in the negative wire and plate. In the gap there are some +𝕆D potential and -𝕆D kinetic probabilities of where electrons and protons might tunnel through it.

The coil and the capacitor

The coil reacts against the current with +𝕆D×e𝕒 potential work, so its spin has the opposite sign to the active -𝕆D×e𝕪 kinetic work of the electron current in the gap. In both cases the two spins are there and are subtracted.

Work and impulse do not create each other

In this model impulse and work do not create each other, they are a straight-line reference frame and at 90° a rotational reference frame. Creating each other implies a change over time with impulse only. Also in the rotational reference frame the straight-line reference frame does not appear at any position. It is not then at a definable position in this frame.

The rotating wheel

When there is no force, the photon can appear as a rolling wheel from the side with an integral area. The e𝕪 and -𝕘d Pythagorean Triangle can be regarded as rotating around its -𝕘d axle, the spoke of the wheel is the e𝕪 straight Pythagorean Triangle side of the photon. This has no straight-line reference frame, there is only spin.

The wheel has no velocity

The wheel can be spinning in one place like a rotating projectile, or it can be moving at a velocity in the straight-line reference frame like a spinning projectile fired from a cannon.

The velocity has no wheel

From above with a change of 90° there is a straight-line reference frame, the spoke appears to move backwards and forwards as a particle. This seems to accelerate and decelerate, changing direction like a piston. The e𝕧/-𝕚d inertial velocity of a photon is separate from the rotational reference frame of the -𝕘d frequency of the photon. That allows the photon to be emitted and absorbed with different -𝔾D×e𝕪 light work from atoms without changing its velocity as c.

Frequency can change without affecting the velocity

This frequency can then change without affecting the velocity. For example, two balls can be spinning at different rates but move through the air at the same velocity. This spin is in the rotational reference frame only.

Same velocity different RPM

Two cars can move at the same velocity, one is in first gear and the other is in second gear. The crankshafts rotate at different frequencies in a rotational reference frame. Their pistons move back and forth in different straight-line reference frames, their accelerations and velocities are different for each car.

Different spectrums, same inertial velocity

The photon can be absorbed into an atom when its rotational reference frame is measured by the electron’s rotational reference frame. The electron rotates around an orbital, so it can only be in this rotational reference frame. When it leaves the atom it moves in straight lines, so it can only be in a straight-line reference frame.

Conserving reference frame changes

Because the -𝕘d rotational frequency is the inverse of the e𝕪 wavelength, the rolling wheel can change while conserving the same inertial velocity as c. For example if a rolling wheel has double the spoke length and rolls with half the frequency, its inertial velocity will be unchanged.

FIGURE 31.16 shows the close analogy between induced electric fields, governed by Faraday's law, and induced magnetic fields, governed by the second term in the Ampère-Maxwell law. An increasing solenoid current causes an increasing magnetic field. The changing magnetic field, in turn, induces a circular electric field. The negative sign in Faraday's law dictates that the induced electric field direction is ccw when seen looking along the magnetic field direction.

An increasing capacitor charge causes an increasing electric field. The changing electric field, in turn, induces a circular magnetic field. But the sign of the Ampère-Maxwell law is positive, the opposite of the sign of Faraday's law, so the induced magnetic field direction is cw when you're looking along the electric field direction.

Induction in logic

In this model the straight-line reference frame is deterministic, from that there is an induced rotational reference frame. This is similar to the dictionary definition of induction where a probabilistic inference come from premises. The rotational reference frame is like a set of axions, the straight-line reference frame is in effect deduced with deterministic logic from this.

An induction of inferences

For example the straight-line reference frame has electrons observed with a E𝕐/-𝕠d kinetic impulse, in the rotational reference frame there are -𝕆D kinetic probabilities as an induction of inferences from that. A coil with this rotational reference frame is called an inductor.

Measurements as axioms in logic

These measurements are like axioms in logic which cannot themselves be proven deterministically. These lead to a deductor as a capacitor, the electrons are deducted from one side of the wire and a battery like in bookkeeping. Because they are particles they can be counted like numbers in accounting as deductions.

Godel's theorem

The difference between the two reference frames leads to an interpretation of Godel's theorem. There some things can be true but not proven, as axioms they are assumed and then deterministic logic is derived from them in straight-line reference frame. But there is no way to derive some of the axioms. As Aristotle said, the chain of proof would be endless if it did not stop at assumptions or axioms.

Zeno's arrow

In this model it is like Zeno's arrow, in the rotational reference frame there is no motion so the arrow is stationary. This is like an axiom or starting point for it to begin moving at 90° in the straight-line reference frame. In logic an axiom has no motion, it is where the chain of deterministic logic or mathematics begins.

A moving arrow as a derivative

A moving arrow is in the straight-line reference frame, its velocity is a fractional derivative like e𝕧/-𝕚d. It can collide with other arrows like vectors in a deterministic chain of cause and effect. To stop, its e𝕧 would have to become zero, then the straight-line reference frame would cease to exit.

Stopped axioms cannot be reached

It cannot get to some axioms as they are outside this deterministic chain of logic or mathematics, these are true but not provable in that straight-line reference frame. There are then stopped positions, like with Zeno's stationary arrow, that the moving arrow cannot get to.

An unstoppable force overshoots an immovable position

It would overshoot these positions because it would become an immovable position when it is an unstoppable force or velocity. The process of Godel's theorem occurs over time with cause and effect. This computation cannot reach some axioms which are not in this deterministic path.

The halting problem

It is also like the halting problem in computation, that occurs with a straight-line reference frame over time. This cannot calculate whether it can halt or not, like Zeno's arrow it moves forever unless it reaches an axiom that tells it to halt. For example if a particular answer or limit is reached as an axiom, then it can stop.

A limit cannot be calculated

It cannot calculate what that limit is, such as in calculating π or an infinite sequence summing to 2. If it reaches 3 in calculating π that can be an arbitrary axiom telling the computation to halt. This also happens in renormalization in physics, calculating the mass of an electron leads to infinities by getting closer to it. Having a cutoff point of where the particle is observed stops this calculation.

An infinite loop does not begin or end

An infinite loop is in a rotational reference frame where it connects back to the beginning of the computation like circular logic. In a sense this does not halt as it does not have a beginning, any position in the circular code can be regarded as a starting point and assigned a probability.

> If a changing magnetic field can induce an electric field and a changing electric field can induce a magnetic field, what happens when both fields change simultaneously? That is the question that Maxwell was finally able to answer after he modified Ampère's law to include the displacement current, and it is the subject to which we turn next.

Combining two reference frames

In this model, Maxwell's equations are a combination of line integrals in a rotational reference frame, with fractions in a straight-line reference frame.

> ### 31.4 Maxwell's Equations
>
> James Clerk Maxwell was a young, mathematically brilliant Scottish physicist. In 1855, barely 24 years old, he presented a paper to the Cambridge Philosophical Society entitled "On Faraday's Lines of Force." It had been 30 years and more since the major discoveries of Oersted, Ampère, Faraday, and others, but electromagnetism remained a loose collection of facts and "rules of thumb" without a consistent theory to link these ideas together.
>
> Maxwell's goal was to synthesize this body of knowledge and to form a *theory* of electromagnetic fields. The critical step along the way was his recognition of the need to include a displacement-current term in Ampère's law.

Maxwell's theory of electromagnetism is embodied in four equations that we today call **Maxwell's equations.** These are

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$	Gauss's law
$\oint \vec{B} \cdot d\vec{A} = 0$	Gauss's law for magnetism
$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_{\text{m}}}{dt}$	Faraday's law
$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} + \epsilon_0 \mu_0 \frac{d\Phi_{\text{e}}}{dt}$	Ampère-Maxwell law

The Lorentz force law

The Lorentz force law says that there are two Roy electromagnetic forces, from magnetism and electric charges. This begins with the -𝕠d×e𝕪/-𝕠d kinetic momentum as q, in this model that is a combination of an integral and a derivative. This momentum can be changed by two forces, here the electric charge come from E𝕐/-𝕠d as the kinetic impulse. This force only moves in straight lines, unlike charges move together and unlike charges repel each other.

The Biot Savart law and a rotational reference frame

That is added here with a velocity times B⃗ as the magnetic force. This velocity is the e𝕪/-𝕠d kinetic velocity, for that to change with magnetism this model has e𝕪/-𝕆D as kinetic work. That comes from the Biot Savart law where a particle can be moving with a velocity, according to the angle of the magnetic field this can either turn the particle or it is not affected. That force is then in a rotational reference frame.

Forces from two reference frames

Together the particle is acted on in two reference frames, in the straight-line reference frame it can be accelerated and in the rotational reference frame a torque is exerted on it. These can be seen with a planet orbiting a star, from above it might appear as a circular orbit in a rotational reference frame with +𝕀D×e𝕙 gravitational work. The Lorentz forces are in relativity in between gravity and inertia.

The circle becomes an ellipse

Turning 30°, the orbit appears as an ellipse, this is also rotating a conic section from a circle to an ellipse. It cannot become a parabola at 45° because that would mean the planet escaped the gravitational field of the star by rotating the reference frame. This ellipse is then a combination of the straight-line and rotational reference frames, that would apply to an electron in an elliptical orbital.

The circle becomes a straight-line

When turned 90° the planet appears to move back and forward with an acceleration like a spring, this is like the Coulomb force as the Eℍ/+𝕚d gravitational impulse. It is also like a planet moving past a star with no rotation, if the acceleration is strong enough this would leave the gravitational field of the star.

Inertia becomes stronger than gravity

That would be where the E𝕍/-𝕚d inertial impulse of the planet was stronger than the Eℍ/+𝕚d gravitational impulse of the star.

A hyperbolic trajectory

When that happens, the planet would have a hyperbolic orbit at 90° in the rotational reference frame. That would come from the -𝕀D×e𝕧 inertial work of the planet being stronger than the +𝕀D×e𝕙 gravitational work of the star.

The hyperbola approaches the parabola

Rotating this reference frame, the hyperbola would narrow to approach a parabola, it could not reach this because that would mean rotating the reference frame would have the star capturing the planet. In between the straight-line and rotational reference frames there is a parabola, that is where one Pythagorean Triangle side is squared and the other remains linear as $y=x^2$.

> Maxwell's claim is that these four equations are a *complete* description of electric and magnetic fields. They tell us how fields are created by charges and currents, and also how fields can be induced by the changing of other fields. We need one more equation for completeness, an equation that tells us how matter responds to electromagnetic fields. The general force equation
>
> $$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \qquad \text{(Lorentz force law)}$$
>
> is known as the *Lorentz force law.* **Maxwell's equations for the fields, together with the Lorentz force law to tell us how matter responds to the fields, form the complete theory of electromagnetism.**

Combining Roy and Biv

In this model Newton's laws of motion relate to inertia, that comes from the -𝕚d and e𝕧 Pythagorean Triangle. The laws of gravity come from the +𝕚d and e𝕙 Pythagorean Triangle. Maxwell's equations are from -𝕠d and e𝕪 Pythagorean Triangles as electrons, some are from -𝕆D×e𝕪 kinetic work and some from the E𝕐/-𝕠d kinetic impulse. The +𝕠d and e𝕒 Pythagorean Triangles as protons attract the electrons to give a circuit.

> Maxwell's equations bring us to the pinnacle of classical physics. When combined with Newton's three laws of motion, his law of gravity, and the first and second laws of thermodynamics, we have all of classical physics—a total of just 11 equations.
>
> While some physicists might quibble over whether all 11 are truly fundamental, the important point is not the exact number but how few equations we need to describe the overwhelming majority of our experience of the physical world. It seems as if we could have written them all on page 1 of this book and been finished, but it doesn't work that way. Each of these equations is the synthesis of a tremendous number of physical phenomena and conceptual developments. To know physics isn't just to know the equations, but to know what the equations *mean* and how they're used. That's why it's taken us so many chapters and so much effort to get to this point. Each equation is a shorthand way to summarize a book's worth of information!

Gauss's law

In Gauss's law, charged particle have a E𝔸/+𝕠d potential impulse and a E𝕐/-𝕠d kinetic impulse, with this model there is no electric field only particles. This is referred to as an electric flux around charges, in this model that is a magnetic field in a rotational reference frame. A straight-line reference frame is a straight-line in between charges, they can accelerate each other over time.

Faraday's law

In this model a changing magnetic field appears like a series of separate rotational reference frames that are quantized. These can be in a straight-line or curve, then it can appear to be an electric field.

Gauss's law for magnetism

With Gauss's law for magnetism there are no north and south poles in this model, electrons spin in one direction. Because they can be viewed from above or below this can appear to be clockwise or counterclockwise.

Ampere Maxwell law first half

With the Ampere Maxwell law a current is in a straight-line reference frame. When this current goes through a coil there is a rotational reference frame measured at 90°.

Ampere Maxwell law second half

When the electric current changes, such as by closing a switch, then there is an increased current in the straight-line reference frame as impulse. In the rotational reference frame at 90°, there are quantized changes in the work done as a magnetic field.

Lorentz force law first half

Here an electric force comes from impulse, there is no electrical field because that would be an integral. Instead electric charges can collide as particles. This Coulomb force cannot turn a charge with torque, it can only accelerate it in a straight-line.

Lorentz force second half

The magnetic force can only turn a charge with a torque, it cannot accelerate a charge in a straight-line.

Let's summarize the physical meaning of the five electromagnetic equations:

- **Gauss's law:** Charged particles create an electric field.
- **Faraday's law:** An electric field can also be created by a changing magnetic field.
- **Gauss's law for magnetism:** There are no isolated magnetic poles.
- **Ampère-Maxwell law, first half:** Currents create a magnetic field.
- **Ampère-Maxwell law, second half:** A magnetic field can also be created by a changing electric field.
- **Lorentz force law, first half:** An electric force is exerted on a charged particle in an electric field.
- **Lorentz force law, second half:** A magnetic force is exerted on a charge moving in a magnetic field.

These are the *fundamental ideas* of electromagnetism. Other important ideas, such as Ohm's law, Kirchhoff's laws, and Lenz's law, despite their practical importance, are not fundamental ideas. They can be derived from Maxwell's equations, sometimes with the addition of empirically based concepts such as resistance.

Maxwell's equations as work and impulse

In this model Maxwell's equations reduce to work and impulse with the +𝕠d and e𝕒 Pythagorean Triangle protons and the -𝕠d and e𝕪 Pythagorean Triangle electrons. Newton's laws are no simpler here, they are reduced to work and impulse with the +𝕚d and e𝕙 Pythagorean Triangle for gravity and the -𝕚d and e𝕧 Pythagorean Triangle for inertia.

Zeno's stationary arrows

The rotational reference frames in Roy electromagnetism give quantization, this is where there is no time and so there can not be a continuous change. Zeno's arrow would appear as a series of stationary arrows in a line, the force moving it would be work over a distance.

Stationary electrons

In an atom the electrons are stationary as they do not change over time, they are only probability densities separated like the arrows. If an electron absorbs a photon it jumps to the next orbital, there is some impulse because it has been observed that this change takes some time.

Zeno's points on a line

This also relates to Zeno's paradox as points on a line. The quantized points would be separated with no continuous lines between them, this would be in a rotational reference frame where the points were like circles. They have no length and like an electron are points only.

A circle as a point or a line

When a circle is measured in a rotational reference frame, there is no way to observe the straight radius and so there cannot be a size. When it is observed in a straight-line reference frame at 90° it appears as a continuous line as with Zeno, there is no way to measure a point.

Zeno's arrow cannot stop

In the straight-line reference frame there is only a line, Zeno's arrow cannot stop at a point so it is in continuous motion. The line has no points in it, there is only a displacement from the start to the end of the line over a time with impulse.

An arrow is a line

The arrow itself is also referred to as a line by Zeno, because the arrow is contained in the line from its tip to its end it cannot move. In the moving straight-line reference frame the arrow has a limit of being stationary, that would be observed by a second arrow moving parallel to it at the same velocity. But then if it appears stationary, then so is the second arrow. The two arrows could only move by jumping together from one quantized position to the next like a movie frame.

A continuous spectrum like a line

The line is like a continuous spectrum of e𝕪/-𝕘d photons, such as from collisions in blackbody radiation. It was assumed that atoms would emit light as an oscillator with an infinite number of frequencies. This is in a straight-line reference frame where there is a motion in an oscillator so it can change an infinitesimal amount with its frequency.

A discrete spectrum as a series of points

As the wavelength decreases in this model it is like a line getting shorter, it approaches a point in size. Then the rotational reference frame becomes stronger, the spectrum becomes discrete like the movie frames. An oscillator is frozen with one value, then it jumps to the next value like the movie frames.

Achilles and the tortoise

In another of Zeno's paradoxes, Achilles is running after a tortoise. He has a faster velocity, but he cannot reach the same position as the tortoise. In the straight-line reference frame, Achilles cannot be stationary nor can the tortoise. If Achilles gets next to the tortoise, then it will appear stationary to him, but then he cannot pass the tortoise. If so then his velocity could not have been faster than the tortoise's. This is like Zeno's arrow but where it can move but cannot stop.

The barn paradox in special relativity

A person is on a long pole moving towards a barn at 99% of the inertial velocity of c. The pole is too long for the barn, the front end would hit the back of the barn before the back end was inside it. Yet the person finds the pole fits inside the barn after all.

Observing the time in the straight-line reference frame

In the straight-line reference frame they look along the pole and can observe its velocity relativity to the approaching barn. There is a clock gauge on the back of the barn wall, this is at 90° to the pole's motion. It is in a rotational reference frame, but here the clock is used to observe the time in the straight-line reference frame.

The clock is running slower

He knows it would take 2 seconds to get the trailing end of the pole in the barn but the front end would hit the barn wall in 1 second. These times are not accurate, they are used here for illustration. The clock on the barn wall appears to be running slower, they observe that 2 seconds have passed before they hit the barn wall. They conclude that the back end must have gotten into the barn before they hit the barn wall.

A clock at the barn entrance

Looking back along the pole there is another clock on the barn entrance wall, this is also at 90° to them. As they enter the barn they observe the clock is running more slowly, 2 seconds pass, in this time they observe the back end of the pole inside the barn.

Stationary movie frames

In the rotational reference frame another person looks from above, at 90° to the pole's motion. They cannot observe the pole's motion, so it appears to be stationary at different points like movie frames.

The pole is half the length

They know it would take 2 seconds for the pole to fit inside the barn, they measure the pole as being half its original length in each frame. In one of these frames the pole is inside the barn before it hits the back wall. Here the pole is in the straight-line reference frame, but it is only used to measure the distance not the displacement.

The second hand is frozen

As a variation of this there is a moving clock at 90° to the measurer. The second hand on the clock appears to be frozen, this is because speed and velocity cannot be observed in the rotational reference frame. It appears with different orientations in each movie like frame.

The rotating spoke on a photon

This is like the rotating ey spoke on the ey×-gd photon as a rolling wheel. Here the clock need not be rolling, instead the second hand is rotating.

The clock hand is half the length

The measurer knows the width of the clock, and that it is twice as wide as to be able to fit into the barn. Nonetheless they measure the second hand when it points forward and backward in two movie frames. It is half its previous length, they conclude the whole clock would fit inside the barn.

The clock as an ellipse

Also the circular clock gauge appears as an ellipse, as if it has rotated to one side with the minor axis being small enough to fit inside the barn. The major axis points up and down, this is not contracted, it remains twice as long as needed to fit inside the barn.

A hyperbola and hyperspace

This would have a limit as a line, becoming a straight-line reference frame. Beyond this would be a hyperbola, like a hyperspace past c. A circle is a rotational reference frame as a conic section, different circles represent electron orbitals. As these circular orbitals increase there are also more elliptical orbitals, a combination of a rotational and straight-line reference frame.

A hyperbola as a straight-line reference frame

This makes electrons act more like particles with an impulse. At the ionization boundary this turns 90° to become a hyperbola as a straight-line reference frame. The electron then acts like a particle outside these orbitals.

Two sides of a reference frame

When the observer was in the straight-line reference frame, they could observe the clock gauge at 90° to the pole's motion. This is because each reference frame is composed of a straight Pythagorean Triangle side like the pole and a spin or time Pythagorean Triangle side like the clock's rotation. When they were in the rotational reference frame they could measure the length of the pole or second hand, this was at 90° to them.

Changing the velocity

When the second hand pointed upwards it was not contracted, that is because the pole was not moving in that direction. When it was at 45° to the wall it appeared more contracted, when pointed at or away from the wall it was measured to be half its length when pointing upwards.

The displacement is observed

Even though both Pythagorean Triangle sides are in each reference frame, the result is different. This is because, in the straight-line reference frame, the displacement over time determined whether the pole would fit inside the barn. That would be observing the E𝕍/-𝕚d inertial impulse.

The torque or probability is measured

In the rotational reference frame the turning of the clock's second hand gave its length contraction, if the clock hand stopped then there was no way to measure this. So, the torque in turning the second hand was the force here as -𝕀D×e𝕧 inertial work. Also, there would be a -𝕀D inertial probability is which direction the second hand would be pointing.

The length is not the displacement

The length contraction of the pole would allow it to fit inside the barn, it is not necessary to observe the pole's displacement as it moves. Its length would be short enough, so this is no longer an observation of the pole's E𝕍/-𝕚d inertial impulse. Instead, the amount of work required to move the pole half the distance would be halved.

Displacement and position

This is like the difference between a displacement and length on a ruler, according to this model. Moving from a 1 centimeter mark to a 2 centimeter mark is a displacement like the pole's motion across the barn. The positions of the 1 and 2 centimeter marks are not a displacement, there is no motion described between them. The pole was contracted in the rotational reference frame, but this was not observed with a displacement. Instead, it was like a contracted ruler where the marks were closer together.

Torque and time

In the rotational reference frame there would have been a torque between the successive movie frames, otherwise the second hand would not have moved. This need not have been a constant turning rate, it would have been accelerating and decelerating in between each movie frame. Torque is required to give the duration between one number on the clock face and the next. If there is no duration then it can only describe one instant of time there.

Different torques with the same orientation

An instant of time was of no use in measuring the second hand in the movie frames. For example, it might have been turning twice as fast, and yet shown the same second hand orientations. The duration of time would have been half as much with this double speed, the torque in starting and stopping the second hand would also have been doubled. Only by measuring the torque of the second hand could there be an accurate measurement, the clock's velocity could not be observed in the rotational reference frame.

The barn paradox in general relativity

This would come from the +𝕚d and e𝕙 Pythagorean Triangle as gravity, proportionally also the +𝕠d and e𝕒 Pythagorean Triangle as the proton. There is a silo on a heavy planet, this gives an e𝕙 height contraction and +𝕚d gravitational time slowing on the surface. A rocket moves down towards the surface to go into the silo. As with the pole, it is too long to fit into the silo. Instead of the pole moving at close to c, here it is assumed the planet is very dense.

Observing the silo clock

The rocket moves downwards in the straight-line reference frame with the silo under it. This is like the pole approaching the barn. It observes a clock at the bottom of the silo, the crew have calculated at their inertial velocity only half the rocket will fit into the silo. They have their onboard clock which is their proper time. It is assumed the rocket can move downwards at a constant inertial velocity, hitting the floor without damage.

Slower gravitational time

As they descend, but still at a large e𝕙 height, they observe the clock in the silo is moving more slowly than their own on board. It should take a second for the rocket to reach the bottom of the silo, leaving the upper half protruding from the top. Instead, the clock shows it takes 2 seconds to reach the bottom, they conclude the silo appears to be twice its original depth and the rocket should fit inside.

A contracted height

A second rocket remains in orbit, they are in a position at 90° to the rocket's descent. This is a rotational reference frame, they are outside the planet's gravitational field. The measure the rocket as being only half the length it was before it began its descent. They conclude the rocket should completely fit into the silo. For example, the silo could be on the equator, the second ship is far away above the north pole.

Redshifted and curved light

The light would also be redshifted and curved in climbing out of the gravitational well. As the rocket becomes further away then the time the light takes to come from the rocket increases. That would appear as if the clock on the rocket is slowing down, but this would be corrected for.

The light would also be in a curved path as it would need to move parallel to the equator before reaching the second rocket, this would also be corrected for.

> It's true that Maxwell's equations are mathematically more complex than Newton's laws and that their solution, for many problems of practical interest, requires advanced mathematics. Fortunately, we have the mathematical tools to get just far enough into Maxwell's equations to discover their most startling and revolutionary implication—the prediction of electromagnetic waves.

The gap in the capacitor

In this model the electromagnetic waves are in rotational and straight-line reference frames. The wave component here is only in the rotational reference frame, the electric fields in this model are also magnetic fields. The discovery of electromagnetic waves came from something being in the gap of a capacitor, here this is from work and impulse.

Emitting a discrete spectrum

In the rotational reference frame these photons would be quantized like movie frames, when electrons are placed in the gap there would be the emission and absorption of e𝕪×-𝕘d photons. That comes from the voltage between the plates as the +𝕆D potential and -𝕆D kinetic differences, these are the same as in an atom giving quantized electron orbitals.

Emitting a continuous spectrum

The electrons are also accelerated by the plates in a straight-line reference frame. This comes from a current between the plates mediated by the electrons in it. That causes the electron to emit and absorb e𝕪/-𝕘d photons with a continuous spectrum. Some electrons would be in this rotational reference frame as waves, others would be in the straight-line reference frame as particles. This acts like a blackbody where electrons are emitted with both kinds of spectrums.

Ionized photon emissions

There can also be whole atoms in this gap, then the voltage causes them to ionize with a discrete spectrum such as with a neon lamp. Some electrons would also be emitted and absorbed as the neon atoms collided with each other.

Two spectrums in the gap

From the side at 90° the electrons would be measured only in movie frames as quantized levels. The e𝕪×-𝕘d photons emitted would have a discrete spectrum. In the straight-line reference frame these would be accelerated electrons with a continuous spectrum. The two reference frames would be told apart by whether work was being measured or impulse was being observed. That would be a change over a distance or of time.

The blackbody

A blackbody has a combination of a discrete and a continuous spectrum, at lower frequencies the e𝕪 distance between these levels is smaller as the left side of the blackbody going to zero. This decreases like the side of a normal curve or inverse exponential. The right side is from the straight-line reference frame, this is a continuous spectrum becoming an exponential decay curve.

Two kinds of photons emitted

In this model there would be two kinds of photons emitted from the blackbody, depending on the reference frame. Those in a rotational reference frame come from quantized electron orbital changes. They are emitted as -𝔾D×e𝕪 light work, but those e𝕪×-𝕘d photons can collide with atoms to also have a e𝕐/-𝕘d light impulse. Other e𝕪/-𝕘d photons are emitted from electron collisions, these can be absorbed into atoms where the remaining frequency is emitted as Raman radiation.

Emitted as light work or impulse

Each photon is the same, they are emitted either as -𝔾D×e𝕪 light work or a e𝕐/-𝕘d light impulse. With -𝔾D×e𝕪 light work they transmit the change in -𝔾D light torque as electrons change orbitals, this gives them the discrete spectral frequencies. The e𝕐/-𝕘d light impulse comes from atomic and electron collisions from the heat of the blackbody, these have a continuous spectrum.

Temperature and the spectrum type

The heat of the blackbody comes from the e𝕪 temperature in a straight-line reference frame. When this increases the continuous spectrum is more common as electrons are liberated from atoms. This can be difficult because of the Fermi energy in the atoms, at lower temperatures the photons are emitted in the rotational reference frame. The changing temperature shifts the shape and distribution of the blackbody curve.

Photon inertial velocity

In both cases the photons are the same, they have the same inertial velocity in a straight-line reference frame. This comes from the constant e𝕪 and -𝕘d Pythagorean Triangle area, when the -𝕘d Pythagorean Triangle side is larger there is more -𝔾D×e𝕪 light work from electron orbitals.

The photoelectric effect

This higher -𝕘d frequency light also liberates more electrons with the photoelectric effect. When the intensity of the photons is increased, but not the frequency, then the -𝔾D×e𝕪 light work does not change and no more electrons are emitted by them. In the blackbody the lower frequency photons do not liberate more electrons and create electron collisions with a continuous spectrum.

> Maxwell developed his four equations as a mathematical summary of what was known about electricity and magnetism in the mid-19th century: the properties of *static* electric and magnetic fields plus Faraday's discovery of electromagnetic induction. Maxwell introduced the idea of *displacement current*—that a changing electric flux creates a magnetic field—on purely theoretical grounds; there was no experimental evidence at the time. But this new concept was the key to Maxwell's success because it soon allowed him to make the remarkable and totally unexpected prediction of **electromagnetic waves**—self-sustaining oscillations of the electric and magnetic fields that propagate through space without the need for charges or currents.
>
> Our goals in this section are to show that Maxwell's equations lead to a *wave equation* for the electric and magnetic fields and to discover that all electromagnetic waves, regardless of frequency, travel through vacuum at the same speed, a speed we now call the *speed of light*. A completely general derivation of the wave equation is too mathematically advanced for this textbook, so we will make a small number of assumptions—but assumptions that will seem quite reasonable after our study of induced fields.

Free space with photons and gravis

In this model free space would contain the four central Pythagorean Triangles, e𝕪×-𝕘d photons, e𝕣×+𝕘d virtual photons, +𝕘d×e𝕓 gravis, and -𝕘d×e𝕧 iners. Each can transmit changes as fields with work, or particles with impulse, e𝕣×+𝕘d is added to e𝕪×-𝕘d, it cannot be observed or measured by itself. -𝕘d×e𝕧 is subtracted from +𝕘d×e𝕓, also cannot be observed or measured by itself.

Two observable and measurable Pythagorean Triangles

That leaves two observable and measurable central Pythagorean Triangles, the e𝕪×-𝕘d photon and +𝕘d×e𝕓 gravis as fields, the e𝕪/-𝕘d photon and e𝕓/+𝕘d gravi as particles.

Probable e𝕪×-𝕘d and +𝕘d×e𝕓 locations

e𝕪×-𝕘d and +𝕘d×e𝕓 still have constant Pythagorean Triangle areas, they are measured in various positions according to a probability density. It does not mean they are everywhere filling Biv space-time, the most highly probable measurement is along a straight path from their source. This is because e𝕪 and e𝕓 are straight rulers for measurements, they can also be measured on a geodesic like with gravitational lensing around galaxies.

Motions from torque

These fields are in rotational reference frames, do not use time in terms of a starting and final time to be observed. Their motions can be described with a torque, such as the -𝕆D kinetic torque of an electron emitting -𝔾D×e𝕪 light work as it drops to a different torque of a lower orbital. The +𝕘d×e𝕓 gravis can also be a +𝔾D gravitational torque, for example neutron stars spiraling in towards each other.

Destructive interference in path integrals

With these fields there is a -𝔾D light probability in the photon being measured on either side of the path of least action. This decreases as an inverse square on either side, they interfere destructively to produce the central path.

A field becoming particles

When a photon is measured closer to a proton's gravitational field, it can be observed as an electron and positron in the straight-line reference frame. The area of the e𝕪 and -𝕘d Pythagorean Triangle would then be the difference between the electron and positron, similar to a change in an electron orbital. If the electron and positron recombine, this is like an electron joining with a proton emitting its -𝕆D kinetic torque as -𝔾D×e𝕪 light work photons.

Path of least action

Then one probable path might be measured even when not on this path of least action. Here action would be from work and impulse, also referred to as energy. The e𝕪 and -𝕘d Pythagorean Triangle has a constant area and does not have an additional area to be measured far off this path. This follows from a conservation of work and impulse, combined as a conservation of energy.

Probability and time

There is an uncertainty in between work and impulse, or position and time. For example, in Blackjack shuffling the cards occurs over time in a straight-line reference frame. This can scatter some of the patterns in the cards, with a deterministic collision between them in the shuffling process.

A long time shuffling

If this continues for a long time the cards become increasingly random in a rotational reference frame. Then when the cards are dealt in a straight-line reference frame they are evenly distributed like a gradient or probability density. This is like closing a switch in a circuit, the -𝕆D kinetic probabilities in a current change which can produce a magnetic field. When this is closed for a long time the -𝕆D kinetic gradient in the circuit becomes constant and the magnetic field disappears.

Tired light

When the e𝕪/-𝕘d photons and e𝕓/+𝕘d gravis are observed they are in a straight-line reference frame, their changes would be observed over time in light years for both as c. In the rotational reference frame they are not moving, without outside influences the angles θ in them does not change. In a gravitational well photons can be redshifted, also when the +𝕚d and e𝕙 Pythagorean Triangle as gravity approaches its maximum e𝕙 height. The +𝕘d×e𝕓 gravis would be affected electromagnetically by the proton, changing its angle θ.

The Doppler shift and the Hubble constant

The photon appears to develop an increasing redshift when the +𝕚d and e𝕙 Pythagorean Triangle has a large e𝕙 height approaching the CMB. This is because it is affected by gravity as the +𝕚d and e𝕙 Pythagorean Triangle, such as with a bent path around a star. It is also redshifted in climbing out of a gravitational well, such as with an event horizon.

The Hubble constant as a square

With a greater e𝕙 height above the observers and measurers, or a greater distance from them, the photon is redshifted with its e𝕪 wavelength according to the Doppler shift. This appears as a Eℍ/+𝕚d gravitational impulse acceleration, here v is e𝕙/+𝕚d and bringing D as e𝕙 over to the left-hand side makes it Eℍ/+𝕚d as a constant.

Squared constants

This is a squared constant like μ and ϵ, here the Hubble constant is proportional to ϵ. It acts over a longer distance than Roy electromagnetism, it is also connected to c. There would be another constant here proportional to μ, that comes from -𝔾D×e𝕪 light work where e𝕪×-𝕘d photons lose their -𝕘d rotational frequency as a square as the e𝕙 height increases.

Work and impulse both decrease

Both decrease, this is not the same as a constant Pythagorean Triangle changing work and impulse inversely to each other. The long distance is observed as a redshift with a square, it is also measured as a frequency change as a square. They need to remain inverses of each other, this appears as if the Pythagorean Triangle area has decreased.

Both reference frames change

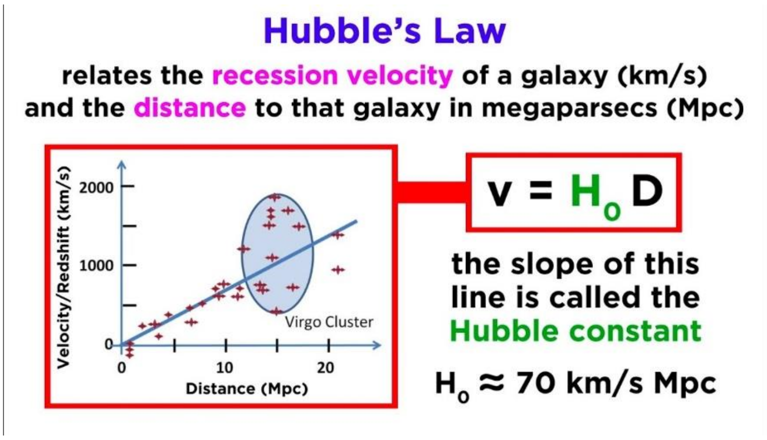
This is because the straight-line and rotational reference frames have each changed, like a photon climbing out of a gravitational well. That can be represented by e𝕐/-𝕘d light impulse where the photon particle redshifts, it can also be a geodesic around the gravitational source. Then the change in -𝕘d photon frequency corresponds to the different e𝕙 heights above a planet for example.

A change in the angle θ must be a force

The +𝕚d and e𝕙 Pythagorean Triangle must be observable as a Eℍ/+𝕚d gravitational impulse, and measurable as +𝕀D×e𝕙 gravitational work. The longer distance or e𝕙 height to the CMB then must itself change photons in their work and impulse. When different heights are observed or measured, this is comparing two values of work or impulse. In between there must be a redshift or photon frequency change.

Comparing reference frames

It is like comparing two reference frames, the work or impulse in moving from one to the other must be included. A redshift can only be observed by comparing it to another redshift value, there is then a difference in the E𝕳/+𝕚d gravitational impulse between these two +𝕚d gravitational times.



The CMB

When a rocket moves to the CMB there would be a contraction in the increments of e𝕙 height along this path. This is from the rotational reference frame where it began its journey. This appears as a contraction of Biv space-time, conversely from the CMB to the local reference frame there appears to be an expansion of Biv space-time.

A contraction in the geodesic

This is like a rocket moving into an intense +𝕚d gravitational field around a black hole, as it gets closer to the event horizon it is measured as having a decreasing e𝕙 height. The +𝕀D×e𝕙 gravitational work increases as a +𝕀D square. That can be regarded as a contraction of Biv space-time around the black hole with the gravitational geodesic. In general relativity this contraction gives a gravitational attraction, then relativistic effects of a e𝕙 height contraction and +𝕚d gravitational time slowing.

An implosion or explosion

In this model gravity moves backwards in time towards the past. Conversely when the time direction is reversed, the rocket would appear to be pushed away from the black hole with an expansion of the geodesic. As the rocket moves towards the CMB it appears to contract or implode with its e𝕙 height. The galaxies around it appear like an explosion that occurred going forwards in time as the big bang.

An explosion in any direction

This is the same in any direction, the rocket has its e𝕙 height contracting because of the increase of e𝕙 in the +𝕚d and e𝕙 Pythagorean Triangle. This decreases the angle θ, so the E𝕳/+𝕚d gravitational impulse is observed to increase exponentially like an explosion.

Moving to the CMB

As E𝕳 increases as the gravitational displacement, then the +𝕚d gravitational time slows down. When e𝕙 approaches its maximum around the CMB this results in a frozen surface like an event horizon. If a rocket moved to the CMB area instantaneously, according to this model, it would appear like local Biv space-time with normal galaxies. It would also appear like this around the proposed time of the big bang 18 billion years ago.

Kinetics as the inverse of gravity

When gravity moves backwards in +𝕚d time it appears like a contraction, or big crunch where the universe would contract back down to a singularity. The inverse of this appears like a Biv space-time expansion as if from a E𝕐/-𝕠d kinetic impulse and -𝕆D×e𝕪 kinetic work. This is because the -𝕠d and e𝕪 Pythagorean Triangle electron has active forces which are the inverse of gravity. This explosion would appear to be from photons, the contraction from gravis as inverses. The explosion then appears to be from a large amount of energy and heat which becomes galaxies.

An explosion against gravity

When time is reversed, in this model gravity expands outwards, so the big bang appears as an explosion going forwards in time with inertia. This appears as gravity trying to hold the smaller universe together against the kinetic and inertial force of the big bang. If this gravity was stronger, then the universe would contract again into another singularity. Here inertia is stronger going forwards in time, that would appear as the universe continues to expand forever.

Galaxies coalescing after the big bang

In the big bang hypothesis galaxies coalesced under this local gravity from the big bang, as the explosive force was spread over a larger distance according to the inverse square law. One problem with this is that galaxies close to this big bang have been shown to be of normal size. In this model galaxies can form from dust coalescing at any time, not from the big bang.

A steady state

This model gives the same kind of redshifts as with the Hubble constant. There is no big bang and explosion, so galaxies were never formed in one. Instead, there would be a steady state where Hydrogen is used up in stars and replenished in some way. Here matter would fall into the event horizons in the center of galaxies, this area has a denser +𝕚d gravitational geodesic from so many stars there.

Hydrogen forming in black holes

This allows stars and gases to flow in an out of the event horizons, the tidal forces tear apart some heavier elements to produce new Hydrogen. These black holes are relatively stable, if matter falls into them then it would have enough inertia for some to move out again as a balance.

The CMB and general relativity

In this model the CMB would appear as a e𝕙 height in a +𝕚d gravitational geodesic, this is often modeled with a rubber mat like in the diagram. Closer to this there would be a e𝕙 height contraction as in general relativity, that would be like the straight lines radiating away from the center. These lines are like rulers in the rotational reference frame with +𝕀D×e𝕙 gravitational work.

Approaching the event horizon

With this +𝕀D×e𝕙 gravitational work the angle θ dilates toward a maximum, then large increases in the +𝕀D gravitational torque and probability only have a small change in the e𝕙 height. This approaches a limit like an event horizon. Above this there is a e𝕙 height where the CMB would be. Below it would be like inflation in the big bang hypothesis.

Gravitational time as the circles

The slowing of +𝕚d gravitational time would be represented by the distance in between the circles. A circle is like a +𝕚d gravitational clock gauge, the distance in between the rings can be regarded as a second of time. When they get closer together this becomes like a half second, then smaller increments like slowing time.

Creating new hydrogen

In the straight-line reference frame this would appear like falling towards the CMB, the time slows until the rocket would appear to reach the CMB wall of a e𝕙 height contraction. This is like where heavier elements would be torn apart by a galactic black hole, when it moved outwards it would reform as Hydrogen to make new stars in an eternal cycle. There would be a strong inertial rotation, the -𝕀D×e𝕧 inertial work of the atoms allow them to move in and out of this event horizon.

A CMB above a black hole

In conventional physics the CMB is where Hydrogen atoms would form, being able to emit photons for the first time. The process is the same in this model, the CMB is created by approaching the maximum e𝕙 height of the +𝕚d and e𝕙 Pythagorean Triangle from gravity. With an event horizon there is a kind of CMB where the heavier atoms are torn apart and reformed as Hydrogen.

The quark gluon plasma

There may also be a quark gluon plasma in the black hole at lower e𝕙 heights, this goes down until it reaches the e𝕙 limit.

The mass of the proton in a dense center

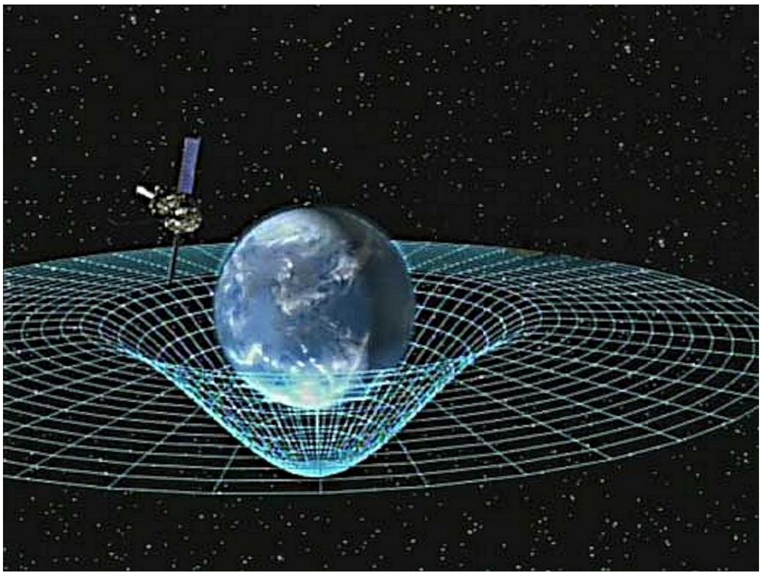
The same happens in protons according to this model, the main mass of the proton is in a dense center. That is proportional to the limit of e𝕙, deeper than the radius of the proton in Roy electromagnetism. The proton has gravity because the +𝕚d and e𝕙 Pythagorean Triangle is proportional to the proton's +𝕠d and e𝕒 Pythagorean Triangle, the lower limit of gravity makes this denser center.

The planet as the singularity

There would be no actual planet like in the diagram, but the big bang model could portray this planet as being like the singularity before it exploded. Then it would be a small point at the center of the mat. This larger planet could be regarded as like the expanding universe soon after the big bang. Then there is a e𝕙 height contraction and slower +𝕚d gravitational time around it.

The CMB as many blackbodies together

The CMB level around an event horizon would have a blackbody curve, like the CMB 18 billion years ago. This Hydrogen creates new stars, which also have blackbody curves. Together this gives galaxies beyond the distant CMB, together they give it the same blackbody curve as an individual star. The CMB in this model would be where distant galaxies are contracted into a wall that has this blackbody curve, they would remain unaffected by this. Those galaxies would be similar to local ones, with similar sizes and characteristics.



Light from another galaxy

In the space between stars and galaxies there would be these two active central Pythagorean Triangles, these are the e𝕪 and -𝕘d Pythagorean Triangle photons and the +𝕘d and e𝕓 Pythagorean Triangle gravis. They can be measured with work such as in a double slit experiment, also as impulse by trying to observe one of the slits.

Changing the reference frame of the other galaxy

Light from another galaxy can be measured as giving an interference pattern, if one of the slits is observed then this changes 90° to a straight reference frame and the interference pattern disappears.

Gravis in a double slit experiment

There would also be +𝕘d×e𝕓 gravitational fields according to this model, these transmit changes in the +𝕚d and e𝕙 Pythagorean Triangle gravity and -𝕚d and e𝕧 Pythagorean Triangle inertia for example in between stars and planets. These could in principle be measured with a double slit experiment, matter should be able to shield a screen from these fields except for a double slit. In the rotational reference frame, there would be an interference pattern of gravis on a screen.

Not shielding against gravity

This is not the same as shielding against gravity, the gravis would be absorbed by protons like photons are absorbed by electrons. That should allow some to go through double slits in an experiment.

Observing gravis with no interference pattern

Observing one slit should produce an e𝔹/+𝕘d gravi impulse similar to gravitons. These would be in the straight-line reference frame, the interference pattern would disappear as with photons from that galaxy.

Line integrals and impulse

In (31.22) there are line integrals, these are associated with work and rotational reference frames here. On the right-hand side they can correspond to an impulse as a straight reference frame. The first equation has an electrical field, in this model both equations would be magnetic fields.

Comparing reference frames

The left-hand side would be -𝔾D×e𝕪 light work in empty space as photons, in the rotational reference frame there is no motion. The photon is measured like a stationary rolling wheel, the squared constant force here is μ. On the right-hand side there is the straight-line reference frame with a e𝕐/-𝕘d light impulse, the squared constant force here is ϵ. These can change inversely to each other as the angle θ of the e𝕪 and -𝕘d Pythagorean Triangle photon changes. Because these are inverses, they do not change the e𝕧/-𝕚d inertial velocity of c.

An electron’s squared forces

The -𝕠d and e𝕪 Pythagorean Triangle electron also has ϵ and μ as squared forces, with -𝕆D×e𝕪 kinetic work there is μ and with the E𝕐/-𝕠d kinetic impulse there is ϵ. When an electron is accelerated this is with ϵ in the straight-line reference frame. The rotational reference frame changes along with this so the Pythagorean Triangle area remains constant.

A rocket in both reference frames

In the rotational reference frame a rocket can then accelerate with -𝕆D×e𝕪 kinetic work from a rocket. This is measured as changes in positions like movie frames without time. In the straight-line reference frame time is observed with continuous changes in displacement. That is because a displacement includes the positions in between, measuring those individual positions must be quantized even if the separate positions were irregular.

Quantization is regular from multiplication

They are regular in this model because work only has whole numbers, it cannot contain derivative fractions because they are impulse only. So the fractions in between these whole numbers are in the straight-line reference frame, they can change continuously because they cannot jump with quantized amounts as in multiplication.

Photons transmit changes in work or impulse

In this model the photon transmits the changes in between iotas, these can be electrons in atoms with work. Then that is a rotational reference frame, the photon must be able to transmit those changes with μ to another atom. It can also transmit the changes in a straight-line reference frame, the electron outside the atom is no longer a wave. The photon can then collide with the electron, it transmits the change in ϵ from there to another electron in the straight-line reference frame.

ϵ and μ are inverse forces

If instead the photon is absorbed by an electron in an orbital, then μ is the inverse of ϵ. That comes from the constant Pythagorean Triangle area of an electron, whether inside or outside the atom. The inverse relation of the Pythagorean Triangle sides comes from the constant Pythagorean Triangle area, this allows for the particle/wave duality to be conserved.

A photon has a ratio of ϵ and μ

As the e𝕪 and -𝕘d Pythagorean Triangle photon moves it contains this ratio of the two Pythagorean Triangle sides, with a constant Pythagorean Triangle area. This can be regarded as ϵ×μ, the inverse ratio maintains the constant Pythagorean Triangle area. This is not the same as 1/(√ϵ×√μ) in conventional physics which represents a fraction of c as its inertial velocity. √ϵ has been inverted, so 1/√ϵ would be e𝕪 in this model and 1/√μ would be -𝕘d.

Squared constants with work and impulse

The e𝕪 side when squared gives a e𝕐/-𝕘d light impulse with ϵ. The -𝕘d side when squared gives -𝔾D×e𝕪 light work with μ. As inverses the ratio of these forces can change without affecting the inertial velocity of c.

The photon's inertial velocity does not change

That allows the photon to have its inertial velocity of c to be conserved, also because α is ≈ 1/137 of c then the photons can remain able to be absorbed with quantization in the atom. This also allows for the photon to slow down in a denser medium, around +𝕚d gravitational masses. This is because the photon transmits changes between the -𝕠d and e𝕪 Pythagorean Triangle electron and -𝕚d and e𝕧 Pythagorean Triangle inertia.

Gravity slows the photon's frequency

Gravity is the inverse of inertia, so a stronger +𝕚d gravitational field slows down the -𝕘d rotational frequency of the photon. In -𝔾D×e𝕪 light work this imparts a -𝔾D light torque onto the photon in its rotational reference frame. Then it curves around a star or can be trapped in the photosphere around an event horizon.

A rocket moves towards an event horizon

From sufficiently outside the event horizon's gravitational field, a rocket can move to a lower e𝕙 height. In the straight-line reference frame this redshifts the photons coming from the rocket. In the rotational reference frame the +𝕀D gravitational torque on the rocket curves the photon paths, that slows their -𝕘d rotational frequency to conserve the changes. Observing at 90°, this curve appears as a straight-line where the photon is accelerating upwards slowed by the E𝕙/+𝕚d gravitational impulse of the event horizon.

A past and future of photons

In this model the straight-line reference frame has c as e𝕧/-𝕚d. Because this is a derivative fraction, then different fractions can be compared as pasts and futures. To change a fraction impulse is needed in this model, otherwise they are like stationary points in Zeno's line paradox. With e𝕧/-𝕚d this would be the E𝕍/-𝕚d inertial impulse. The photon can then collide with an electron, then deterministically transmit this change over time to a second collision. This motion would be observed as light years.

Light years and light meters

In the rotational reference frame, the photons are emitted and absorbed by electrons in atoms. These change from one quantized position to another as whole numbers, multiplied like -𝕘d×e𝕪. Instead of light years these would be light meters, a light year would be approximately 9 quadrillion meters so it might be referred to as a light quad to use an equivalent value in meters to one light year.

A photon in the present

This light quad has no past or future, it refers only to a present in a rotational reference frame. The photon is like a stationary wheel or vortex, the ratio of the e𝕪 spoke to the -𝕘d axle gives the size and area of the wheel and its frequency. Because it is only in the present it does not change over time, such as with tired light, without an external force.

The rolling wheel

When the wheel is rolling in a straight-line reference frame it always moves at c, this is because the radius e𝕪 and frequency -𝕘d are inverses.

The gravi

In this model the +𝕘d×e𝕓 gravi is also like a rolling wheel, e𝕓 is the spoke and -𝕘d is the rotational frequency. This transmits gravitational changes in the e𝕙 height or e𝕓 depth of for example a planet. The rotation of its moon might move their center of mass, the change in the planet's inertia would subtract a different amount of -𝕀D×e𝕧 inertial work from its +𝕀D×e𝕙 gravitational work over a period of rotation.

The gravi moves at c

This would be transmitted as +𝔾D×e𝕓 gravi work, it would be measured as a stationary +𝕘d×e𝕓 wheel. In the straight-line reference frame, it would also move at e𝕙/+𝕚d as c. This is the same c value as e𝕧/-𝕚d in inertia, that makes the two inverses and balance each other such as in a photosphere.

α and c

This comes from α as the ground state of the Hydrogen atom, for simplicity. As $e^{-\mathbb{o}d}$ where d=1, the -𝕆D×e𝕪 kinetic work of the electron would be measured as a whole number of 1. Conversely the +𝕆D×e𝕒 potential work of the electron would also be 1 to balance this. Because the -𝕚d and e𝕧 Pythagorean Triangle inertia is proportional to the -𝕠d and e𝕪 Pythagorean Triangle electron, this gives a proportional value of α to -𝕀D×e𝕧 inertial work.

Gravity and inertia both use c

That conserves the -𝕚d inertial mass the electron as it changes orbitals. Inversely to this, the +𝕚d gravitational mass is also 1 with respect to this ground state. That is proportional to 1 with the proton's +𝕆D×e𝕒 potential work. With α as ≈ 1/137, the same in Roy electromagnetism and Biv space-time, then c is the same for both inertia and gravity.

> To begin, we will assume that electric and magnetic fields can exist independently of charges and currents in a *source-free* region of space. This is a very important assumption because it makes the statement that **fields are real entities;** they're not just cute pictures that tell us about charges and currents. The source-free Maxwell's equations, with no charges or currents, are
>
> $$\oint \vec{E} \cdot d\vec{A} = 0 \qquad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt}$$
> $$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \oint \vec{B} \cdot d\vec{s} = \epsilon_0 \mu_0 \frac{d\Phi_e}{dt} \qquad (31.22)$$
>
> Any electromagnetic wave traveling in empty space must be consistent with these equations.

Electromagnetic particle/wave photons

In this model, an electromagnetic wave has a straight-line and rotational reference frame. This gives the photon a particle/wave duality, the particle from the straight-line and the wave from the rotational reference frame. They do not induce each other, that came from Maxwell where a magnetic field appeared to induce an electric field. Instead it transmits changes in between electrons, its constant Pythagorean Triangle area means the two reference frames are connected together at 90°.

The Structure of Electromagnetic Waves

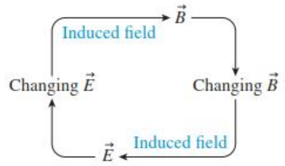


FIGURE 31.18 Induced fields can be self-sustaining.

Faraday discovered that a changing magnetic field creates an induced electric field, and Maxwell's postulated displacement current says that a changing electric field creates an induced magnetic field. The idea behind electromagnetic waves, illustrated in **FIGURE 31.18**, is that the fields can exist in a self-sustaining mode *if* a changing magnetic field creates an electric field that, in turn, happens to change in just the right way to recreate the original magnetic field. Notice that it has to be an *electromagnetic* wave, with changing electric *and* magnetic fields. A purely electric or purely magnetic wave cannot exist.

You saw in Section 30.6 that an induced electric field, which can drive an induced current around a conducting loop, is *perpendicular* to the changing magnetic field.

Transverse work and longitudinal impulse

In this model the induced magnetic field is in the rotational reference frame. Perpendicular to this there is the electric straight-line reference frame that observes particles not fields. The rotational reference frame of -𝔾D×e𝕪 light work is a transverse wave, the e𝕐/-𝕘d light impulse of a straight-line reference frame is a longitudinal impulse.

Moving at 90°

These are transmitted in a third 90° direction as the photon moves. This also has a rotational or straight-line reference frame, the photon can then react with -𝕀D×e𝕧 inertial work or have an E𝕍/-𝕚d inertial impulse in Biv space-time.

And earlier in this chapter, when we introduced the displacement current, the induced magnetic field in a charging capacitor was *perpendicular* to the changing electric field. Thus we'll make the assumption that **$\vec{E}$ and $\vec{B}$ are perpendicular to each other** in an electromagnetic wave. Furthermore—we'll justify this shortly—**$\vec{E}$ and $\vec{B}$ are each perpendicular to the direction of travel.** Thus an electromagnetic wave is a *transverse wave,* analogous to a wave on a string, rather than a sound-like longitudinal wave.

We will also assume, to keep the mathematics as simple as possible, that an electromagnetic wave can travel as a *plane wave,* which you will recall from Chapter 16 is a wave for which the fields are same *everywhere* in a plane perpendicular to the direction of travel. **FIGURE 31.19a** shows an electromagnetic plane wave propagating at speed v_{em} along the x-axis. $\vec{E}$ and $\vec{B}$ are perpendicular to each other, as we've assumed, *and* to the direction of travel. We've defined the y- and z-axes to be, respectively, parallel to $\vec{E}$ and $\vec{B}$. Notice how the fields are the same at every point in a yz-plane slicing the x-axis.

The e𝕪 and -𝕘d Pythagorean Triangle box

The Gaussian box has a 90° angle between the E⃗ or e𝕪 Pythagorean Triangle side and the B⃗ or -𝕘d Pythagorean Triangle side. These are the e𝕪 and -𝕘d Pythagorean Triangle photon Pythagorean Triangle sides, each corner of the box can be regarded as being connected to e𝕪 and -𝕘d/

Observing and measuring the box

Here photons can be regarded as entering and exiting the box, in the straight-line reference frame they would be observed with a e𝕐/-𝕘d light impulse. That would use the -𝕘d or B⃗ side to observe the time these photons entered and exited. They can also be measured with -𝔾D×e𝕪 light work using the e𝕪 box side as a distance. Then -𝔾D is the light probability of whether the photon is measured inside or outside the box.

Pythagorean Triangle area and side ratio

Because the e𝕪 and -𝕘d Pythagorean Triangle photon has a constant area, the number entering and exiting is conserved. This area is an integral in the rotational reference frame, connected to this is the Pythagorean Triangle side ratio as inverses in the straight-line reference frame. When these sides are inverses, the area is constant. This allows for the photons to be observed or measured as they go through the box.

Vectors and scalars

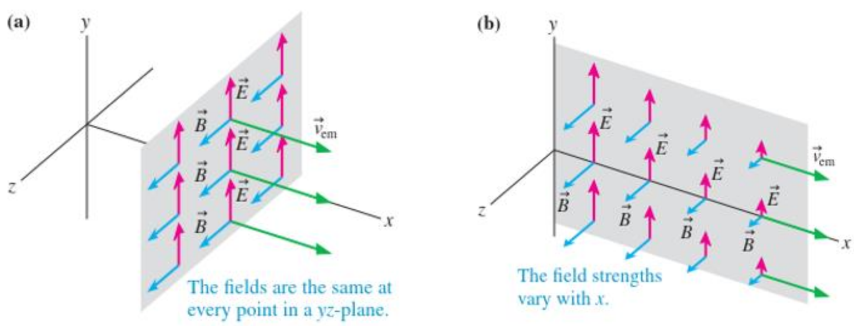
In this model a straight-line reference frame uses force vectors, they are observed with time as a scalar. This is because the time is a rotating vector, it cannot be observed as a vector itself. In the rotational reference frame torque is a scalar, in the sense that its rotation cannot be a vector in a direction. This is measured with a straight-line vector like a ruler. In conventional physics time and mass are scalars, as they are here. As a magnitude they would not be the length of a vector, but the time that is observed as the vector moves.

Now that we know something about the structure of the wave, we can start to check its consistency with Maxwell's equations. **FIGURE 31.20** shows an imaginary box, a Gaussian surface, centered on the x-axis. Both electric and magnetic field vectors exist at each point in space, but the figure shows them separately for clarity. $\vec{E}$ oscillates along the y-axis, so all electric field lines enter and leave the box through the top and bottom surfaces; no electric field lines pass through the sides of the box.

Three axes at 90°

Here the yz plane is analogous to the e𝕪×-𝕘d Pythagorean Triangle area, a point on this plane would be e𝕪 using the y axis. At 90° to y there is z as -𝕘d, this can be the rotational axle of the photon so that y turns around z at 90°. At 90° to both y and z is x, this would be the direction of the photon's motion with an e𝕧/-𝕚d inertial velocity. The origin can also be regarded as the corner of the box to observe and measure photons as they enter and exit.

FIGURE 31.19 An electromagnetic plane wave.



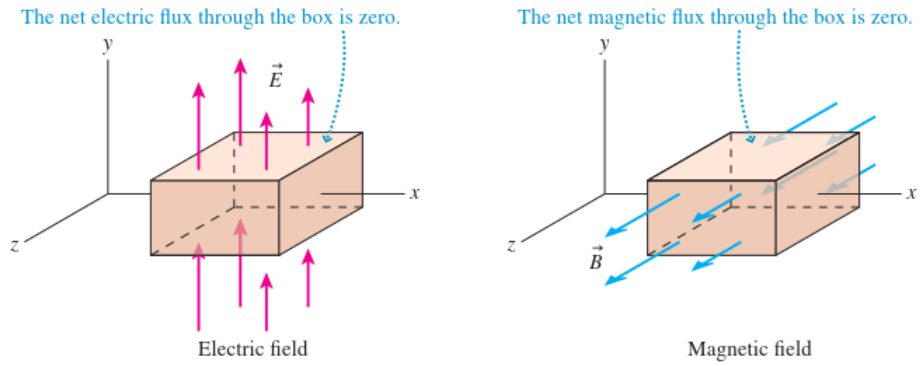
From the axis origin

Taking the axis origin, a photon can go through the box on the x axis. The z axis is then moving along the x axis in the rotational reference frame, while remaining at 90° to x. As z moves, the y axis would be moving up and down like a piston in the straight-line reference frame. This is like a longitudinal wave at 90\degrees to z as the transverse wave. Here y is not a wave, the acceleration up and down comes from impulse.

Different reference frames in the box

In the diagrams E⃗ can be regarded as e𝕪 moving up the y axis. B⃗ moves along the z axis. Each e𝕪 and -𝕘d Pythagorean Triangle photon is conserved in number, as one enters another would exit the box. Moving upwards on y can be the e𝕐/-𝕘d light impulse where photons are observed as particles. Moving horizontally on z can be -𝔾D×e𝕪 light work where photons are measured as waves.

FIGURE 31.20 A closed surface can be used to check Gauss's law for the electric and magnetic fields.



Electric charge and magnetic fields

In this model the vertical photons would be observed as particles in the straight-line reference frame. The horizontal photons would be measured as waves in the rotational reference frame. The straight-line reference frame is not an electric field here, in this model the electric field in Maxwell's equations is a magnetic field.

> Because this is a plane wave, the magnitude of each electric field vector entering the bottom of the box is exactly matched by an electric field vector leaving the top. The electric flux through the top of the box is equal in magnitude but opposite in sign to the flux through the bottom, and the flux through the sides is zero. Thus the *net* electric flux is $\Phi_e = 0$. There is no charge inside the box, because there are no sources in this region of space, so we also have $Q_{in} = 0$. Hence the electric field of a plane wave is consistent with the first of the source-free Maxwell's equations, Gauss's law.

Snapshot measurements

Here the magnetic flux is in the rotational reference frame, there is no continuous motion in and out of the box. Instead, there would be snapshot measurements of where the magnetic field is likely to be. This can vary, for example there might seem to be more going into the box than going out, but with electrons this would have a lower -𝕆D kinetic probability.

Destructive interference of probabilities

Also, there can be a lower -𝕆D kinetic probability of more coming out of the box than going in. These two kinetic probabilities tend to interfere destructively like with path integrals, the most kinetically probable measurements are where an equal number of -𝕠d and e𝕪 Pythagorean Triangle electrons go in and out of the box. This is not measured with time, but in snapshots the normal distribution would show this as an average.

Parts of a wave

The wave here would be in the rotational reference frame. There cannot then be part of a wave going into the box and a different segment of a wave going out, that would appear like an ocean wave for example. This would need the wave to move over time, in this model that only happens with particles in a straight-line reference frame. Those particles can be observed and counted so the numbers are conserved over time.

A flux in a rotational reference frame

Gauss's net flux here would only apply to rotational reference frames, so there can be a low probability of asymmetric measurements at positions inside the box. This is like a box where gas molecules flow in one side and out the other. In the straight-line reference frame there are collisions between the molecules, over time the numbers going in and out match. These collisions are chaotic, they result in a scattering of concentrations of molecules into a smoother gradient of pressure everywhere.

Boltzmann's and Planck's constant

This is from Boltzmann's constant as e𝕪 /-𝕆d ×-𝕠d, in the rotational reference frame the -𝕆D kinetic probabilities of the molecules follow a normal distribution. In the straight-line reference frame this occurs from the particle collisions and scattering. These two reference frames are inverses of each other, the inverse of e𝕪 /-𝕆d ×-𝕠d is -𝕠d ×e𝕐/-𝕠d as Planck's constant.

Photon collisions

As with the gas molecules, photons can be observed and measured going in and out of the box. In the straight-line reference frame, they collide with electrons in the air. That gives a scattering of e𝕪/-𝕘d photons like with the gas molecules. These photons are associated with -𝕠d ×e𝕐/-𝕠d as Planck's constant, they are emitted when an electron particle is observed in an atom at a quantized orbital.

Gas molecules in the box

The interactions between gas molecules in the box can also be described by the Boltzmann constant, then as the gas moves the quantized molecular bonds tend to form between other molecules. This can act like a drag or friction.

Gas condensation and torque

This could make the gas condense into a liquid, instead here it would impart a -𝕆D kinetic torque in between the molecules in the rotational reference frame. That creates a normal distribution of molecular velocities.

Light work in the box

In the box e𝕪×-𝕘d photons also do -𝔾D×e𝕪 light work in the rotational reference frame. This -𝔾D light torque is the same as a light probability, the photons can interfere with each other constructively and destructively. As snapshots of measurements, there would be an average -𝔾D light probability of the photons entering and exiting with the same probability densities.

The double box experiment

This can be compared to the double slit experiment, as a double box experiment. There would be a barrier around each box so no photons would be able to go in between the boxes. Then it would be expected that the flux would go through the two boxes the same as it going through a double slit.

Comparing a light source and a screen

The photons would be observed or measured on a screen, the amount of light being emitted from a lamp would be calculated. If the two match then overall the light's magnetic flux is the same in entering and exiting the boxes.

An electric field at 90°

This creates a problem with the electric field, it is said to be at 90° to the magnetic field. The same double box experiment can be done with electrons. If there is an electric field, then the electrons should be measurable as both kinds of fields. There is no other 90° to see this electric field, there is only the direction of motion at 90° to both.

Capacitors and inductors

The electric field in conventional physics is associated with a capacitor, the magnetic field with the coil of an inductor. These oppose each other in a circuit, Maxwell said that the gap in a capacitor would have an electric field in it, he called it a displacement current. This model has a magnetic field in the gap not an electric field. This is because the rotational reference frame of a field is at 90° to the straight-line reference frame of a particle.

Electrons in the double box experiment

In the double slit experiment, electrons from a circuit can be sent through the air and through these double slits. If attempts are made to observe which slits the electrons go through, like which of the double boxes, then they do not make an interference pattern on the screen. They act like particles like they often do in a circuit. These cannot jump over the gap in a capacitor, they cannot be an electric field because there is no interference pattern from a field.

Which box the electrons go through

When there is no observation of which slit, or which box, the electrons go through, then they make an interference pattern on the screen. This would be from the magnetic field of the electrons, that is known to be a field and act like a wave.

Capacitors in parallel

This double box experiment is like capacitors in parallel, they are next to each other so an electric field should create an interference pattern with neighboring capacitors. The anode of the capacitor has electrons like with a cathode ray tube, inside this there should be an electric field going to the cathode. If there is no electric field, the electrons would move as particles in a straight-line reference frame in the capacitor and cathode ray tube.

Interference patterns between capacitors

With two capacitors in parallel, then trying to observe which one the electrons in the current go through should remove an interference pattern between them. With no observation, the two capacitors should interfere with each other constructively and destructively. The double slit experiment was not known of in Maxwell's time, after Planck it was shown that electrons and photons have a particle/wave duality. This is not an electric field/magnetic field duality.

Entering one box and exiting the other

With the double box experiment, the flux is supposed to be the same entering and entering the boxes. Two boxes should be the same as cutting one box into two pieces. If photons and electrons can be measured as fields, then they might enter one box and exit the other. To see if this happens then one box can be observed, but then the interference pattern on the screen disappears.

The exact same argument applies to the magnetic field. The net magnetic flux is $\Phi_m = 0$; thus the magnetic field is consistent with the second of Maxwell's equations.

Suppose that $\vec{E}$ or $\vec{B}$ had a component along the x-axis, the direction of travel. The fields *change* along the x-axis—that's what a traveling wave is—so it would not be possible for the flux through the right face to exactly cancel the flux through the left face at every instant of time. An x-component of either field would violate Gauss's law by creating a net flux when there are no enclosed sources. Thus our claim that an electromagnetic wave must be a transverse wave, with the fields perpendicular to the direction of travel, is a requirement of Gauss's law.

Faraday’s law and a closed curve

Faraday’s law refers to a closed curve, this is in a rotational reference frame. There is then a -𝕆D kinetic torque where the electrons are turned through this curve. They would do this as electron waves with a magnetic field.

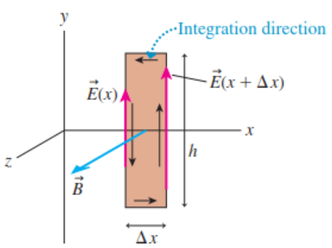
The rectangle is not a curve

The rectangle below is not a curve, the area would be an integral in the rotational reference frame if it was surrounded by a wire loop. Looking edge on to this loop it would appear like a single line in a straight-line reference frame. It would be expected that a E𝕐/-𝕠d kinetic impulse, such as from moving a bar magnet, would have electrons being observed as particles in this straight-line section.

A magnetic field from a bar magnet

This magnetic field can come from a bar magnet in a stationary position. If the bar magnet is moved to a different position over time, that creates a E𝕐/-𝕠d kinetic impulse in the edges of the rectangle if it is a wire loop. That is not an electric field in this model, in the straight-line reference frame at 90° the electrons move as particles. This is a change with respect to time like dΦ/dt but as electron particles. Then dΦ would be an electrical displacement E𝕐 not -𝕆D as a magnetic torque.

FIGURE 31.21 Applying Faraday’s law.



Faraday’s Law

Gauss’s law tells us that an electromagnetic wave has to be a transverse wave. What does Faraday’s law have to say? Faraday’s law is concerned with the changing magnetic flux through a closed curve, so let’s apply Faraday’s law to the narrow rectangle in the xy-plane shown in FIGURE 31.21. We’ll assume that Δx is so small that $\vec{B}$ is essentially constant over the width of the rectangle.

The magnetic field $\vec{B}$ is perpendicular to the rectangle, so the magnetic flux is $\Phi_m = B_z A_{rectangle} = B_z h \Delta x$. As the wave moves, the flux *changes* at the rate

$$\frac{d\Phi_m}{dt} = \frac{d}{dt}(B_z h \Delta x) = \frac{\partial B_z}{\partial t} h \Delta x \qquad (31.23)$$

Motion of a magnet

Here the line integral is with respect to a distance $\vec{s}$, that would be e𝕪 with -𝕆D×e𝕪 kinetic work. That can be measured at 4 sides of the rectangle, that remains in a rotational reference frame. At 90° to this is the straight-line reference frame, where $\vec{B}$ is shown in the z direction. There would be motion in this direction, like an electromagnet being switched on or a bar magnet moving over time.

Line integrals and points

In this model a line integral combines a stationary and a rotational reference frame. The line comes from Zeno’s points on a line, it is continuous. A point has an area around it because it is not a line, this is like a probability density of the points with work. If the points are evenly spaced then this is like quantization.

Overlapping point distributions

If not, then additional points can be included to make them all quantized. This is not necessary as there can be multiple fields overlapping with interference, then the points are measuring different integrals.

Two reference frames in a line integral

The line integral then combines two references frames, work and impulse, like with energy. The change in the electric field as an integral over a distance s here is work. Then the electrical field comes from the spin Pythagorean Triangle side, so it must be a magnetic field here.

Rotating around the rectangle

Here the line integrals rotate around the rectangle, this is in a rotational reference frame. The sign would remain the same in this model, for example -𝕆D×e𝕪 kinetic work would always be negative. With +𝕆D×e𝕒 potential work this is positive from the protons, that can be opposed to -𝕆D×e𝕪 kinetic work.

The ordinary derivative dB_z/dt, which is the full rate of change of B from all possible causes, becomes a partial derivative $\partial B_z/\partial t$ in this situation because the change in magnetic flux is due entirely to the change of B_z with time and not at all to the spatial variation of B_z.

According to our sign convention, we need to go around the rectangle in a counterclockwise direction to make the flux positive. Thus we must also use a counterclockwise direction to evaluate the line integral:

$$\oint \vec{E}\cdot d\vec{s} = \int_{\text{right}} \vec{E}\cdot d\vec{s} + \int_{\text{top}} \vec{E}\cdot d\vec{s} + \int_{\text{left}} \vec{E}\cdot d\vec{s} + \int_{\text{bottom}} \vec{E}\cdot d\vec{s} \qquad (31.24)$$

Integral at 90° to the line

The integral here begins on the left edge, then over a distance x+Δx the change is measured as the integral area. There would be a -𝕠d Pythagorean Triangle side at 90° to all points on this line, pointing inwards into the rectangle. That is like inside a coil in an inductor, if there was a current moving around the rectangle that would create a -𝕆D kinetic torque at 90° to it. Changing the wire loop dimensions by Δx would change the -𝕆D kinetic probability and torque inside it as an inverse square law.

Δx and e𝕪 as infinitesimals

Δx would be an infinitesimal, with -𝕆D×e𝕪 kinetic work the positions of e𝕪 are on a straight-line like the lines around the rectangle. An increment of e𝕪 is an infinitesimal, many of these would be a series of points not a line.

No straight-line boundaries on a field

With the whole rectangle, the change in distance as e𝕪 here would be at 90° along z not x, like $\vec{B}$ which is e𝕪 in this case. Then the integral rectangle is -𝕆D as a kinetic probability, it has no boundaries as straight lines unless there is a wire loop.

Moving in the x direction

If the loop was moving in the x direction instead, and there was a magnet pointing along z, then moving the loop would create a E𝕐/-𝕠d kinetic impulse in the wire. Inside the loop would be the changing -𝕆D kinetic probability from the magnet, as this changed the electrons would be measured in different positions as a current.

Inducing a current

A magnet can move along z with the rectangle remaining stationary, this changes the -𝕆D kinetic torque and probability density in it. If there is a rectangle there as a wire loop, then this would induce an e𝕪/-𝕚d kinetic current in it.

Measuring a point on z

That current would be at 90° to the magnet's E𝕐/-𝕠d kinetic impulse motion, that would come from moving the magnet along z as a change over time. The integral area of the rectangle is in a rotational reference frame, at 90° to the motion of the magnet along z. That does -𝕆D×e𝕪 kinetic work.

Iron filing measuring a magnetic torque

This is like iron filings on paper. At 90° to a magnet they show its magnetic field. The iron filing pattern is in circles in a rotational reference frame. Looking edge on to the paper, this would be in a straight-line reference frame. Then as the iron filings moved, they would be observed to accelerate with a E𝕐/-𝕠d kinetic impulse.

The paper as the rectangle

The paper can be regarded as being like the rectangle, the iron filings represent what the magnetic field is doing. The shape of the iron filings does not correspond to the rectangular piece of paper, the sides of that are a straight-line reference frame. When the magnet moved along z it could come closer to the paper, then the iron filing might move as the stronger -𝕆D kinetic probability overcome friction on the paper.

Moving the paper along the x axis

Moving the magnet on the x axis, the iron filings would move along the paper in that direction. Looking edge on in the straight-line reference frame, the iron filings would be accelerating then decelerating with a E𝕐/-𝕠d kinetic impulse and E𝕍/-𝕚d inertial impulse.

Moving the paper does work

The edge of the paper is analogous to (31.25), The change in the magnetic field is like moving the paper a distance of Δx. This does -𝕆D×e𝕪 kinetic work in moving some of the iron filings to a more kinetically probable position.

Along the left edge of the loop, at position x, E has the same value at every point. Figure 31.21 shows that the direction of $\vec{E}$ is *opposite* to $d\vec{s}$; thus $\vec{E}\cdot d\vec{s} = -E_y(x)ds$. On the right edge of the loop, at position $x + \Delta x$, $\vec{E}$ is *parallel* to $d\vec{s}$, and $\vec{E}\cdot d\vec{s} = E_y(x+\Delta x)\,ds$. Thus the line integral of $\vec{E}\cdot d\vec{s}$ around the rectangle is

$$\oint \vec{E}\cdot d\vec{s} = -E_y(x)h + E_y(x+\Delta x)h = [E_y(x+\Delta x) - E_y(x)]\,h \qquad (31.25)$$

Derivatives in calculus

In this model the calculus equation below would not be used, this has Δx in the denominator. Instead, this would be with respect to time as -𝕠d with the -𝕠d and e𝕪 Pythagorean Triangle electrons. This is an integral that comes from the -𝕠d and e𝕪 Pythagorean Triangle, the infinitesimal e𝕪 is like Δx and -𝕠d is like a fluxion.

A change in a derivative needs a force

A change in the derivative needs a force, this is observed over an instant of time as the fluxion. Then the difference between x and Δx would be a displacement as E𝕐. When Δx goes to zero then the displacement force disappears, then E𝕐 becomes e𝕪.

Δx as points on a line

In this model then, x would be a point and adding Δ x to it would be another point. These still remain separated even though Δx is infinitely small. This is the same as with Zeno's points on a line, the points are like Δx, there can always be more Δx points added between them. It never becomes a lines, here then E𝕐 is a line as a displacement between two points.

Separating derivatives and integrals

In this model the Pascal's Triangle calculus is used, each cell in Pascal's Triangle is a derivative multiplied by an integral. For example in line 3, there is x^3, $3x^2y$, $3xy^2$, y^3. $3x^2y$ is composed of $3x^2$, which is the derivative of x^3 to its left, and y^1. Moving one cell to the right $3x^2$ becomes 6x, then the integral of y is $1/2y^2$, multiplying these together gives $3xy^2$ which is the correct value. The next derivative of x is 6, the next integral of $1/2y^2$ is $1/6y^3$. Multiplying this by 6 gives y^3.

Creating the concept of an infinitesimal

In this model the calculus process of using Δx, which goes to zero, is used to remove the integral in the Pythagorean Triangles cell. Then only derivatives need to be calculated. A similar process can remove the derivative so the integral can be calculated.

Defining y

The function below can then be $3x^2$, taking the derivative needs to get the correct answer 6x here. This is the same process as moving a cell to the right and then removing or ignoring the integral section. Using x and y in the Pythagorean Triangle that is done by setting y to equal an infinitesimal. This allows the integral to not change the function.

An infinitesimal definition

The next cell in the row changes by this infinitesimal Δy here, $3x^2$ times y becomes as before 6x times $1/2y^2$. Then as an infinitesimal y^2 is the same as $1/2y^2$, also the same as y. As an infinitesimal Δy is in a sense the same as zero, but multiplying by zero would make 6x also zero here. That means y=1 or y=0 cannot be used here, but the custom definition of an infinitesimal works. The result is that the derivative of $3x^2y$ here becomes 6x.

The Pascal's Triangle calculus

It seems likely Newton knew this as he invented calculus and used Pascal's triangle. This also connects to probability because the cells relate to permutations and combinations. A line can give probabilities which goes to a normal curve distribution. Horizontally it becomes a Taylor series to calculate derivatives to different approximations.

The Taylor Maclaurin series

This also gives the Taylor series, the denominator is like the increasing coefficient of the y integral in moving to the right one cell at a time. This can be used for example with the third row of the Pascal's Triangle calculus, here f(a) is a function set to zero which is called a Taylor Maclaurin series. The variables here are f(a) as a, and x. Calling a f(a) allows for it to be separated from the integral in each cell.

A function like an infinitesimal

That gives a^3, $3a^2x$, $3ax^2$, x^3. The first cell as a^3 is f(a), it is set to zero but it still acts like a function to avoid multiplying by zero. The next cell is the derivative of a^3 which is $3a^2$, this is multiplied by the integral as 1/1! times x. Then the next cell is added as the second derivative of a^3 which is 6a, that is multiplied by the integral of x as ½!x^2. To this is added the third derivative which is 6, that is multiplied by the integral as 1/3!x^3.

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\cdots,$$

The Taylor series with complex numbers

The Taylor series can also use a nonzero a here, for example by making x=0. Then the Pascal's Triangle calculus has (a-ai) at the top as a complex number. Using xi as an imaginary number instead of x, along with ai instead of a, then xi-ai would be an imaginary number here. The real numbers a give the derivatives as straight Pythagorean Triangle sides and a straight-line reference frame. The integrals as ai give spin Pythagorean Triangle sides and a rotational reference frame.

Obscure and Intangible numbers

Instead of using complex numbers, this model would use Obscure and Intangible numbers. These would be from the +𝕠d and e𝕒 Pythagorean Triangle proton, the -𝕠d and e𝕪 Pythagorean Triangle electron, the 𝕠d- and 𝕞 Pythagorean Triangle neutrino, the e𝕪 and -𝕘d Pythagorean Triangle photon, the e𝕒 and +𝕘d Pythagorean Triangle virtual photon, the +𝕘d and e𝕓 Pythagorean Triangle gravi, the e𝕧 and -𝕘d Pythagorean Triangle iner, the +𝕚d and e𝕙 Pythagorean Triangle as gravity, the 𝕚d and e𝕥 Pythagorean Triangle as the space-time part of the neutrino, and the -𝕚d and e𝕧 Pythagorean Triangle as inertia.

Rows of Pythagorean Triangles

Each of these has the first term as the straight Pythagorean Triangle side, the second as the rotational Pythagorean Triangle side. This works like a+bi as a complex number. For example with gravity the first three rows would be e𝕙+𝕚d, Eℍ +𝕚d×e𝕙 +𝕀D, e$𝕙^3$ +𝕚d×Eℍ, +𝕀D×e𝕙, +$𝕚d^3$, and so on with additional rows. When these terms in a row are summed with the Taylor series, the positive sign comes from the cells here. Using for example e𝕧-𝕚d then the terms would be negative after the first one.

A function like an infinitesimal

Here the Taylor Maclaurin series has a function acting like an infinitesimal, the calculus definition used Δx which also acted like an infinitesimal. In both cases they act like zero, but they obey the rules of multiplication and division. Here the function is kept separate, it can then be calculated and reduced to a value before using it in this equation.

Separating the integrals to use derivatives

The same can be done to separate the integrals from the derivatives. Then the integrals as x/1! + x^2/2!+x^3/3!+... become ∫(x)+∫∫(x)+∫∫∫(x)+... . That allows for the rules of derivatives and integration to be the same as in conventional calculus. When combined they give the binomial theorem going to the normal distribution in the rows. Vertically the cells are linear as logarithms.

The exponential function in the Pascal's Triangle calculus

The Pascal's Triangle calculus here would also give the exponential function for example. This is a sequence of the integral parts of each cell. In this model x here would be -𝕠d for example, then the sequence gives the spin Pythagorean Triangle side. When this is divided by the hypotenuse it gives an angle of the Pythagorean Triangle, that is sinθ in this model.

The exponents from the Taylor series rows

When the inverse of this is used then it becomes derivatives, the exponents reduce by one in each term, the coefficient increases as n!. That gives the straight Pythagorean Triangle side number in the exponent such as e𝕪, together each term would be multiplied together to give the cells of the Pascal's Triangle calculus from $e^{e𝕪-𝕠d}$.

Constant area trigonometry

Because there is a constant Pythagorean Triangle area, the two sides change inversely to each other, In constant area trigonometry, the hypotenuse is no longer constant to allow for the area instead to be constant. This does not change the angles in trigonometry, nor the changing from cosθ to sinθ as derivatives and integrals.

The exponent from the Maclaurin series

Because the Pythagorean Triangle sides here are inverses, then the exponent such as $e^{e𝕧\text{-}𝕚d}$ works differently to the Euler equation. In that the Pythagorean Triangle is inscribed in a circle, the hypotenuse remains constant, and the exponents as complex number rotate the hypotenuse in the circle. When the Pythagorean Triangle sides are inverses they go into the exponent with the formula below.

Tracing a spiral

That means the circle's hypotenuse is not constant, as the angle changes it traces a logarithmic spiral. When the -𝕚d and e𝕧 Pythagorean Triangle is used, this has the right angle at the origin. Then a hyperbola is traced out at points on its hypotenuse. The area in the hyperbola gives e like the series below.

The Maclaurin series of the exponential function e^x is

$$\sum_{n=0}^{\infty}\frac{x^n}{n!}=\frac{x^0}{0!}+\frac{x^1}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}+\cdots$$

$$=1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}+\cdots.$$

Expressed in sines and cosines

This can be expressed in sines and cosines. The first cell in the Pascal's Triangle calculus row is from the straight Pythagorean Triangle side, for example $e𝕧^3$. This continues on to give cosθ with the add cells. On the right-hand side the cell is $\text{-}𝕚d^3$, going to the left as even numbers these give sinθ.

The exponents connect to the angle θ

That makes e𝕧-𝕚d as $e^{e𝕧\text{-}𝕚d}$ equal to cosθ+isinθ in conventional trigonometry, here isinθ can be written as cosθ-𝕚sinθ. If the +𝕚d and e𝕙 Pythagorean Triangle was used as gravity this would be +𝕚sinθ. When added together the odd and even terms alternate to make this value of $e^{e𝕧\text{-}𝕚d}$ an angle θ in cosθ-𝕚sinθ.

The same in constant area trigonometry

Here this is in constant area trigonometry, the e𝕧 and -𝕚d Pythagorean Triangle sides vary inversely so the hypotenuse is not a constant. The angle is the same as is the formula with sines and cosines.

The product rule for derivatives

In conventional calculus taking the derivative of a product xy gives dxy+xdy. The second row of the Pascal's Triangle calculus is $x^2+2xy+y^2$. Then the central term 2xy, moving this to the right takes the derivative of 2x to give 2. Then the integral of 2y is $1/2\times 2y^2$ which is the third term in the row. Going to the left the same way, because this is derivatives for both x and y, this gives the derivative of 2xy with respect to y as 2y, then the integral is $½\times 2x^2$ or x^2 the first term in the row. So taking the central term, the product rule adds the terms on both sides.

The Product Rule

$$F(x)=f(x)\cdot g(x)$$

$$F'(x)=f'(x)\cdot g(x)+g'(x)\cdot f(x)$$

OR

$$F(x)=(first)\cdot(second)$$

$$F'(x)=(first)'\cdot(second)+(second)'\cdot(first)$$

The product rule for integration

This is written as $\int xdy=xy-\int ydx$. This becomes $xy=\int xdy+\int ydx$ as with derivatives. The dx part is treated like a derivative, so this combines derivatives and integrals as in the Pascal's Triangle calculus cells. Starting again with 2xy, this can come from x^2, moving this to the right gives $2x\times y$. When starting from y^2, moving to the left gives $2y\times x$. Conversely the integral for 2xy is multiplied by the derivative of y, this gives $x^2\times 1$. The integral for 2y is multiplied by the derivative for x which gives $y^2\times 1$.

Differentiation and integration in opposite directions

The answer is the same, the direction is the opposite here as with derivatives. They start from x^2 and y^2 to arrive at 2xy. Here 2xy is the starting point to arrive at x^2 and y^2. That allows for the derivative to be taken to arrive at the central Pascal's Triangle calculus cell as a derivative. Then the integral is taken to arrive at the integral cells, and so on back to differentiating. These are at 90° to each other because the Pythagorean Triangle sides are at 90°.

Lumping and splitting

The straight-line reference frame here comes from differentiating, this is a splitting up into objects that are separate. The same definition is in common language. When these are put back together they function as a whole, this is called integration in a rotational reference frame.

$$\int u\,dv=uv-\int v\,du$$

the product rule for differentiation

where it comes from: $$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$$

put into reverse $$\int\frac{d}{dx}(uv)=\int(u\frac{dv}{dx}+v\frac{du}{dx})$$

$$uv=\int u\frac{dv}{dx}+\int v\frac{du}{dx}$$

and then rearranged $$\int u\frac{dv}{dx}=uv-\int v\frac{du}{dx}$$

Approximating a function

The Taylor Maclaurin series here would grow larger where $[f(a)+x]^n$ is the top of the Pascal's Triangle calculus. This gives a way of approximating the overall function of a and x, by continuing this along the row this can be stopped to give a reasonable approximation.

Fractional dimensions

In this model the exponent of a row is usually an integer, this is quantized with the rotational reference frame. When the exponent is a fraction, this is where fractals can be modeled. This uses fractional derivatives and integrals.

Moving off the row

In this model other polynomials can be created by moving off a single row, for example $a^3+4a^3x+10a^4x^2+6a^2x^2+x^7$. Each can be regarded as a multiplied derivative and integral, then the derivative function f(a) can be calculated separately to the integral. This should make it easier to find the roots of the polynomial. Where the coefficient is different this can be multiplied to make it fit to a suitable cell for the exponents.

Pythagorean Triangles and the Pascal's Triangle calculus

In this model the top of the Pascal's Triangle calculus is each Pythagorean Triangle. For example (e𝕪-𝕠d)^n gives permutations and combinations, the rows approach the normal curve as an inverse exponential. The columns approach the exponential curve.

NOTE $E_y(x)$ indicates that E_y is a function of the position x. It is *not* E_y multiplied by x.

You learned in calculus that the derivative of the function $f(x)$ is

$$\frac{df}{dx} = \lim_{\Delta x \to 0}\left[\frac{f(x+\Delta x) - f(x)}{\Delta x}\right]$$

We've assumed that Δx is very small. If we now let the width of the rectangle go to zero, $\Delta x \to 0$, Equation 31.25 becomes

$$\oint \vec{E} \cdot d\vec{s} = \frac{\partial E_y}{\partial x} h \Delta x \qquad (31.26)$$

No derivatives of both position and time

In this model there cannot be a derivative of both position and time. with respect to position that would be an integral such as -𝕆D×e𝕪 kinetic work. With respect to time there would be a derivative such as the E𝕐/-𝕠d kinetic impulse. In (31.27) E would be proportional to x because both are straight Pythagorean Triangle sides, B is proportional to t because they are both spin Pythagorean Triangle sides.

Removing the integral

The area hΔx of the rectangle is like the integral in the Pascal's Triangle calculus that is removed, that only leaves the derivatives here.

Calculus and the Pythagorean Equation

This model has a different version of calculus, that is because the integral areas and slopes come from Pythagorean Triangles. When the integral is taken this squares a Pythagorean Triangle side, that happens in the Pythagorean Equation. When the derivative is taken this is a small Pythagorean Triangle tangent to a curve, this would have a squared straight Pythagorean Triangle side as impulse. Together they are squaring both Pythagorean Triangle sides which gives a squared hypotenuse.

We've used a partial derivative because E_y is a function of both position x and time t.
Now, using Equations 31.23 and 31.26, we can write Faraday's law as

$$\oint \vec{E} \cdot d\vec{s} = \frac{\partial E_y}{\partial x} h\Delta x = -\frac{d\Phi_m}{dt} = -\frac{\partial B_z}{\partial t} h\Delta x$$

The area $h\,\Delta x$ of the rectangle cancels, and we're left with

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad (31.27)$$

Equation 31.27, which compares the rate at which E_y varies with position to the rate at which B_z varies with time, is a *required condition* that an electromagnetic wave must satisfy to be consistent with Maxwell's equations.

Measuring a wave's position with a probability density

In (31.28) this is at 90° to the previous rectangle. Because this is with respect to time it would be the E𝕐/-ⓓd kinetic impulse of electrons, with light it would be the e𝕐/-𝕘d light impulse. The flux is changing because a single electron or photon particle can go through the rectangle. A wave has an uncertainty as its boundaries are not defined like a particle, part of one wave might enter the rectangle while part of another leaves it. This cannot be defined by time, a position on the rectangle can have a probability density associated with it. Then these can be added to give an average position of the flux.

The Ampère-Maxwell Law

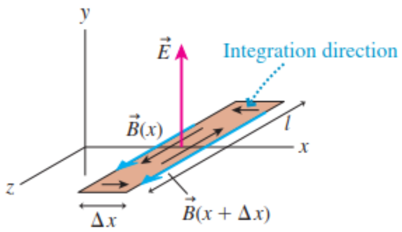
We have only one equation to go, but this one will now be easier. The Ampère-Maxwell law is concerned with the changing electric flux through a closed curve. **FIGURE 31.22** shows a very narrow rectangle in the xz-plane. The electric field is perpendicular to this rectangle; hence the electric flux through it is $\Phi_e = E_y A_{\text{rectangle}} = E_y l\,\Delta x$. This flux is changing at the rate

$$\frac{d\Phi_e}{dt} = \frac{d}{dt}(E_y l\Delta x) = \frac{\partial E_y}{\partial t} l\Delta x \qquad (31.28)$$

Changing electric flux as a magnetic field

In the diagram the change with respect to a distance is in the rectangular integral as -𝕆D×e𝕪 kinetic work. Changing 90° to the straight-line reference frame gives the E𝕐/-ⓓd kinetic impulse change with respect to -ⓓd kinetic time. However in this model the electric flux through a closed curve is the magnetic field as well, the curve has a -𝕆D kinetic torque with electrons. Each electron can be measured in a rotational reference frame, or observed in a straight-line reference frame. The rectangle can then be changed 90° to the one below, also at 90° to the previous rectangle.

FIGURE 31.22 Applying the Ampère-Maxwell law.



The same line integral

The line integral here is the same as the previous one. In this model that is because they are both magnetic fields. Here Δx again goes to zero, that removes the integral from the Pascal's Triangle calculus. If the integral remains, then permutations and combinations can be used with the impulse and work here.

Permutations and combinations

Permutations would be used where there are collisions in a straight-line reference frame. These can happen only one way as the particles cannot go through each other. Combinations occur with waves in a rotational reference frame, the permutations can be added to as there is no deterministic sequence with particles. The waves can pass through each other so that they combine with constructive and destructive interference.

Rotational reference frames in rows

Multiplying the derivatives and integrals in cells also allows for a rotational reference frame as an overall row. This is the binomial theorem of probability approaching a normal curve. That comes from squaring the spin Pythagorean Triangle sides such as -𝕠d as an exponent. The formula for e is also derived from a row with a Taylor series above. These combinations then connect to the integrals here.

Vertical columns as straight-line reference frames

Vertically the columns are linear as logarithms, so they are exponentials. The permutations can be like collisions in a chain reaction leading to exponential growth over time in a straight-line reference frame.

Columns and rows at 90°

At 90° to this is the rotational reference frame of the rows, the normal curve is an inverse exponential. This inverse comes from the Pythagorean Triangle used have sides as inverses of each other.

Entropy

Entropy is also in the Pascal's Triangle calculus, this is the number of possible paths. The cells can be regarded as being like branches of a tree, there are different routes through the branches to get to a particular cell. With more paths the entropy is higher, that happens when the allowed rows are wider with probability. The scattering of the branches selected occurs vertically in a straight-line reference frame deterministically.

The Taylor series with sines and cosines

The Taylor series also gives an infinite series for sinθ and cosθ in the Pythagorean Triangle used, for example with the +𝕠d and e𝕒 Pythagorean Triangle proton. Starting from the left, the odd terms give cosθ and the even terms give sinθ. These correspond to the angle θ in the Pythagorean Triangle used, if this angle changes then the d:e ratio in the +𝕠d and e𝕒 Pythagorean Triangle would change also in the Taylor series for cosθ and sinθ.

Differentiating and integrating sines and cosines

These are composed of derivatives and integrals, so cosθ and sinθ can change into each other by moving one cell left or right in a row. This corresponds to an additional derivative, and one less integration, in moving to the right for example.

The line integral of $\vec{B} \cdot d\vec{s}$ around this closed rectangle is calculated just like the line integral of $\vec{E} \cdot d\vec{s}$ in Figure 31.21. $\vec{B}$ is perpendicular to $d\vec{s}$ on the narrow ends, so $\vec{B} \cdot d\vec{s} = 0$. The field at *all* points on the left edge is $\vec{B}(x)$, and this field is parallel to $d\vec{s}$ to make $\vec{B} \cdot d\vec{s} = B_z(x)\,ds$. Similarly, $\vec{B} \cdot d\vec{s} = -B_z(x + \Delta x)\,ds$ at all points on the right edge, where $\vec{B}$ is opposite to $d\vec{s}$. Thus, if we let $\Delta x \to 0$,

$$\oint \vec{B} \cdot d\vec{s} = B_z(x)l - B_z(x + \Delta x)l = -[B_z(x + \Delta x) - B_z(x)]l = -\frac{\partial B_z}{\partial x} l \Delta x \quad (31.29)$$

Inverses in the electromagnetic wave

Taking this as a constant c^2, that can be on the right-hand side of the equation. The -𝕆D×e𝕪 kinetic work and E𝕐/-𝕠d kinetic impulse would be on the left-hand side. In this model the two would be written as inverses, so as -𝕆D increases then E𝕐 decreases inversely as the square of the e𝕪 photon spoke.

The ratio of ϵ:μ

In the photon this would be E𝕐/-𝔾D=c^2=ϵ×μ. If E𝕐 increases then the e𝕐/-𝕘d light impulse is stronger in the straight-line reference frame, photons are observed more as particles. If -𝔾D increases then -𝔾D×e𝕪 light work increases in the rotational reference frame as waves. The inertial velocity of c does not change because the two squared forces are at 90° to each other.

Changing this ratio in two reference frames

This is like in the rolling wheel model, the wheel can double in size as E𝕐 increases, then the rotational frequency squared as -𝔾D halves. In the straight-line reference frame the wheel moves at the same inertial velocity. In the rotational reference frame the wheel rotates half as fast tracing out a circumference twice as large. This comes from 2πr when the radius e𝕪 is doubled. That sweeps out the same circumference, for example initially a full circle was completed, then with double the radius half the circle is completed.

The circle's area as work or impulse

Instead of being an area, πE𝕐 as πr^2 can be regarded as a force vector times π. Then the rotational frequency -𝕘d can be drawn as the radius r, then the -𝔾D×e𝕪 light work has the integral area of the circle as a -𝔾D light torque. These become inverses of each other, so doubling the e𝕐/-𝕘d light impulse would have -𝔾D×e𝕪 light work.

Equations 31.28 and 31.29 can now be used in the Ampère-Maxwell law:

$$\oint \vec{B} \cdot d\vec{s} = -\frac{\partial B_z}{\partial x} l \Delta x = \epsilon_0 \mu_0 \frac{d\Phi_e}{dt} = \epsilon_0 \mu_0 \frac{\partial E_y}{\partial t} l \Delta x$$

The area of the rectangle cancels, and we're left with

$$\frac{\partial B_z}{\partial x} = -\epsilon_0 \mu_0 \frac{\partial E_y}{\partial t} \quad (31.30)$$

Equation 31.30 is a second required condition that the fields must satisfy.

Rearranging the wave equation

In equation (31.31) this can be rearranged to give $\partial x^2/\partial t^2 = v^2$, the displacement D comes from the separation of the squares of time and distance. In this model displacement is moving from a starting to a final position, the actual distance between these positions. For example, in moving between two positions on a ruler, displacement refers to the interval between them.

Motion in between two points

Also the motion in between two points implies an acceleration or force, to leave the first position requires a force otherwise the two positions would not be connected. This happens in work according to this model, the positions are used to measure torque at them. In impulse the displacement is observing the motion of a particle in between those positions over time.

The wave equation at c

In (31.34) the velocity squared is substituted by $1/\epsilon\mu$ which is c^2, then the displacement is substituted here by E. That would represent the photon as a wave moving at c. In this model x^2 would come from the e𝕐/-𝕘d light impulse of the photon, $1/t^2$ would come from -𝔾D/e𝕪 light work. This would give E𝕐/-𝔾D, that would be proportional to E𝕍/-𝕀D as the inertial velocity c^2 here.

Displacement as E and B

In (31.32) the displacement D is substituted by two forces as E and B. These would be the electric displacement here as E𝕐, B would be -𝔾D as the light torque or probability of the photon. That would also be the magnetic component of its electromagnetism. Taking the numerators, and moving B to the left-hand side gives E/B=c^2=E𝕍/-𝕀D. That means the electric force E as E𝕐 is proportional to the squared inertial displacement E𝕍, the magnetic force B is proportional to the squared -𝕀D inertial mass.

Changing division to multiplication

Because the e𝕪 and -𝕘d Pythagorean Triangle as the photon has a constant area, also with the -𝕚d and e𝕧 Pythagorean Triangle, that gives an equation E/B=E𝕐/-𝔾D∝E𝕍/-𝕀D. When the division is substituted by multiplication this gives an integral field as E𝔹=-𝔾D×E𝕐∝-𝕀D×E𝕍.

Pythagorean Triangle area and side ratios

This is allowed with this model because now it refers to the areas of the Pythagorean Triangles, not the ratios of the sides. Because the Pythagorean Triangles are the same this is now in the rotational reference frame. Now with the constant Pythagorean Triangle area, if -𝔾D doubles for example then E𝕐 halves, the photon can then change its frequency and wavelength inversely to each other. This still equals -𝕀D×E𝕍 so the photon still moves at c^2.

A constant velocity

When the photon moves near a +𝕚d gravitational field it is slowed down, so here E𝕍/-𝕀D changes its ratio just as E𝕐/-𝔾D did when the photon changed its frequency and wavelength. This means that c is no longer a constant, when it is slowed down near an event horizon the photon's frequency and wavelength are not changed in its own reference frame.

The inertial velocity changes

So in the first case the frequency and wavelength changed without the inertial velocity changing. Now the inertial velocity is changing without the frequency and wavelength changing. This follows from the constant Pythagorean Triangle areas.

Faster than c

In this model the photons can be slowed to under 1/c approaching the limit of the -𝕚d and e𝕧 Pythagorean Triangle angle θ. They can also increase to greater than c without them changing in their own reference frame. A rocket would then move faster than c without light changing in between people on the rocket.

Changing the frequency and wavelength

When the frequency and wavelength of a photon changes, this requires a force. That can be the e𝕐/-𝕘d light impulse in the straight-line reference frame such as a photon colliding with an electron. It can also be through -𝔾D×e𝕪 light work when a photon is absorbed and emitted from an atom. The e𝕐/-𝕘d light impulse changes with 1/ϵ or E𝕐 here as a squared constant, -𝔾D×e𝕪 light work changes with μ or 1/-𝔾D.

Changing the inertial velocity

In (31.34) then ϵ and μ are squared forces, they are not limits but the strengths of how these forces change. In this model c comes from α as ≈ 1/137, so ≈ 137 times the inertial velocity α is c. This is a constant here because it transmits information in between electron orbitals as quantized increments of α as $e^{-𝕠d}$. So c can change here, it can slow near an event horizon and distant galaxies can be moving greater than c.

Redshift

This is done by work and impulse as well. The event horizon has a strong Eℍ/+𝕚d gravitational impulse, this pulls the photon downwards with its e𝕐/-𝕘d light impulse changing. As Eℍ increases then E𝕐 decreases inversely, if the photons were emitted from a rocket, then this would be observed as a redshift from a sufficient distance.

Slowed frequency

When +𝕀D×e𝕙 gravitational work increases this is in the rotational reference frame, the photons are curved around the event horizon with a +𝕀D gravitational torque. This decreases the -𝔾D light torque or probability of the photon.

The Wave Equation

In « **Section 16.4**, during our study of traveling waves, we derived the *wave equation:*

$$\frac{\partial^2 D}{\partial t^2} = v^2 \frac{\partial^2 D}{\partial x^2} \qquad (31.31)$$

There we learned that any physical system that obeys this equation for some type of displacement D can have traveling waves that propagate along the x-axis with speed v.

If we start with Equation 31.27, the Faraday's law requirement for any electromagnetic wave, we can take the second derivative with respect to x to find

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial^2 B_z}{\partial x \partial t} \qquad (31.32)$$

Ocean wave impulse

An increase in the E𝕐/-𝕠d kinetic impulse in a straight-line reference frame, decreases e𝕪/-𝕆D kinetic work in the rotational reference frame. An ocean wave would increasingly be longitudinal in force more like a tsunami with this E𝕐/-𝕠d kinetic impulse. The wave would have a lower -𝕆D kinetic torque, being formed by the straight-line displacement change of an earthquake.

Ocean wave torque

When a wave increases in the rotational reference frame, the -𝕆D kinetic torque increases. An ocean wave like this would have a shorter distance between waves, a deeper trough, and cause ships to roll more with this torque.

You've learned in calculus that the order of differentiation doesn't matter, so $\partial^2 B_z/\partial x\,\partial t = \partial^2 B_z/\partial t\,\partial x$. And from Equation 31.30,

$$\frac{\partial^2 B_z}{\partial t\,\partial x} = -\epsilon_0\mu_0\frac{\partial^2 E_y}{\partial t^2} \qquad (31.33)$$

Substituting Equation 31.33 into Equation 31.32 and taking the constants to the other side, we have

$$\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{\epsilon_0\mu_0}\frac{\partial^2 E_y}{\partial x^2} \quad \text{(the wave equation for electromagnetic waves)} \qquad (31.34)$$

A particle equation

In this model the wave equation is actually a particle equation, that is because the terms are divided. It can be rewritten as E/B=E𝕐/-𝔾D∝E𝕍/-𝕀D becoming E𝔹=-𝔾D×E𝕐∝-𝕀D×E𝕍 as a wave equation. It is still a combination of impulse as particles and work as waves here, but multiplication gives fields in which waves travel.

E/B and EB

This can be seen with E/B compared to E𝔹, with a given inertial velocity of c the ratio of the electric to the magnetic aspects of a photon should not change. That would mean a photon would need to slow down when its electric part of the wave reduced, but it does not.

The rolling wheel model

The rolling wheel model of a photon is used here, the wheel has a radius of e𝕧 and a rotational frequency of -𝕚d to give e𝕧/-𝕚d as c. However -𝕚d×e𝕧 can vary with a constant Pythagorean Triangle area, then if e𝕧 doubled and -𝕚d halved the wheel would still roll at c. This allows for the e𝕧 wavelength and -𝕚d inertial mass of the photon to vary inversely.

The photoelectric effect

When the photon has a higher -𝕘d rotational frequency, it can knock electrons out of atoms with the photoelectric effect. This is as if they have a greater -𝕚d inertial mass. This is like where the -𝕚d×e𝕧 wheel has half the e𝕧 spoke and twice the -𝕚d rotational frequency, it can do more -𝕀D×e𝕧 inertial work in moving the electrons even though the inertial velocity of the photon is the same.

Equation 31.34 is the wave equation! And it's easy to show, by taking second derivatives of B_z rather than E_y, that the magnetic field B_z obeys exactly the same wave equation.

As we anticipated, Maxwell's equations have led to a prediction of electromagnetic waves. Referring to the general wave equation, Equation 31.31, we see that an electromagnetic wave must travel (in vacuum) with speed

$$v_{em} = \frac{1}{\sqrt{\epsilon_0\mu_0}} \qquad (31.35)$$

The constants ϵ_0 and μ_0 are known from electrostatics and magnetostatics, where they determined the size of $\vec{E}$ and $\vec{B}$ due to point charges. Thus we can calculate

$$v_{em} = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 3.00 \times 10^8 \text{ m/s} = c \qquad (31.36)$$

Different rolling wheels

Radio waves and X-rays have different frequencies, this is like the -𝕚d×e𝕧 rolling wheel having a different angle θ but moving at the same inertial velocity of c. Here c is fixed by its transferring changes in between atoms, as electrons emit and absorb -𝔾D×e𝕪 light work. These changes are according to the squared constant force as μ.

Colliding wheels

The photon also has a e𝕐/-𝕘d light impulse in colliding with electrons, when its e𝕐/-𝕘d light impulse changes then this is according to ϵ. To conserve ϵ and μ as inverses, the electron's changes in inertial velocity are also like a rolling wheel. Here it acts like a particle outside the atom, so the wheel grows with E𝕐 increasing when photons increase its inertial velocity.

Faster electrons

The electron moves as if its e𝕪 spoke as the wheel radius has increased. It has absorbed this change from the photon which loses some e𝕐/-𝕘d light impulse to increase the electron's E𝕐/-𝕠d kinetic impulse. Because the electron is a particle outside the atom in a straight-line reference frame, it cannot absorb a photon but only collide with it. Because the electron is moving faster, it cannot be absorbed by an atom.

Lost impulse of the photon

This increases the -𝕘d rotational frequency of the photon, that allows it to be absorbed into an atom as -𝔾D×e𝕪 light work. Its e𝕐/-𝕘d light impulse is weaker after giving some E𝕐/-𝕠d kinetic impulse to the electron. The photon acts as if it has slowed enough by losing E𝕐 to be absorbed by the atom, this is like a meteor losing enough inertia to be captured by a planet's gravitational field.

A higher orbital and torque

Conversely when an electron moves to a higher orbital its e𝕍/-𝕚d inertial velocity slows as its -𝕀D inertial torque increases. This is like a rocket needing a higher -𝕀D inertial torque to moving higher in a planetary orbit. At its maximum e𝕙 height in the orbit, the rocket moves most slowly with the lowest E𝕍/-𝕚d inertial impulse and the highest -𝕀D×e𝕧 inertial work.

Coulomb's law

In this model Coulomb's law does not refer to waves and work, only particles and impulse with ϵ. This is like the electrostatic forces from free electrons as particles, they repel each other and attract protons. That connects to changes in photons with a e𝕐/-𝕘d light impulse.

α in the atom defines c

In an atom, photons are emitted and absorbed in quanta with -𝔾D×e𝕪 light work using μ. The photon must then transmit changes according to ϵ and μ for impulse and work to be conserved. The ratio of these forces is defined in the ground state of Hydrogen as α or $e^{-𝕠d}$, that is proportional to the e𝕧/-𝕚d inertial velocity of the electron there.

> This is a remarkable conclusion. Coulomb's law and the Biot-Savart law, in which ϵ_0 and μ_0 first appeared, have nothing to do with waves. Yet Maxwell's theory of electromagnetism ends up predicting that electric and magnetic fields can form a self-sustaining electromagnetic wave *if* that wave travels with speed $v_{em} = 1/\sqrt{\epsilon_0\mu_0}$. No other speed will satisfy Maxwell's equations.
>
> Laboratory measurements had already determined that light travels at 3.0×10^8 m/s, so Maxwell was entirely justified in concluding that light is an electromagnetic wave. Furthermore, we've made no assumption about the frequency of the wave, so apparently electromagnetic waves of any frequency, from radio waves to x rays, travel (in vacuum) with speed c, the speed of light.

E and B as two reference frames

In this model B is the spin component of the oscillation in the rotational reference frame, E moves back and forth in this oscillation like a spring in the straight-line reference frame. B in the rotational reference frame would connect to sinθ and wine waves. E in the straight-line reference frame would connect to cosθ and moving in a straight-line.

Fractions as altitude

In this model there are two reference frames, the straight-line reference frame uses fractions from derivatives. The sequence 1/2, 1/3, 1/4, ... can be regarded as the e𝕒 altitude above the proton.

The quantum numbers as inverses

This decreases further away as inverses to the quantum numbers n=1,2,3,4,... These are inverses of each other, the fractions connect to the inverse square law where the E𝔸/+𝕠d potential impulse decreases as a square at higher altitudes. These would then be squares of the square root Pythagorean Triangle sides.

Squared from work

The inverse of these would then be 1, √2, √3, √4,... which when squared would be integral areas from +𝕆D×e𝕒 potential work around the proton. In between them there is a small difference proportional to Γ.

Summing the columns

This comes from the hyperbola, the area under it up to 1 on the horizontal axis is e. Here the axes would come from the -𝕠d and e𝕪 Pythagorean Triangle for the electron and the -𝕚d and e𝕧 Pythagorean Triangle for inertia. The horizontal axis would be e𝕪 and proportionally e𝕧. The vertical axis would be -𝕠d and -𝕚d respectively.

Tracing out the hyperbola

The hyperbola is traced out as a tangent to the -𝕠d and e𝕪 Pythagorean Triangle and -𝕚d and e𝕧 Pythagorean Triangle as they change their angle θ, their right angles are in the origin.

The difference as Γ

When the fractions are columns inside the area of the hyperbola this sequence does not sum to the area of the hyperbola. The difference is known to be Γ, with π/4 as a quarter of the circle corresponding to the hyperbola this can also be filled with columns in the same way. Then instead of the axis being e𝕪 and e𝕧 it would be e𝕒 from the proton and e𝕙 from gravity. The other axis would be +𝕠d and proportionally +𝕚d.

Γ as the difference

Here Γ in both cases represents the difference between an integral area, which would be in the rotational reference frame, and the columns which would be in the straight-line reference frame.

Areas under the hyperbola

With the fractions up to 1 as ½, 1/3, ¼, 1/5,... the difference between the two is Γ. The hyperbola is here in an -𝕠d and e𝕪 Pythagorean Triangle or -𝕚d and e𝕧 Pythagorean Triangle as reference frames. Then the spin axis is -𝕠d or -𝕚d. The integral area under the hyperbola can be broken up into separate integral areas, these extend up to the hyperbolic curve unlike the columns.

An infinite series not integral areas

These are not fractions, the areas are added together. These are different from the fraction which were all fractions of one, the integral areas can add up to larger values than one. Because the columns are impulse they are not areas, they are lines separated by instants of time. This sequence is infinite, it is like an infinite series summing to an area like the formula for e.

The derivative of the exponent

These areas would be the logarithmic exponent in the rotational reference frame as integrals, they are quantized multiplied areas from the spin Pythagorean Triangle axis. That is because they cannot be fractions, if they were then they would be the same as at least one column.

Discrete and continuous spectra

Changing 90° makes these as a derivative fraction as ½, 1/3, ¼, 1/5,... in the straight Pythagorean Triangle side axis. An exponent that was a fraction would then become a whole number, not part of the sequence of fractions. These are then different, the integral areas are like quantized values in the rotational reference frame, as with a discrete spectrum. The fractions can change continuously like a continuous spectrum in the straight-line reference frame.

The same in calculus

That is the same as the conventional calculus rule where the logarithmic exponent has a fraction as its derivative. Here the exponent would come from squaring the spin Pythagorean Triangle side to give integrals. The curve comes from the torque from the -𝕆D×e𝕪 kinetic work and -𝕀D×e𝕧 inertial work with the hyperbola. The columns come from the E𝕐/-𝕠d kinetic impulse and E𝕍/-𝕚d inertial impulse.

Integrating the columns

In this model that explains the process of integration by dividing it into columns, their width goes to zero so that they become the integral area such as with the hyperbola. They are integrated to become these areas as the exponents, but here there is a difference between the sum of the columns and the hyperbolic area as Γ.

Dividing up a circle and a parabola

The same would then apply to dividing up a parabola or circle with these columns, they approach the integral area but the difference would also be proportional to Γ. Here this follows from the Pythagorean Triangles used, the integral area comes from -𝕆D×e𝕪 kinetic work and -𝕀D×e𝕧 inertial work. This remains different from the columns which sum with the E𝕐/-𝕠d kinetic impulse and E𝕍/-𝕚d inertial impulse.

Tangents as derivatives

The columns can also be regarded as being on the hyperbolic curve, as tangents they would give the ratio of the slope as -𝕠d/e𝕪 and -𝕚d/e𝕧. That gives the definition of the derivative, as a small area this would be at an instant of time not a position like a fluxion. The difference in area between the integral and the column or tangent would come from Γ.

The circle quadrant

With the circle a quadrant contains π/4 as an integral area, that is part of a conic section at 90° to the Pythagorean Triangle containing the hyperbolas. This quadrant can be the +𝕠d and e𝕒 Pythagorean Triangle proton and proportionally the +𝕚d and e𝕙 Pythagorean Triangle as gravity.

Extending to any curve

Now the tangent derivative is on the circle, this shows the connection of derivatives and integrals extends from the hyperbola to the circle. Generalizing this process would allow any curve to be differentiated and integrated the same way.

The circle quadrant

The circle as in calculus can be divided into vertical rectangles corresponding to ½, 1/3, ¼, 1/5,… As these increase in number the rectangles approach the quarter circle integral in area as with those under the hyperbola. Their width goes to an instant of time with a E𝔸/+𝕠d potential impulse and Eℍ/+𝕚d gravitational impulse. The area of the quadrant would come from +𝕆D×e𝕒 potential work and +𝕀D×e𝕙 gravitational work.

The complete circle

Multiplied by four this gives an integral field area as a circle, like the ground state for example. This integral is connected to the e𝕒 altitude as the series of fractions, also proportionally the e𝕙 height.

Sweeping across the radius

The difference between the two would again be related to Γ because this is the same as changing the logarithm base from e to π. The connection of the derivative fraction as a tangent, to the integral areas, continues with this change of base.

A change over time or distance

Moving across these columns can be regarded as a change of impulse over time, the columns are then changing like squared force vectors. Conversely the changing of the areas must occur as quantized integers because they cannot be fractions. These cannot change over time because the spin Pythagorean Triangle side is squared as a torque. The straight Pythagorean Triangle side is not squared and acts like a ruler to measure positions on it with work.

The Heisenberg uncertainty principle

In between there is an uncertainty which is related to the Heisenberg uncertainty principle. While the derivative is the inverse of the integral, which follows from the constant Pythagorean Triangle areas, as they change there is a minimum difference between them from Γ.

Changing the base from e to π

When the base of the logarithm is changed from e to π this applies to a circle. Then it changes from the -𝕠d and e𝕪 Pythagorean Triangle electron and -𝕚d and e𝕧 Pythagorean Triangle inertia, to the +𝕠d and e𝕒 Pythagorean Triangle proton and +𝕚d and e𝕙 Pythagorean Triangle gravity.

Other bases

By changing the base this can also apply to an ellipse or parabola. Other curves can be modeled by changing the base, that gives a different shape to the curve while retaining the relationship between the derivative tangents and integrals. For example, a complicated curve could be broken up into segments with different bases to model their derivatives and integrals.

Chaos

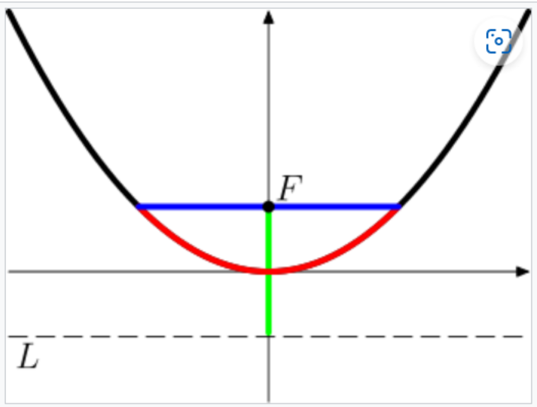
In this model δ is the first Fiegenbaum number, this is a proportional change between the sizes of cascades in chaos. At 90° to this would be the second Fiegenbaum number β, this is ≈ √(2π). When δ is divided by β this is α ≈ √δ/√β, this is like √ϵ and √μ as fractions giving c. Here they correspond to α which is an inertial velocity ≈ 1/137 of c. when multiplied together as √δ√β that would be ≈ 137 times greater which is c. This is like the values of √ϵ for e𝕧 in the ground state as a length, then √μ in the denominator would be -𝕚d. Then in the ground state √ϵ/√μ would be proportional to the inertial velocity of α as well, when written as 1/(√ϵ√μ) that is also c.

δ and β converge

As δ and β converge to these squared constants, they give a series of fractions like those fractions as columns under the hyperbola. As squared constants they would also represent forces in this model, they can vary as square roots like √ϵ and √μ.

The parabolic constant

This is called κ here, it is the ratio of the latis rectum to the focal length. This is similar to the parabolic-like shapes in chaos, the two lines below are at 90° to each other. When they are divided, they would be changing in a straight-line reference frame like a series of cascades. When multiplied they give an integral area in the rotational reference frame of the parabola.



The universal parabolic constant is the red length divided by the green length.

The parabola and quarks

When half the green line is multiplied by the blue line, this gives a rectangle containing the parabola. Then the area of the parabola is 2/3 of the rectangle also as a constant in the rotational reference frame. This would lead to the 2/3 charge of the +𝕠2/3 up quark, the remainder of the rectangle would then be the -𝕠1/3 down quark.

Quarks as particles and waves

The quarks can then move like parabolas inside a proton as a constant, that would be as particles in the straight-line reference frames. As waves they would have quantized areas of 2/3 and 1/3, the difference between them is 1 as a quantized value. These are connected by gluons with this quantized value of 1.

Two constants with quarks

There are then two constants with the quarks, the first is the ratio of the e𝕒 altitude they move to and the distance they move. In this model the quarks can move between the three orthogonal +𝕠d and e𝕒 Pythagorean Triangles in the proton in parabolic trajectories. When measured as waves they give an integral rectangle from this height and distance, that is also a constant giving the 2/3 and 1/3 values.

The Koide formula

In this model the Koide formula has three parabolic areas in the numerator here. These are mass which are measured as -𝕆D×e𝕪 kinetic work. Alternatively they can be observed as the inverse E𝕐/-𝕠d kinetic impulse, a higher mass would have a lower impulse under a the same force for each.

Total kinetic probability

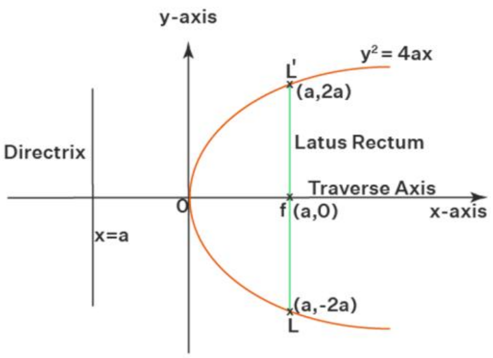
In the denominator the three spin Pythagorean Triangle sides are added together, when squared they give -𝕆D×e𝕪 kinetic work and -𝕀D×e𝕧 inertial work as the overall probabilities of their masses. With work there is no time, the three -𝕚d inertial masses here give the relative probabilities which are equal. That is because the three generations have been generated.

Adding three rectangles

In the numerator there are three values of -𝕆D×e𝕪 kinetic work, they are also a -𝕆D kinetic torque in a Parabola. This would be the integral area of the parabola, 2/3 of the area of the rectangle it is inscribed in. The denominator would be the sum of these three rectangles, the spin Pythagorean Triangle sides are the latis rectums. The e𝕪 straight sides are the directrix, each is proportional to its latis rectum.

Different rectangle areas from torque

The area is not the same for each rectangle because in this model, the second and third generations are made by an additional torque to a neighboring orthogonal Pythagorean Triangle.



The κ rectangle

This can be drawn as a rectangle where the three -𝕚d inertial masses make a horizontal line. Then the e𝕪 vertical sides are drawn so that each has the correct side ratios from the parabolic constant. The total -𝕆D kinetic probability is the horizontal line squared. This could also be drawn as three orthogonal Pythagorean Triangles from the origin, then the -𝕚d inertial mass values extend on an axis. The angles between these and a (1,1) vector according to Foote is 45°.

A torque between the three generations

They can also be drawn connected to each other, first the -𝕚d inertial mass of the electron is drawn as an axis. Then the -𝕚d inertial mass of the muon is drawn orthogonal to this. The τ electron has its -𝕚d inertial mass at 90° to both connected to the muon Pythagorean Triangle side. In between these a line can be drawn as the vector sum of all three spin sides. To change 90° in between each generation there would be a -𝕆D kinetic torque, the three spin sides can change by using quaternions.

Vector summation

In this model vector sums are only used with straight Pythagorean Triangle sides, the same can be done with three e𝕪 straight sides and then they are vector summed. When squared this gives a force vector. The three lepton masses would then have their associated e𝕪 straight sides individually squared as parabolas to give 2/3. It is the same answer because κ constrains the angle θ of each -𝕠d and e𝕪 Pythagorean Triangle.

$$Q = \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2} = 0.666661(7) \approx \frac{2}{3}$$

Deriving quark and lepton masses from κ

In this paper the masses of all the quarks and leptons are derived from 2/3. This can occur from the parabolic constant κ as a fraction giving parabolas. Then when multiplied together they give a rectangle, the parabola is 2/3 of its area. When the strange quark is negative it becomes +𝕠1/3, then it adds to the -𝕠1/3 down quark to leave 2/3 with the +𝕠2/3 up quark. This allows for a parabola of 2/3 to occur as +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work where the mass is the torque of the quarks in the nucleus. These are bound together with gluons with a value of 1 as +2/3 and -1/3 equaling 1.

> The main point in this descent is that we have produced a tuple not yet in the literature, the one of strange,charm,and bottom. How is it?
>
> Closer examination shows that the reason of the miss is that in order to meet (2), the value of $\sqrt{s}$ must be taken negative. But this is a valid situation, according Foot interpretation and the parametrisation (3)
>
> Of course, once we are considering negative roots, the equation (4) is not the only possible matching. But the possibilities are nevertheless reduced by the need of a positive discriminant in the equation and by avoiding to come back to higher values, above the mass of the bottom quark. Also, one we have recognised the sign of $\sqrt{s}$, the validity of the two next steps in the descent, up and down, is unclear. We will come back to these two quarks in the next section.
>
> Another important observation is that $(-\sqrt{m_s}, \sqrt{m_c}, \sqrt{m_b})$ is on the opposite extreme of Foot's cone respect to $(\sqrt{m_\tau}, \sqrt{m_\mu}, \sqrt{m_e})$, making an angle of almost ninety degrees.

Quarks and the rectangle area

Here all six quark masses are derived from the lepton mass, this uses the κ constant. These also have +𝕠2/3 charges which are 2/3 as the three upper quarks, the remaining part of the rectangle is -𝕠1/3 as the three lower quarks.

Charge as 1

These are +𝕚d gravitational and -𝕚d inertial masses. The charge of the electron as -𝕠d, d=1 comes from the change of the -𝕠1/3 down quark to an +𝕠2/3 up quark in Roy electromagnetism. The difference between the two is 1, that is emitted as the -1 electron leaving the +1 proton.

Quark lepton ratio

In Biv space-time the change is in 1/3 increments, the -𝕚1/3 down quark as inertial mass changes to an +𝕚2/3 up quark as gravitational mass. The first three quarks give a value of Q, there is a lepton value for Q 1/3 of this which adds to 1.

Another three generations

When the strange quark is negative here, that can have it with the top and bottom quarks in another three orthogonal Pythagorean Triangles. That would be three generations in another way, the sequence is then -𝕠2/3, -𝕠1/3 and +𝕠2/3.

> Furthermore, the parameters of mass and phase of this quark triple[2] seem to be three times the ones of the charged leptons: we have $M_q = 939.65$ MeV and $\delta_q = 0.666$, while in the leptons $M_l = 313.8$ MeV and $\delta_l = 0.222$, about 12.7 degrees.
>
> We could take seriously both facts and use them to proceed in the reverse way: take as only inputs the mass of electron and muon, then recover M_l and δ_l, multiply times three to get the parameters of the opposite tuple and then the masses of strange, charm and bottom, and then use the ladder up and down to recover the previous table. It is impressive:

Inputs	Outputs
$m_e = 0.510998910 \pm 0.000000013$	$m_\tau = 1776.96894(7)$ MeV
$m_\mu = 105.6583668 \pm 0.0000038$	$m_s = 92.274758(3)$ MeV
$M_q = 3M_l$	$m_c = 1359.56428(5)$ MeV
$\delta_q = 3\delta_l$	$m_b = 4197.57589(15)$ MeV
	$m_t = 173.263947(6)$ GeV
	$m_u = 0.0356$ MeV
	$m_d = 5.32$ MeV

A projectile fired by a cannon

Here the parabolic constant is associated with a projectile fired from a cannon. In the rotational reference frame from the side the projectile in moved downwards from the +𝕀D×e𝕙 gravitational work of a planet. It moves horizontal at a constant velocity, with a different initial velocity and gravity this constant is maintained.

The length of the hyperbola

It is similar to the ratio of δ and β in chaos. It is also like the ratio of a radius to a circle's circumference with π. With the ratio of the area e to the axis under a hyperbola, this is an integral from work as a square like the area of the circle. The axis would be proportional to the length of the hyperbola above it.

The conic sections and chaos

These are all conic sections, the Fiegenbaum number proportions also connect to the connect sections but in chaos.

The cosmological constant and Γ

In this model the cosmological constant would come from Γ as well. The expansion of the universe appears to occur in the straight-line reference frame from an explosion. When this is compared to hyperbolic geometry in special relativity there would be a difference between the two from Γ. The -𝕀D×e𝕧 inertial work from this would appear as if the universe would continue to expand by a small amount as the cosmological constant from Γ.

The Hubble controversy

Conversely looking back towards the maximum e𝕙 height of the +𝕚d and e𝕙 Pythagorean Triangle, this appears as a Eℍ/+𝕚d gravitational impulse proportional to the amplitudes of similar stars and galaxies. With +𝕀D×e𝕙 gravitational work this can be extrapolated from the CMB and its waves like sound. These would also appear to deviate from each other as in the Hubble controversy.

General relativistic effects

There is also a +𝕚d gravitational time slowing closer to the CMB, when observed in reverse the universe appeared to expand more quickly. With +𝕀D×e𝕙 gravitational work there is a e𝕙 height contraction, the explosion of the big bang would appear to be expanding as Biv space-time because the e𝕙 height is dilating and the +𝕚d gravitational time is speeding up. The speed of light would be constant in this area, just as it slows near an event horizon it appears to be speeding up closer to the CMB going forwards in time.

The singularity

This would reach a limit where the infinite contraction to a singularity ended. Instead, it would appear as a circle such as the CMB. This limit of the +𝕚d and e𝕙 Pythagorean Triangle as gravity is like the limit of the +𝕠d and e𝕒 Pythagorean Triangle proton as the ionization boundary. Up to this level there is a series of fractions, these are electron orbitals with the proton.

Quantized redshifts

With gravity these are quantized levels like those seen inside galaxies. Also, in this model galaxies can be redshifted in quantum intervals from +𝕀D×e𝕙 gravitational work.

The limit of the fractions

When this limit of reached there can be no more observations and measurements, it is like where the electron leaves the atom. In the CMB this also appears to happen, the protons and electrons are all separated as a plasma. That allows a final emission of photons to be observed and measured. Beyond Γ there would be the circle, the e𝕙 height is made up of the fractions of this distance as a continual change.

The event horizon as a circle

The event horizon would also have these fractions as distances with its Eℍ/+𝕚d gravitational impulse. This would be proportional to the electron mass in size, it is larger in this model because of the many stars in the galaxy. Where gravity is seen as a depressed rubber sheet in general relativity, here each star depresses the rubber may so it is much lower in the center.

Neutron stars

This becomes much lower than an individual star could make it, that allows for neutron stars and black holes to be much larger. The neutron star is then like neutrons in an atom, the additional gravity in this depression allows for more atoms to be compressed into neutrons.

The bottom of the depression

The bottom of the depression has these black holes, they are where the quantized size of this circle expands. These can be moved around, for example in a collision of galaxies. Then more stars and gas might be absorbed on one side and emitted as jets of Hydrogen replenishing the used-up Hydrogen in the galaxy.

Reaching the rotational reference frame

The circle is in a rotational reference frame, matter falling into the event horizon is in the straight-line reference frame of fractional distances. In this rotational reference frame a measurer would not be changing over time, there would be +𝕀D×e𝕙 gravitational work where the +𝕀D gravitational probability densities were higher in some positions.

The rocket freezes in time and height

This rotational reference frame is measured in general relativity when a rocket might fall into the event horizon. +𝕚d gravitational time slows and stops when it reaches this +𝕀D gravitational torque and probability. This has a e𝕙 height contraction so that the fractional distances change, that also slows the observation of the falling rocket. When it reaches the rotational reference frame the rocket appears to stop, frozen in time.

Above the rocket under the event horizon

At this event horizon there would be a Γ related boundary on both sides that would have to be traversed. Matter would not experience this boundary in its rotational reference frame except for tidal effects from +𝕀D×e𝕙 gravitational work. When inside the boundary light from outside would be like a spaceship traveling at above c. Matter near the event horizon would have similar e𝕙 height contractions to the rocket from +𝕀D×e𝕙 gravitational work.

A sine wave from a rolling wheel

In (31.37) there is a sine wave from the rolling wheel, the tip of the spoke traces out the sine wave shape. The frequency ω comes from -𝕘d in the photon. Here x would correspond to the length of the e𝕪 spoke, double this as the diameter would be the wavelength between two wave crests.

The ratio of the frequency and wavelength

In this model sinθ would come from the rolling wheel as B, the angle θ gives the ratio of the -𝕘d rotational frequency and e𝕪 wavelength. This would be the in the rotational reference frame, the straight-line reference frame is from cosθ. That is like looking at the sine wave from above, it is observed as moving like a piston moving backwards and forth longitudinally.

Longitudinal impulse

That is also like jerking an elastic string, this would be propagating forward as a compression and expansion. With sound waves they move with the particles of air in the straight-line reference frame colliding with each other.

Eddy currents

In the rotational reference frame at 90° it is like a rolling wheel creating eddy currents as the sound wave moves, these change like quantized movie frames not with a continuous motion. This also happens under an ocean wave from the torque of the wave. The phase of the wave at a position gives the probability density and torque.

A copper plate between magnetic poles

When a copper plate it moved in between two magnetic poles, there is +𝕆D×e𝕒 potential work done with reacts against the plate's motion. This creates a +𝕆D potential torque as eddy currents resisting the motion. These are measured from above, at 90° along the plate's motion there is a E𝔸/+𝕠d potential impulse which decelerates its motion.

Combined into a cosine

In (31.38) the two are combined into a cosine as the straight-line reference frame, here this would only be the electric charge.

Connecting *E* and *B*

The electric and magnetic fields of an electromagnetic wave both oscillate, but not independently of each other. The two field strengths are related. E_y and B_z both satisfy the same wave equation, so the traveling waves—just like a wave on a string—are

$$E_y = E_0 \sin(kx - \omega t) = E_0 \sin\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right]$$
$$B_z = B_0 \sin(kx - \omega t) = B_0 \sin\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right] \qquad (31.37)$$

where E_0 and B_0 are the amplitudes of the electric and magnetic portions of the wave and, as for any sinusoidal wave, $k = 2\pi/\lambda$, $\omega = 2\pi f$, and $\lambda f = v = c$. These waves have to satisfy Equation 31.27; thus

$$\frac{\partial E_y}{\partial x} = \frac{2\pi E_0}{\lambda}\cos\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right] = -\frac{\partial B_z}{\partial t} = 2\pi f B_0 \cos\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right] \qquad (31.38)$$

No longitudinal or transverse waves

In this model photons do not move as either longitudinal or transverse waves. These wave shapes can only occur with a force, that would change the photon. The rolling wheel model can rotate without a force, drawing a sine wave from the tip of the e𝕪 spoke is only in comparison to a horizontal line of the photon’s path.

A rotating wheel with no forces

A rotating wheel can be moving at a constant inertial velocity with no forces, if a point at the tip of a spoke traces out a sine wave it does not mean that point is accelerating up and down. When viewed from above this point appears to move from side to side like a piston, this would also be at 90° to the wheel’s motion.

Opposing inverse forces

These can be regarded as opposing forces canceling each other out. The piston motion of the e𝕐/-𝕘d light impulse is in a straight-line reference frame, the torque of the rolling wheel makes a sine wave in the rotational reference frame.

A and B at 90°

In this model E and B are at 90° to each other giving the straight-line reference frame and rotational reference frame respectively. There is a third direction at 90° to these, that is where the e𝕐/-𝕘d light impulse in a straight-line reference frame or -𝔾D×e𝕪 light work in a rotational reference frame would occur. That does not come from the cross product here.

31.6 Properties of Electromagnetic Waves

It had been known since the early 19th century, from experiments with interference and diffraction, that light is a wave, but no one understood what was “waving.” Faraday speculated that light was somehow connected to electricity and magnetism, but Maxwell was the first to understand not only that light is an electromagnetic wave but also that electromagnetic waves can exist at any frequency, not just the frequencies of visible light.

In the previous section, we used Maxwell's equations to discover that:

1. Maxwell's equations predict the existence of sinusoidal electromagnetic waves that travel through empty space independent of any charges or currents.
2. The waves are transverse waves, with $\vec{E}$ and $\vec{B}$ perpendicular to the direction of propagation $\vec{v}_{em}$.
3. $\vec{E}$ and $\vec{B}$ are perpendicular to each other in a manner such that $\vec{E} \times \vec{B}$ is in the direction of $\vec{v}_{em}$.
4. All electromagnetic waves, regardless of frequency or wavelength, travel in vacuum at speed $v_{em} = 1/\sqrt{\epsilon_0 \mu_0} = c$, the speed of light.
5. The field strengths are related by $E = cB$ at every point on the wave.

Only one vector

In this model E⃗ as e𝕪 in the e𝕪 and -𝕘d Pythagorean Triangle photon can be regarded as a vector. B would be spin, so it is not a vector. It can be drawn as a straight-line in a Pythagorean Triangle, is operation is like looking down a line at a rotating point. The photon would have -𝕘d as this spin side rotating like an axle. A rolling wheel would have this rotating point as the -𝕘d Pythagorean Triangle side.

Neutrino spin

A neutrino moves with this at 90° it appears like an arrow vector as 𝕞 which is spinning like a bullet as it moves. The spin Pythagorean Triangle there would be 𝕠d and 𝕚d, there is no positive or negative sign. At 90° to +𝕠d and -𝕠d this acts like a zero as 𝕠d.

Motion in the third reference frame

In this model there are three orthogonal Pythagorean Triangles with a photon. There is a rotational reference frame from the side, the rolling wheel appears like a vortex doing -𝔾D×e𝕪 light work. Changing 90° to a straight-line reference frame this appears as a piston motion from above, without forces it refers to different phase orientations of the e𝕪 spoke. The third orthogonal reference frame is the motion of the photon in Biv space-time as a e𝕧/-𝕚d inertial velocity or a -𝕚d×e𝕧 inertial momentum.

FIGURE 31.23 illustrates many of these characteristics of electromagnetic waves. It's a useful picture, and one that you'll see in any textbook, but a picture that can be very misleading if you don't think about it carefully. First and foremost, $\vec{E}$ and $\vec{B}$ are *not* spatial vectors. That is, they don't stretch spatially in the y- or z-direction for a certain distance. Instead, these vectors are showing the field strengths and directions along a single line, the x-axis. An $\vec{E}$ vector pointing in the y-direction is saying, "At this point on the x-axis, where the tail is, this is the direction and strength of the electric field." Nothing is "reaching" to a point in space above the x-axis.

Polarization angles

The direction of x is in a straight-line reference frame, this is because it is an inertial velocity derivative of e𝕧/-𝕚d. At 90° to this is the rotational reference frame, the photon can then be rotated to different polarization angles.

Destructive interference on either side of x

This extends outwards from x with different -𝔾D light probability values. At equal distances to either side the light probabilities interfere destructively, this gives the highest -𝔾D light probability of the photon being measured along this line. It can also be curved in a rotational reference frame such as light going around a black hole. Then the light probability with a lower e𝕙 height is closer to the black hole, this is contracted and so the photon moves along this geodesic.

Destructive interference around a black hole

When the -𝔾D light probabilities are measured on either side of the photon they are no longer the same. The +𝕀D gravitational probability is stronger on the black hole side, this reduces the -𝕀D inertial probability of the photon on that side. The destructive interference occurs more on the black hole side and so the photon moves in a curved arc.

A greater gravitational probability

The same would occur with matter near the event horizon. The +𝕀D×e𝕙 gravitational work might act on a rocket, the +𝕀D gravitational probability is larger when e𝕙 is smaller because of the constant Pythagorean Triangle area. Then the +𝕀D gravitational probability is larger under the rocket, so it is more probably measured at lower heights.

Higher probability of a lower height

The photon is also more probably measured at lower e𝕙 heights because it is proportional to the proton and electron quantization levels. When the electrons are compressed closer to the proton in this gravitational field, that is the same higher +𝕀D gravitational probability of their being closer to the nucleus.

A proportional electron and photon arc

The electrons would then move downwards in an arc that is proportional to the lower photon arc. This translates as smaller -𝕆D×e𝕪 kinetic work for the electrons, the difference between the orbitals is smaller and so the -𝔾D×e𝕪 light work of the photons is also smaller when they move lower in this arc.

Changing the height of the reference frame

In the same reference frame photons then do the same -𝔾D×e𝕪 light work between electrons in orbitals. For example, far above the black hole, a laser emits photons that are absorbed by atoms. Taking the same laser and atoms to near the event horizon, the same laser still has its photons absorbed, the relative frequencies of the photons to the electrons is the same even though the electron orbitals have a much lower e𝕒 altitude.

The same reference frame

The electron orbits are compressed downwards, the photons transmit changes between the laser and atoms as if they had curved downwards towards the event horizon. The laser and atoms have the same rotational reference frame from gravity, so do the photons. This would occur at any e𝕙 height which can only occur if the atom orbitals are compressed downwards like the oath of the photons.

Height contraction along a geodesic

The e𝕙 height contraction is the same for the laser and electrons with +𝕀D×e𝕙 gravitational work, the photons move along a gravitational geodesic with this e𝕙 height contraction. The e𝕒 altitude contraction above the protons is proportional to the e𝕙 gravitational contraction.

A greater probability of being higher

Conversely when matter is moved further away from a black hole, the electrons around the protons have a higher -𝕆D kinetic probability of being at a greater e𝕒 altitude above the nucleus. This e𝕒 altitude is proportional to the e𝕙 height, so the electrons move further away from the nucleus.

Probability as torque

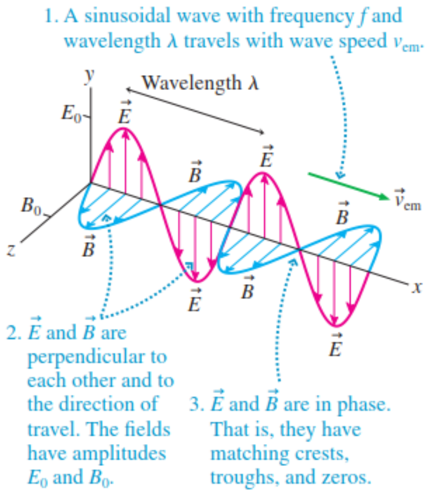
The photons tend to resume their straight-line reference frame motion past the event horizon. The -𝔾D light probabilities on each side of its path again destructively interfere equally to give a straight-line. Because torque is the same as probability, an equal probability on each side of the photon means the -𝔾D light torque is canceled out on both side. Near the event horizon the greater -𝔾D light probability on one side was the same as a greater -𝔾D light torque on that side.

Second, we're assuming this is a *plane wave*, which, you'll recall, is a wave for which the fields are the same *everywhere* in any plane perpendicular to $\vec{v}_{em}$. Figure 31.23 shows the fields only along one line. But whatever the fields are doing at a point on the x-axis, they are doing the same thing everywhere in the yz-plane that slices the x-axis at that point. With this in mind, let's explore some other properties of electromagnetic waves.

Two reference frames

In this model the e𝕪 straight Pythagorean Triangle side can trace out a sine wave. At 90° to this would be a cosine wave as below, but here this would be in a straight-line reference frame as a spring like motion back and forth.

FIGURE 31.23 A sinusoidal electromagnetic wave.



The Poynting vector

Here the cross product is used to give a vector at 90° to both E and B. In this model the cross-product vector would be a spin Pythagorean Triangle side, that cannot move in a straight-line reference frame. Using μ here is the squared constant from work in a rotational reference frame, that is associated with sinθ and sine waves, also with the cross product.

Energy and Intensity

Waves transfer energy. Ocean waves erode beaches, sound waves set your eardrums vibrating, and light from the sun warms the earth. The energy flow of an electromagnetic wave is described by the **Poynting vector** $\vec{S}$, defined as

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \qquad (31.39)$$

Power as impulse

Here power is referred to as joules per second, in this model that would be the e𝕐/-𝕘d light impulse. E𝕐 is the light displacement where photon particles are observed in -𝕘d light time. The unit area is a separate variable, this is at 90° to the power direction and so it is in the rotational reference frame as an integral area. When the photons are spread over a larger area the power is reduced inversely to this.

Oscillating with respect to time

The Poynting vector here oscillates with respect to time as the e𝕐/-𝕘d light impulse. As a spring oscillation this would expand and contract back to zero twice a rotation in time.

The Poynting vector, shown in **FIGURE 31.24**, has two important properties:

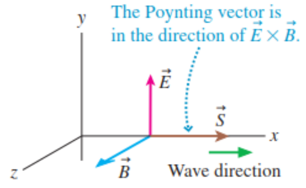
1. The Poynting vector points in the direction in which an electromagnetic wave is traveling. You can see this by looking back at Figure 31.23.
2. It is straightforward to show that the units of S are W/m^2, or power (joules per second) per unit area. Thus the magnitude S of the Poynting vector measures the rate of energy transfer per unit area of the wave.

Because $\vec{E}$ and $\vec{B}$ of an electromagnetic wave are perpendicular to each other, and $E = cB$, the magnitude of the Poynting vector is

$$S = \frac{EB}{\mu_0} = \frac{E^2}{c\mu_0} = c\epsilon_0 E^2$$

The Poynting vector is a function of time, oscillating from zero to $S_{max} = E_0^2/c\mu_0$ and back to zero twice during each period of the wave's oscillation. That is, the energy

FIGURE 31.24 The Poynting vector.



In this model the average comes from the normal curve, that would be -𝔾D×e𝕪 light work in the rotational reference frame. The inverse of this would be the middle of the pulsing of the spring like e𝕐/-𝕘d light impulse.

Pulsing in the straight-line reference frame

This pulsing is in the straight-line reference frame, at 90° the rotational reference frame does not pulse as there is no straight-line motion. Power here is energy per second, in this model that would be the e𝕐/-𝕘d light impulse where E𝕐 is the light displacement force which is transferred per second as 1/-𝕘d. When divided by an area A this is an integral as a rotational reference frame, that is 90° from the e𝕐/-𝕘d light impulse.

An area of photon particles

The e𝕪/-𝕘d photons are observed as particles, they are spread out in this area each with a e𝕐/-𝕘d light impulse. At 90° the area is in the rotational reference frame as the integral where the photons are waves. When the photon particles are counted with their impacts per second on this area, a single impact can be differentiated from the others.

The double slit experiment

With the double slit experiment, observing which slit the e𝕪/-𝕘d photons goes through gives a e𝕐/-𝕘d light impulse. The power of the photons hitting a screen is observed over time. This must involve observing which slit the photon particle goes through, that would be from above and at 90° to the slit material. Then the photon path would be a straight-line going to one slit or the other, and to a screen where it would be observed.

No time in the rotational reference frame

When the viewpoint is changed by 90° this is looking directly at the two slits either before or after they would pass through them. As the rotational reference frame there is no time, there is the -𝔾D light probability of which slit they would go through.

Observing from inside the slit

It is not possible to observe which path, the photon cannot be observed by looking directly at the slits. It also cannot be observed by being in between the slits and the screen. Even if the photon hits the observer there, it is not possible to know the path it took. This path can only be observed at 90° to this as before.

Timing to determine which slit

If the observer was behind the slits looking at both, then they are the same as the screen would measure. They cannot observe the photon in midflight, they have to observe the photon as a particle by it hitting them. To observe which slit the photon went through is at 90°, the time of the photon emission and the time it arrived at the screen will determine which slit the photon went through. This would be where the double slits were offset to one side to make the observation easier. Without observing the time, the instruments must be in both slits to observe the photon there.

Photons colliding with electrons

If there were electron particles around the screen, then the photon particles could collide with the electrons and show a path through one of the slits. That would be observed along the edge of the slit material again in the straight-line reference frame. But this fails at 90° behind the screen, the photons could still go through either slit of they collided with these electrons. They may have bounced off them at a sufficient angle to go through either slit.

flow in an electromagnetic wave is not smooth. It "pulses" as the electric and magnetic fields oscillate in intensity. We're unaware of this pulsing because the electromagnetic waves that we can sense—light waves—have such high frequencies.

Of more interest is the *average* energy transfer, averaged over one cycle of oscillation, which is the wave's **intensity** I. In our earlier study of waves, we defined the intensity of a wave to be $I = P/A$, where P is the power (energy transferred per second) of a wave that impinges on area A. Because $E = E_0 \sin[2\pi(x/\lambda - ft)]$, and the average over one period of $\sin^2[2\pi(x/\lambda - ft)]$ is $\frac{1}{2}$, the intensity of an electromagnetic wave is

$$I = \frac{P}{A} = S_{avg} = \frac{1}{2c\mu_0}E_0^2 = \frac{c\epsilon_0}{2}E_0^2 \qquad (31.40)$$

Intensity and impulse

In this model the intensity comes from the e𝕐/-𝕘d light impulse, this is a distance squared the same as r^2. The spherical area would not be used here, the photon particle is not moving in a rotational reference frame so there is no associated sphere.

Scattered particles

As this distance increases there is a ratio of E𝕐/E𝔸, that would use the altitude squared above a proton. With the photon emission this would spread out as a particle, the inverse square relationship would depend on the -𝕘d light time the photons were scattering as particles.

Emitting light as waves

This can also be measured as -𝔾D×e𝕪 light work, then the area of the photon waves is an integral. There is a -𝔾D light probability that a photon would be measured at an equal e𝕒 altitude above the original proton. With a Hydrogen atom an electron might go up and down in its orbital emitting photon waves. These do not occur over a continuous time, instead there are quantized snapshots like movie frames.

No direction for photon waves

These have no straight-line direction because that would be in the straight-line reference frame. Instead, they have a gradient like a slope around the proton. This would be flat from the +𝕠d and e𝕒 Pythagorean Triangle proton, also from the e𝕪 and -𝕘d Pythagorean Triangle photon. When measured with +𝕆D×e𝕒 potential work the +𝕆D potential probability gradient would decrease according to the inverse square law as the e𝕒 altitude increased.

Probability and area

This does not mean the area of the e𝕪 and -𝕘d Pythagorean Triangle photon expanded to include all this area. Instead, the constant Pythagorean Triangle area has a -𝔾D light probability of being measured on a given boundary. With a larger circular area, the constant Pythagorean Triangle area has a smaller -𝔾D light probability of being measured at a position on this circumference.

A change in altitude

As this e𝕒 altitude increases linearly, the -𝕆D kinetic probability decreases as a square. That comes from the proton controlling the original electron orbitals. This orbital change is mediated by the e𝕪 and -𝕘d Pythagorean Triangle photon, with a greater e𝕒 altitude the photon is less likely to be measured in a given direction.

No directionality of a photon wave

The rotational reference frame of the photon wave means it has no actual directionality, there are -𝔾D light destructive interferences in unlikely direction that cancel out. So the photons would for example be more likely to be measured in the direction a laser is pointed at.

Equation 31.40 relates the intensity of an electromagnetic wave, a quantity that is easily measured, to the amplitude E_0 of the wave's electric field.

The intensity of a plane wave, with constant electric field amplitude E_0, would not change with distance. But a plane wave is an idealization; there are no true plane waves in nature. You learned in Chapter 16 that, to conserve energy, the intensity of a wave far from its source decreases with the inverse square of the distance. If a source with power P_{source} emits electromagnetic waves *uniformly* in all directions, the electromagnetic wave intensity at distance r from the source is

$$I = \frac{P_{source}}{4\pi r^2} \qquad (31.41)$$

Equation 31.41 simply expresses the recognition that the energy of the wave is spread over a sphere of surface area $4\pi r^2$.

A photon has a proportional momentum

In this model Biv space-time is proportional to Roy electromagnetism. The +𝕠d and e𝕒 Pythagorean Triangle proton has a positive +𝕠d Pythagorean Triangle side referred to as a positive charge, it also has a proportional +𝕚d gravitational mass from the +𝕚d and e𝕙 Pythagorean Triangle. The -𝕠d and e𝕪 Pythagorean Triangle electron has a -𝕠d negative Pythagorean Triangle side referred to as a negative charge, it also has a proportional -𝕚d inertial mass from the -𝕚d and e𝕧 Pythagorean Triangle as inertia.

Changing angles θ

In a Hydrogen atom, when a photon is emitted, it changes the angle θ of the -𝕠d and e𝕪 Pythagorean Triangle that emitted it. That changes inversely the angle θ of the +𝕠d and e𝕒

Pythagorean Triangle the electron was orbiting. This angle would not change over time, that would be where the spin Pythagorean Triangle side was unsquared. Instead this side is squared as a torque or probability. This changes according to measuring over a distance.

Conserving Pythagorean Triangle proportions

To conserve these proportions, the e𝕪 and -𝕘d Pythagorean Triangle photon must change the -𝕚d and e𝕧 Pythagorean Triangle inertia of an electron it collides with or is absorbed into. That gives the photon momentum.

Collisions in a straight-line reference frame

When a photon collides with an electron this is in a straight-line reference frame. There is only straight-line motion, there can only be a collision because a particle must have a before and after over time. The only force allowed is impulse, that can only happen with a change in displacement as a squared acceleration.

The photon and electron are motionless

In the rotational reference frame of the photon, and separately the electron, they are each motionless. These are measured as a torque and a probability density. When they approach each other there are snapshots of these different probability densities, there can be many different probable positions. These would tend to cancel out with destructive interference leaving only one measurement for each.

The collision in a rotational reference frame

In the collision the rotational reference frames of the photon and electron are still motionless. The electron can only absorb the photon over a distance like it does in an orbital. This changes the -𝕆D kinetic probability of where the electron is, its e𝕪 position would change like in a collision. The photon transmits this change in position and kinetic probability, so it is measured as having changed its angle θ.

The photon cannot be completely absorbed

The photon must reappear, rather than being completely absorbed. This is because of the quantized nature of the interaction, the electron changed and this must be transmitted to conserve its interactions with other iotas. If not then an electron would appear to spontaneously change, with no conserved photons it would get out of sync with other electrons and atoms.

Colliding electrons

The collision is then the same in the rotational reference frame and the straight-line reference frame. When two electrons collide this also happens, in the rotational reference frame there is a changed -𝕆D kinetic probability in between them. That is mediated by photons appearing between them doing -𝔾D×e𝕪 light work. In the straight-line reference frame this is observed as a collision of elastic particles.

Absorbing a photon

When a photon is absorbed into an electron in an orbital, this is in a rotational reference frame. The photon increases the -𝕆D kinetic torque of the electron which raises it to a higher orbital. There cannot be a remaining photon after this because all the -𝔾D light torque was absorbed in changing the -𝕆D kinetic torque of the electron.

Emitting a photon later

The electron will emit a photon later to return to a lower orbital, this is like the collision between the photon and the electron in free space. The concept of later is in a straight-line reference frame, in the rotational reference frame there would be snapshots of the electron being measured in different orbitals. These need not be arranged in a before and after sequence, the probabilities can occur in different orders though others tend to cancel out destructively. This maintains the correspondence between the electron as a wave and a particle.

Radiation Pressure

Electromagnetic waves transfer not only energy but also momentum. An object gains momentum when it absorbs electromagnetic waves, much as a ball at rest gains momentum when struck by a ball in motion.

Work, impulse and energy

Here there is a change in the -𝕠d×e𝕪/-𝕠d kinetic momentum of an electron. This change in momentum can only occur with either E𝕐 being a square in a E𝕐/-𝕠d kinetic impulse or -𝕆D being a square in -𝕆D×e𝕪 kinetic work. When this impulse and work are in one formula that gives the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy. The e𝕪/-𝕠d term in the kinetic momentum has both the numerator and denominator squared.

A time interval in impulse

This is divided by c which here would be 1/(√ϵ√μ), that would give the ratios of the work and impulse squared forces here. Taking this as a time interval Δt, then the photon would be a particle in a straight-line reference frame. When colliding with an electron in an atom, it would push the electron to a greater e𝕒 altitude. This is like stretching a spring more, or in some cases compressing it more, with the photon impact.

A temporary change in altitude

Like with a rocket this only gives a temporary increase in altitude, a circular orbit would be made elliptical in the rotational reference frame. When there is an equal and opposite reaction from the electron, they would recoil from each other elastically. That would drive the electron back down again, releasing the photon.

The resonant frequency of the spring

There are no actual ellipses or circular orbitals in the straight-line reference frame, edge on the electron appears like a spring moving backwards and forwards. The impact of the photon particle does not change the resonant frequency of the spring, after the elastic collision the spring has a chaotic motion which resumes its usual frequency over time.

The spring constant and quantization

The quantized orbital would then be maintained by the spring constant, the period of its oscillation is inversely proportional to the e𝕒 altitude above the proton. A higher orbital would be like the spring temporarily halved its frequency for example. The photon would act like it was attached to the end of the spring, making it take longer to expand and contract in the straight-line reference frame. When the photon particle was released, the spring would go back to its original frequency. This would be consistent with what is measured in the rotational reference frame.

Suppose we shine a beam of light on an object that completely absorbs the light energy. If the object absorbs energy during a time interval Δt, its momentum changes by

$$\Delta p = \frac{\text{energy absorbed}}{c}$$

F=ma and momentum

Here f=ma is written as Δp or -𝕠d×e𝕪/-𝕠d kinetic momentum changing with respect to time which would give -𝕠d×e𝕪/-𝕆D with e𝕪/-𝕆D kinetic work according to dimensional analysis. Instead this is regarded as a change over time with a E𝕐/-𝕠d kinetic impulse, that shows the inverse relationship of work and impulse.

A change in power over time

The change in power as P is then in a straight-line reference frame as the E𝕐/-𝕠d kinetic impulse of electrons in atoms. These are impacted by photon particles with a e𝕐/-𝕘d light impulse.

This is a consequence of Maxwell's theory, which we'll state without proof.

The momentum change implies that the light is exerting a force on the object. Newton's second law, in terms of momentum, is $F = \Delta p/\Delta t$. The radiation force due to the beam of light is

$$F = \frac{\Delta p}{\Delta t} = \frac{(\text{energy absorbed})/\Delta t}{c} = \frac{P}{c}$$

Radiation pressure from work

Here the radiation pressure is coming from F which is -𝔾D×e𝕪 light work. Then the photon waves would do -𝕀D×e𝕧 inertial work on an object, in the rotational reference frame the object is a series of snapshots of probability densities. The changes in probabilities act as a force, a destructive interference can then appear like a solid object where other objects bounce off it.

Pressure and impulse

In the straight-line reference frame intensity is where the photon particles move in straight lines, they collide with the object so that the E𝕍/-𝕚d inertial impulse of each is conserved. Pressure occurs in a straight-line, a torque in a rotational reference frame can exert pressure such as with centrifugal force. Atoms can be regarded as having a centrifugal force being held by the proton in the rotational reference frame. Then a collision has this torque being proportional to probability like with the photon-electron and electron-electron collisions.

where P is the power (joules per second) of the light.

It's more interesting to consider the force exerted on an object per unit area, which is called the **radiation pressure** p_{rad}. The radiation pressure on an object that absorbs all the light is

$$p_{rad} = \frac{F}{A} = \frac{P/A}{c} = \frac{I}{c} \qquad (31.42)$$

where I is the intensity of the light wave. The subscript on p_{rad} is important in this context to distinguish the radiation pressure from the momentum p.

Photons on springs

In this model the electric charges reverse in a straight-line reference frame, the electrons accelerate back and forth with a E𝕐/-𝕠d kinetic impulse. This would be observed from the side of the diagram, the positive and negative charges would be switching sides. This would cause e𝕪/-𝕘d photon particles to be collided with, the photons can be regarded as being stuck on the ends of springs. It is as if the photons make the electron springs heavier, and so they oscillate over longer distances and more slowly.

Electrons as springs

That causes the electron springs to accelerate to greater e𝕒 altitudes, more of these photon particles are moving in the reversing current. When the photon particles are flung out of the antennae atoms the electron springs contract to a faster frequency with their E𝕐/-𝕠d kinetic impulse.

Electrons as waves

With the rotational reference frame the charges switch sides with an oscillating voltage, from +𝕆D×e𝕒 potential work and the +𝕆D potential difference, to -𝕆D×e𝕪 kinetic work and the -𝕆D kinetic difference. This change in work also changes the -𝕆D kinetic probabilities of the electrons. The quantized changes have e𝕪×-𝕘d photons being measured in different positions as snapshots over a distance, not over time.

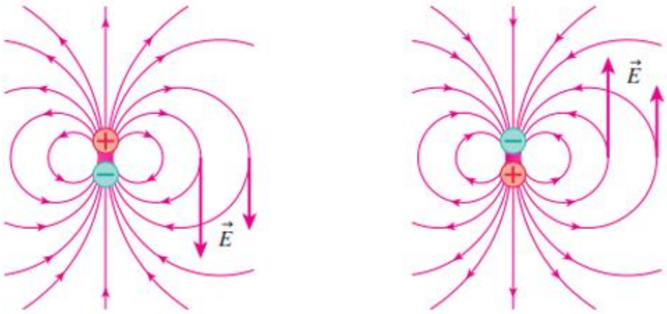
Antennas

We've seen that an electromagnetic wave is self-sustaining, independent of charges or currents. However, charges and currents are needed at the *source* of an electromagnetic wave. We'll take a brief look at how an electromagnetic wave is generated by an antenna.
FIGURE 31.25 is the electric field of an electric dipole. If the dipole is vertical, the electric field $\vec{E}$ at points along a horizontal line is also vertical. Reversing the dipole, by switching the charges, reverses $\vec{E}$. If the charges were to oscillate back and forth,

No electric fields

In this model there would not be an electric field, this is shown below as orthogonal to the proton's +𝕠d potential magnetic field and the electron's -𝕠d kinetic magnetic field. The diagram is in a rotational reference frame because there are curves, here these could only be magnetic fields. The electric charges move with a E𝔸/+𝕠d potential impulse and E𝕐/-𝕠d kinetic impulse, as particles they can collide with each other without needing fields.

FIGURE 31.25 An electric dipole creates an electric field that reverses direction if the dipole charges are switched.



Positive charge on top   Negative charge on top

An alternating current generator

In this model the photon particles move with a e𝕐/-𝕘d light impulse, that comes from the E𝕐/-𝕠d kinetic impulse of the electron particles. The inverse of the photon particles is measured at 90° with photon waves in a rotational reference frame. The oscillation here might come from an alternating current generator. When viewed in a rotational reference frame the center of the generator rotates creating +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work.

Acceleration in the generator

When the generator is viewed at 90° the central axis appears as a straight-line instead of a rotating point. Then the electrons are observed to accelerate from side to side. The electron particles can be regarded as collecting and discarding photon particles, as collisions these can cause the electrons to accelerate like one ball being hit by another.

Collecting and discarding photon particles

When the electrons reach the antennae they encounter an opposing positive charge, they had accelerated towards this. When they decelerate this causes the photon particles to be dislodged and to leave the antennae traveling at c. Then the electrons collect more photon particles from the generator. When the photon particles leave the antennae this is like a scattering as particles collide with each other, they can move in all directions.

Bohm's pilot wave theory

The straight-line reference frame can be regarded as the particle aspect of Bohm's pilot wave theory. This is like a boat moving on a river, from edge on at the surface of the water no waves can be observed. The water molecules can move up and down, and to the rear with an acceleration as a E𝕐/-𝕞d kinetic impulse and E𝕍/-𝕚d inertial impulse.

Particles and pilot waves

From above the same water molecules are measured to be waves, the wake can contain water waves that interfere with each other constructively and destructively. These molecules appear to have a particle and wave nature at the same time and position. Bohm's theory is that the photon and electron would have both these attributes at all times. They would have a position as a particle, then a pilot wave around them changes over time as the particles move.

No fixed position and no fixed time

In this model the positions and time are reversed, also to observe the particle and measure the pilot wave, there needs to be a change in the reference frame of 90°. To observe Bohm's particle it moves over time and has no continuous change in positions, to measure Bohm's pilot wave it changes over positions but has no continuous passage of time.

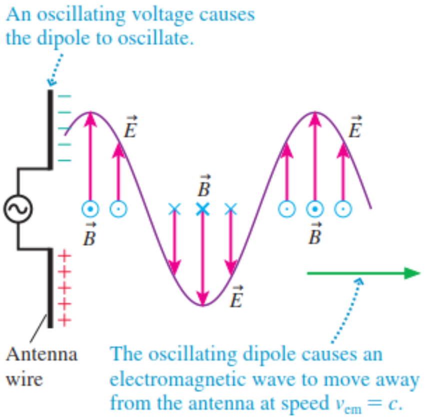
switching position at frequency f, then E would oscillate in a vertical plane. The changing $\vec{E}$ would then create an induced magnetic field $\vec{B}$, which could then create an $\vec{E}$, which could then create a $\vec{B}$, . . . , and an electromagnetic wave at frequency f would radiate out into space.

This is exactly what an **antenna** does. FIGURE 31.26 shows two metal wires attached to the terminals of an oscillating voltage source. The figure shows an instant when the top wire is negative and the bottom is positive, but these will reverse in half a cycle. The wire is basically an oscillating dipole, and it creates an oscillating electric field. The oscillating $\vec{E}$ induces an oscillating $\vec{B}$, and they take off as an electromagnetic wave at speed $v_{em} = c$. The wave does need oscillating charges as a *wave source*, but once created it is self-sustaining and independent of the source. The antenna might be destroyed, but the wave could travel billions of light years across the universe, bearing the legacy of James Clerk Maxwell.

Sine waves as a wheel

In the diagram the sine wave is in a rotational reference frame, this can be drawn by a rolling wheel. It is not rolling in a direction as that can only happen in a straight-line reference frame. A point on the electron spokes of the wheel can be observed at 90° in a straight-line reference frame. They move up and down as in the diagram.

FIGURE 31.26 An antenna generates a self-sustaining electromagnetic wave.



Three reference frames

In this model there are three reference frames at 90° to each other. The photon can be regarded as a rolling wheel, there is no surface it is rolling on. It would be like a thrown coin that is also spinning in free space, away from gravity. The frequency of this spin would be proportional to its inertial velocity.

The mark as a particle or wave

If the coin has one mark on its edge, then observing it with the coin edge's straight-line reference frame, the mark would be oscillating back and forth like a spring. Measuring the coin at 90° it would appear to be a circle in a rotational reference frame, snapshots of it would show the mark tracing out regular points on a sine wave curve. The coin itself is not visible in the straight-line reference frame so there is no rotation. The spring like oscillation is not observable in the rotational reference frame.

Rotating the coin

The coin can also be rotated along its path, if this is done at 90° then the rotational reference frame would appear as the straight-line reference frame edge, and vice versa. There is no change because the two reference frames remain at 90° to each other. This allows for polarization to occur in this model.

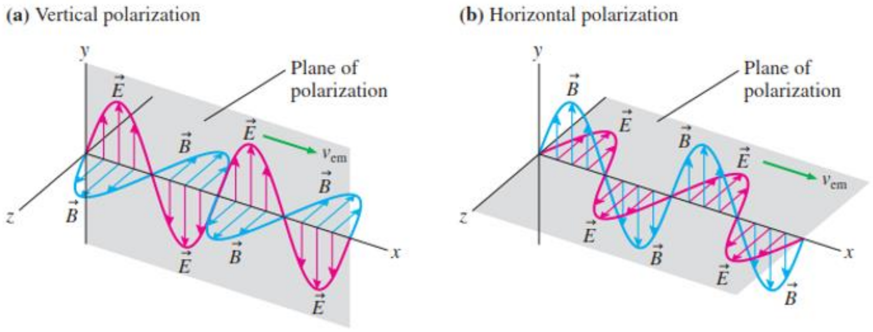
31.7 Polarization

The plane of the electric field vector $\vec{E}$ and the Poynting vector $\vec{S}$ (the direction of propagation) is called the **plane of polarization** of an electromagnetic wave. **FIGURE 31.27** shows two electromagnetic waves moving along the x-axis. The electric field in Figure 31.27a oscillates vertically, so we would say that this wave is *vertically polarized*. Similarly the wave in Figure 31.27b is *horizontally polarized*. Other polarizations are possible, such as a wave polarized 30° away from horizontal.

No E wave

In this model there would only be the blue B wave, at 90° to this there would be an oscillating spring motion.

FIGURE 31.27 The plane of polarization is the plane in which the electric field vector oscillates.



Polarization orientation

Polarization comes from the rotational reference frame, so its direction is random. A single photon has its own polarization angle, these can interfere constructively and destructively with other photons are different angles. When they are at 90° there is no interference with their -𝔾D×e𝕪 light work.

Some wave sources, such as lasers and radio antennas, emit *polarized* electromagnetic waves with a well-defined plane of polarization. By contrast, most natural sources of electromagnetic radiation are unpolarized, emitting waves whose electric fields oscillate randomly with all possible orientations.

A few natural sources are *partially polarized*, meaning that one direction of polarization is more prominent than others. The light of the sky at right angles to the sun is partially polarized because of how the sun's light scatters from air molecules to create skylight. Bees and other insects make use of this partial polarization to navigate. Light reflected from a flat, horizontal surface, such as a road or the surface of a lake, has a predominantly horizontal polarization. This is the rationale for using polarizing sunglasses.

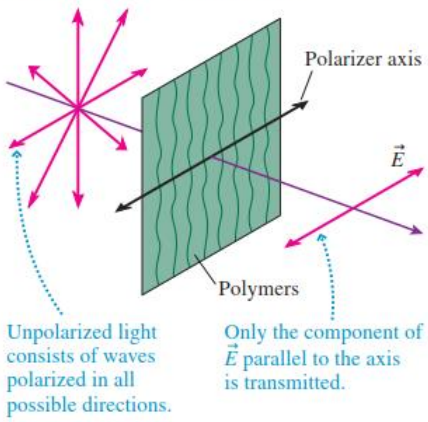
Polarizing filters

In this model one orientation of -𝔾D×e𝕪 light work passes through the vertical slits of the polarizing filter. There is also constructive and destructive interference between the photon waves like in a double slit experiment. At 90° to this the photon is observed as an oscillating spring, this moves electrons as they collide with the photon particles up and down the filter material. These photons are scattered in the material, their rotational reference frames can have them being absorbed in some atoms.

The most common way of artificially generating polarized visible light is to send unpolarized light through a *polarizing filter*. The first widely used polarizing filter was invented by Edwin Land in 1928, while he was still an undergraduate student. He developed an improved version, called Polaroid, in 1938. Polaroid, as shown in **FIGURE 31.28**, is a plastic sheet containing very long organic molecules known as polymers. The sheets are formed in such a way that the polymers are all aligned to form a grid, rather like the metal bars in a barbecue grill. The sheet is then chemically treated to make the polymer molecules somewhat conducting.

As a light wave travels through Polaroid, the component of the electric field oscillating parallel to the polymer grid drives the conduction electrons up and down the molecules. The electrons absorb energy from the light wave, so the parallel component of $\vec{E}$ is absorbed in the filter. But the conduction electrons can't oscillate perpendicular to the molecules, so the component of $\vec{E}$ perpendicular to the polymer grid passes through without absorption. Thus the light wave emerging from a polarizing filter is polarized perpendicular to the polymer grid. The direction of the transmitted polarization is called the *polarizer axis*.

FIGURE 31.28 A polarizing filter.



Two components as reference frames

In this model there is a magnetic rotational reference frame that passes through the filter. The straight-line reference frame moves into the straight lines of the filter material. The sinθ value here would be from the dine wave of -𝔾D×e𝕪 light work, the cosθ value gives the oscillating spring.

Malus's Law

Suppose a *polarized* light wave of intensity I_0 approaches a polarizing filter. What is the intensity of the light that passes through the filter? **FIGURE 31.29** shows that an oscillating electric field can be decomposed into components parallel and perpendicular to the polarizer axis. If we call the polarizer axis the *y*-axis, then the incident electric field is

$$\vec{E}_{\text{incident}} = E_{\perp}\hat{\imath} + E_{\parallel}\hat{\jmath} = E_0 \sin\theta\,\hat{\imath} + E_0 \cos\theta\,\hat{\jmath} \qquad (31.43)$$

Squaring sinθ

In this model the square comes from the -𝔾D light torque of the polarization. To turn the photon to a different polarization this torque might come from reflecting off water for example. As -𝔾D×e𝕪 light work the angle changes as a square, $\sin^2\theta$ would be used here with work. The squared sine here would square the spin Pythagorean Triangle side -𝕘d to give -𝔾D. The hypotenuse would be set as 1 and would not change, here the e𝕪 straight Pythagorean Triangle side would be used like a ruler to measure the photon absorption.

where θ is the angle between the incident plane of polarization and the polarizer axis.

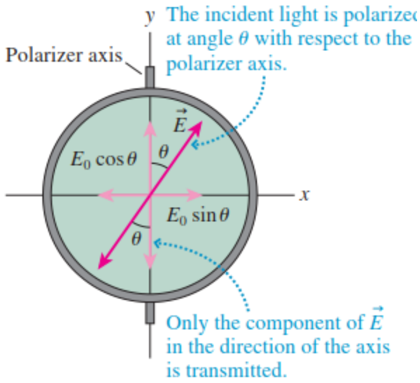
If the polarizer is ideal, meaning that light polarized parallel to the axis is 100% transmitted and light perpendicular to the axis is 100% blocked, then the electric field of the light transmitted by the filter is

$$\vec{E}_{\text{transmitted}} = E_{\parallel}\hat{\jmath} = E_0 \cos\theta\,\hat{\jmath} \qquad (31.44)$$

Because the intensity depends on the square of the electric field amplitude, you can see that the transmitted intensity is related to the incident intensity by

$$I_{\text{transmitted}} = I_0 \cos^2\theta \qquad \text{(incident light polarized)} \qquad (31.45)$$

FIGURE 31.29 An incident electric field can be decomposed into components parallel and perpendicular to a polarizer axis.



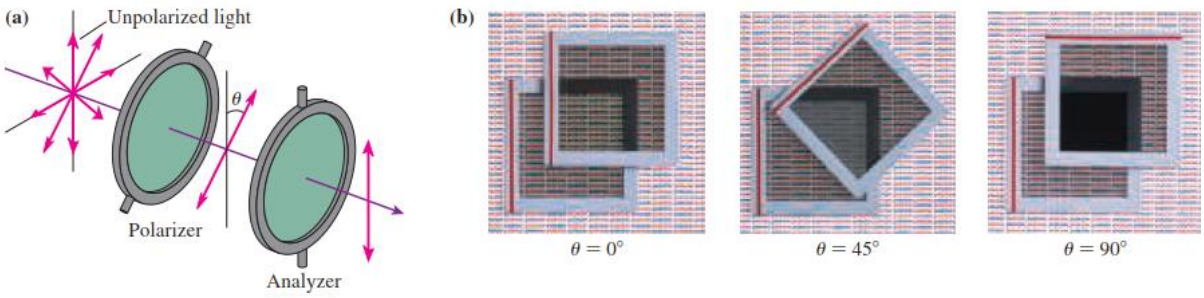
Changing light probability

When the polarizers change in a rotating reference frame, the -𝔾D light probability of a photon getting through them changes as a square.

This result, which was discovered experimentally in 1809, is called **Malus's law.**

FIGURE 31.30a shows that Malus's law can be demonstrated with two polarizing filters. The first, called the *polarizer,* is used to produce polarized light of intensity I_0. The second, called the *analyzer,* is rotated by angle θ relative to the polarizer. As the photographs of **FIGURE 31.30b** show, the transmission of the analyzer is (ideally) 100% when $\theta = 0°$ and steadily decreases to zero when $\theta = 90°$. Two polarizing filters with perpendicular axes, called *crossed polarizers,* block all the light.

FIGURE 31.30 The intensity of the transmitted light depends on the angle between the polarizing filters.



Average intensity

Half of the light is blocked where a normal curve has an average intensity. The filter exerts a torque on the -𝔾D×e𝕪 light work, this also has a normal curve distribution. The average then is in the center of the normal curve as half the e𝕪×-𝕘d photons get through.

Suppose the light incident on a polarizing filter is *unpolarized,* as is the light incident from the left on the polarizer in Figure 31.30a. The electric field of unpolarized light varies randomly through all possible values of θ. Because the *average* value of $\cos^2\theta$ is $\frac{1}{2}$, the intensity transmitted by a polarizing filter is

$$I_{transmitted} = \tfrac{1}{2} I_0 \qquad \text{(incident light unpolarized)} \qquad (31.46)$$

Rotating polarized glasses

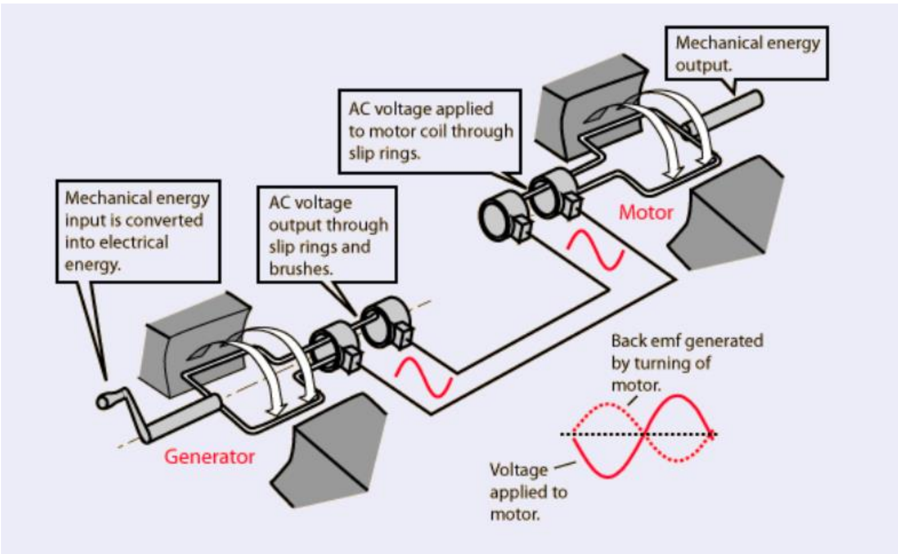
Rotating the polarized glasses occurs in a rotational reference frame.

In other words, a polarizing filter passes 50% of unpolarized light and blocks 50%.

In polarizing sunglasses, the polymer grid is aligned horizontally (when the glasses are in the normal orientation) so that the glasses transmit vertically polarized light. Most natural light is unpolarized, so the glasses reduce the light intensity by 50%. But *glare*—the reflection of the sun and the skylight from roads and other horizontal surfaces—has a strong horizontal polarization. This light is almost completely blocked by the Polaroid, so the sunglasses "cut glare" without affecting the main scene you wish to see.

You can test whether your sunglasses are polarized by holding them in front of you and rotating them as you look at the glare reflecting from a horizontal surface. Polarizing sunglasses substantially reduce the glare when the glasses are "normal" but not when the glasses are 90° from normal. (You can also test them against a pair of sunglasses known to be polarizing by seeing if all light is blocked when the lenses of the two pairs are crossed.)

AC Circuits



AC current in the straight-line reference frame

In this model AC current moves backwards and forwards, or up and down, in a straight-line reference frame. This is like the compression and expansion of a spring in a straight-line reference frame, when viewed at 90° to its motion.

AC current in the rotational reference frame

Viewed at 90° to this the electron waves appear to move as sine waves. There is no acceleration back-and-forth visible here, so there is no visible motion or velocity. The electron appears to be stationary with a rotational frequency of 1/-𝕠d. This is like the generator in the rotational reference frame, it also does not move in a direction but only rotates. Because of this the e𝕪/-𝕠d kinetic current only moves in the straight-line reference frame, in the rotational reference frame there is a -𝕠d×e𝕪 kinetic field from each Pythagorean Triangle that overlap each other.

Photons in the straight-line reference frame

With a photon the e𝕪 spoke would also move back and forth like this in the straight-line reference frame. Here the electron and photon both use the rolling wheel model, the photon transmits differences between one electron to another. In this reference frame it can only collide with an electron as a particle. That is because its interactions can only be described by acceleration and velocity.

Photons cannot collide

Photons cannot collide with each other because they only transmit changes between electrons, not between each other.

Photon and electron entanglement

In the EPR experiment a pair of photons or electrons can be entangled, when one is measured with spin up for example the other always has spin down. The first electron as an -𝕠d and e𝕪 Pythagorean Triangle would be measured with -𝕆D×e𝕪 kinetic work in a clockwise direction for example, the second electron would be measured from the other side with a counterclockwise direction.

Entanglement only measures spin

Because entanglement only measured spin, it can only measure work in the rotational reference frame. In the straight-line reference frame there are only electron particles, the spin there is time as a frequency. That cannot be measured as spin up or down, it is used to observe the E𝕐/-𝕠d kinetic impulse of the electron only.

Stationary entangled electrons

Entanglement then only occurs in the rotational reference frame, there is no time that elapses there as all electrons would appear to be stationary. The two electrons would be like snapshots with no continuous change between them of time, only of the distance between them. Because no time can elapse between the first spin measurement and the second, they are measured to always be the opposite spin.

Bosons

Boson pairs are also like this in orbitals, they are only in the rotational reference frame and time does not pass for them. If they are broken apart by a straight-line reference frame event, such as from a photon particle or a fermion electron, then time can again be observed for them.

Photons as bosons

Photons also have a spin 1 like bosons, this is because they are between the spin ½ of an electron in between one orbital and another. In the straight-line reference frame they can also be between two electron particles in free space, also with spin ½.

The uncertainty principle with entanglement

The straight-line reference frame has its own version of entanglement with elastic collisions. When two electron particles have a known velocity and collide, their kinetic momentum is known but their positions are uncertain. This is because position is only used to measure work, it represents a stationary rotational reference frame. When the velocity of one electron is observed, the other electron must have a known velocity also. That is like entanglement where conversely the spin is known but their velocity is not.

Entangled and enchained

Entanglement in the rotational reference frame is invariable over any distance, the enchained electron particles in the straight-line reference frame are invariant over any time. This continues until one of the electrons collides with another or enters an atom in the rotational reference frame.

The rotational reference frame from above

When the rolling wheel is observed from above, a spot on the circumference would appear to move back-and-forth like the spring. This would be the e𝕪 end of the e𝕪 and -𝕘d Pythagorean Triangle photon, the e𝕪 end of the -𝕠d and e𝕪 Pythagorean Triangle electron.

An ideal spring has no forces

This appears as an impulse force, but an ideal spring does not gain or lose energy. The impulse is the inverse of the -𝕆D×e𝕪 kinetic work done in twisting the spring, so these two forces cancel out.

A generator in a rotational reference frame

The generator spins in a rotational reference frame using magnets and -𝕆D×e𝕪 kinetic work. Looking down the spindle, the middle is usually coils of wire which have a +𝕆D potential and -𝕆D kinetic torque.

The generator as a clock gauge

The generator spins like a clock gauge, this is used to observe the back-and-forth motion of the current in the straight-line reference frame with impulse. The time is where the phase is observed, but here the rotation of the clock is not visible in the straight-line reference frame. This makes time more mysterious as somehow being associated with acceleration and impulse.

The macro world and time

When rotation is visible this is with torque, then there is no time being observed only the force of torque being measured. In the macro world this rotation is visible on clocks because work and impulse are a mixture of measurement and observation there. Then time seems to be associated with clocks, but also with straight-line motion such as a car accelerating.

The clock is not visible in the straight-line reference frame

In the straight-line reference frame time is only in instants, not as a duration between instants. So the clock cannot be visible, there is only one instant on the clock not the duration between them that the hand traverses.

A distance is not visible in the rotational reference frame

In a straight-line reference frame then the rotation of a clock is not visible as then torque would also be observable. In a rotational reference frame a distance is not visible either, that would be in a straight-line reference frame. Instead, there are only positions as snapshots like movie frames of frozen positions rather than continuous movement.

Entanglement and distance

When measuring spin with entangled photons, this can only be measuring the -𝔾D×e𝕪 light work as -𝔾D light torque. Otherwise, this spin would have no effect on the measuring instruments. But then there can be no distance associated with the spin, so it doesn't matter how far apart the entangled photons are.

Enchainment and time

When particles collide, this is predictable similar to with entanglement, If the particles become widely separated, then provided the original momentum is known, observing a first particle with give the velocity of the second particle. This would assume both particles came from a stationary source particle, or one with a known velocity and direction. The time cannot be seen as a duration in the straight-line reference frame here, so it does not matter how long before the collision occurred. This is like with entanglement when the distance does not matter.

The generator spindle

With the generator, the rotation is visible in the rotational reference frame looking down the spindle. At 90° the spindle would be orthogonal to the viewpoint used, then the rotation is not visible.

The frequency in the straight-line reference frame

The current accelerates to the right, then decelerates, it then accelerates to the left, decelerates, and continues this cycle. The frequency is determined by the clock gauge of the generator speed, the electrons move as particles colliding with each other down the wires. This time is usually 50 or 60 Hertz or cycles per second.

The magnetic field around the wires

At 90° to this there is a magnetic field from the generator in the rotational reference frame, this is measured by looking down the wire. That changes with the +𝕆D potential and -𝕆D kinetic voltage or difference, not with time but according to distance. So at different positions on the wire there would be a fixed voltage like a standing wave, if the generator has a constant frequency. There is no time changing in this rotational reference frame.

The magnetic field between wires and plates

This magnetic field can jump to other wires inducing a current. In a capacitor the rotational reference frame with an alternating current, can have the -𝕆D×e𝕪 kinetic work jump over the gap between the plates.

Electron particles jumping a gap

In the straight-line reference frame there would be no current as there can be no E𝕐/-𝕠d kinetic impulse collisions between electron particles. This gave rise to the concept of an electric field, in this model it is the magnetic field in the rotational reference frame.

AC and DC current

An alternating current has two reference frames as does direct current. The rotational reference frame comes from the rotation of the generator, that gives a back-and-forth motion of electron particles. A direct current has a directional flow of electrons in the straight-line reference frame, the rotational reference frame provides the voltage. Around both wires there is +𝕆D×e𝕒 potential work and -𝕆D×e𝕪 kinetic work being done.

Schrodinger’s equation

In Schrodinger’s equation energy is used, with this model that is a combination of work and impulse. This is written in relation to momentum which here is the -𝕠d×e𝕪/-𝕠d kinetic momentum. That has a particle/wave duality where -𝕠d×e𝕪 describes a field and e𝕪/-𝕠d describes the velocity of something which must be a particle.

Separating the straight and spin Pythagorean Triangle sides

Because of this the two variables as the straight and spin Pythagorean Triangle sides can be separated, in this model that becomes work and impulse. The time dependent Schrodinger's equation is in the straight-line reference frame referring to particles. That has h/2m which here is -𝕠d ×e𝕐/-𝕠d as h × ½×1/-𝕠d to give the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy.

The time dependent Schrodinger equation

This would be written has a -𝕠d ×e𝕐/-𝕠d for h where there is the E𝕐/-𝕠d kinetic impulse, dividing by 1/-𝕠d removes this in the numerator. ℏ here us not divided by 2π, that comes from √(2π) as β. Dividing by m gives impulse here, with the separation of variables it gives energy or momentum.

A change in energy over time

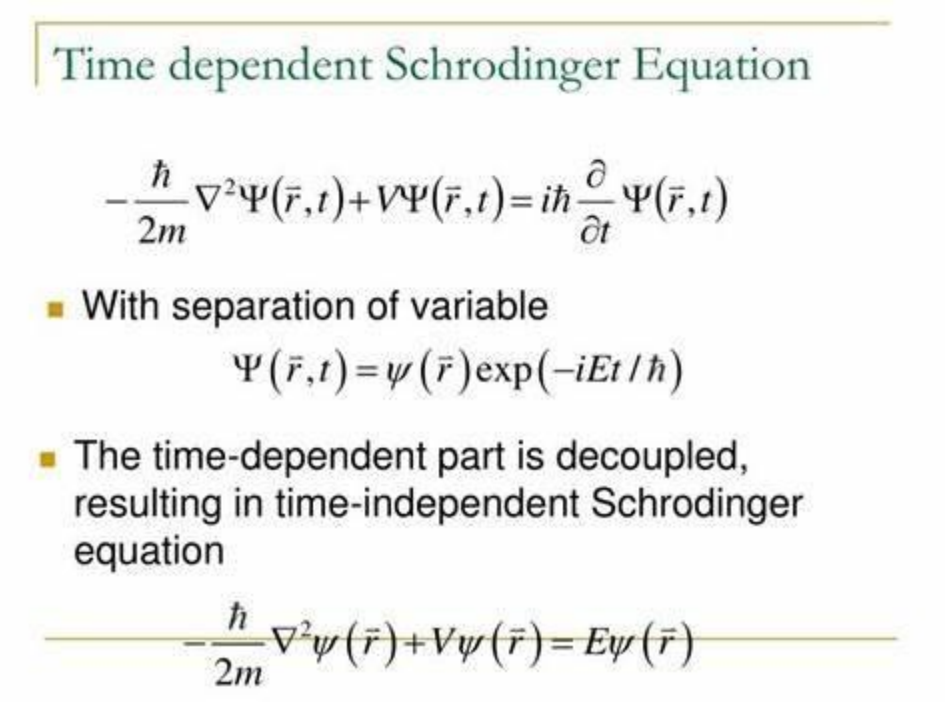
On the right-hand side this is ∂/∂ t as time dependent, that is where the mass -𝕠d is regarded as time. Here h is an observation of an electron as a particle, it has time already in the denominator. This is written instead from the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy multiplied by time as joule seconds to give -𝕠d ×e𝕐/-𝕠d. Then the change in the energy can be regarded as occurring over time.

Time dependent from the derivative

Starting from the -𝕠d×e𝕪/-𝕠d kinetic momentum, taking the derivative is in the straight-line reference frame to give -𝕠d ×e𝕐/-𝕠d. The derivative here comes from the straight-line reference frame.

Energy as an integral of velocity

The time independent Schrodinger equation has the same variables, instead energy is regarded as not changing over time to give EΨ. This is because energy comes from an integral of velocity with the -𝕠d×e𝕪/-𝕠d kinetic momentum. When e𝕪/-𝕠d is integrated with respect to both e𝕪 and -𝕠d it becomes ½×E𝕐/-𝕆D. This integration comes from work in this model, that is also time independent.



Sine waves

The sine wave shape is when the rolling wheel model of the rotational reference frame is compared to a straight-line reference frame of an axis. On a steam train a rotating wheel is connected to a piston at 90° to it. This has the straight-line reference frame as the piston moves with steam pressure back and forth. That is converted in the rotational reference frame to a rotary motion which drives the wheels. While the piston can be regarded as accelerating, the train overall can be at a constant velocity implying there is no actual force.

Pulleys and the rotational reference frame

Pulleys also use the two reference frames, when a rope goes around two pulleys several times there is stronger -𝕆D×e𝕪 kinetic work done by pulling the rope. That reduces the distance the moving pulley would travel compared with pulling on the rope a set distance. Because the distance is shorter the -𝕆D kinetic torque increase to maintain the constant Pythagorean Triangle area. It then becomes easier to turn the pulleys to lift a weight.

Pulleys in the straight-line reference frame

In the straight-line reference frame, from the side the rotation of the pulleys is not visible. Then the rope moves in a zig zag direction which reduces the overall distance the moving pulley can travel. Pulling the rope makes the pulleys move slower towards each other over time, so the E𝕐/-𝕠d kinetic impulse has a stronger E𝕐 kinetic displacement force. That makes it easier to pull up a weight over time in the straight-line reference frame.

Pulling a rope versus gears

Pulling the rope is itself in the straight-line reference frame. Another way of regarding the rotational reference frame is to use a chain and gears instead of pulleys. Then one of the gears can be turned with a -𝕆D kinetic torque, the gearing change increases this torque like gears in a car.

Levers in the rotational reference frame

A level can be seen to rotate from the side in the rotational reference frame. The torque on the fulcrum is where -𝕆D×e𝕪 kinetic work is done by pushing the longer end downwards. This torque is spread out over more positions and so it is easier to exert this force. The side with the weight has this torque spread out of fewer positions, and so the torque is stronger at each position. This makes it easier to move the weight.

Leverage and time

Looking at 90° in the straight-line reference frame from above, the ends of the lever seem to move towards the center at different accelerations. The side moved by hand has its position changing faster over a given time compared to the side with the weight. Because of this the E𝕐 kinetic displacement on this side is larger. The side with the weight has a smaller displacement and moves slower over time, so the force at a given instant must be larger from the constant Pythagorean Triangle area.

The lever in the vertical straight-line reference frame

The lever can also be observed with vertical changes in the straight-line reference frame. Looking along the lever, the end with the hand has a longer displacement between the starting and final positions. The end with the weight has a smaller displacement between the starting and final positions, so the force must be more concentrated there.

32.1 AC Sources and Phasors

One of the examples of Faraday's law cited in Chapter 30 was an electric generator. A turbine, which might be powered by expanding steam or falling water, causes a coil of wire to rotate in a magnetic field. As the coil spins, the emf and the induced current oscillate sinusoidally. The emf is alternately positive and negative, causing the charges to flow in one direction and then, a half cycle later, in the other. The oscillation frequency of the *grid* in North and South America is $f = 60$ Hz, whereas most of the rest of the world uses a 50 Hz oscillation.

The generator's peak emf—the peak voltage—is a fixed, unvarying quantity, so it might seem logical to call a generator an *alternating-voltage source*. Nonetheless, circuits powered by a sinusoidal emf are called **AC circuits,** where AC stands for *alternating current*. By contrast, the steady-current circuits you studied in Chapter 28 are called **DC circuits,** for *direct current*.

The rotating phasor

In the diagram the current changes with an acceleration as impulse, this is in the straight-line reference frame up and down. That is at 90° to the rotation of the phasor, the axle of this rolling wheel would point directly out of the page. This oscillates with the cosine which is at 90° to the sine wave.

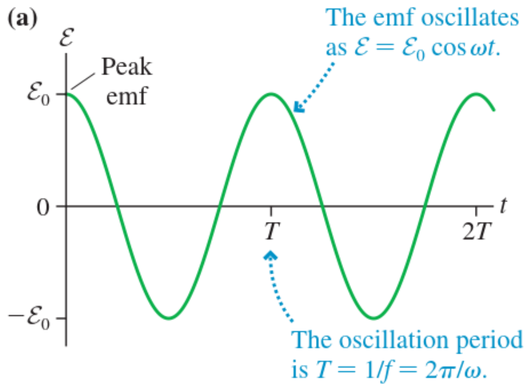
The cosine impulse moves vertically

Here there would not be a cosine wave, instead the straight Pythagorean Triangle side in the cosine as e𝕪/ζ would move up and down on the vertical axis.

The cosine is not a voltage or emf

The cosine here would not be a voltage, it would be an oscillation of power as the E𝕐/-𝕠d kinetic impulse. The -𝕆D kinetic voltage or difference is at a maximum at the top of the sine wave, this is where the maximum -𝕆D kinetic torque reverses the current direction. The maximum current is around zero, the greatest acceleration from the E𝕐/-𝕠d kinetic impulse occurs in between the peak and trough of the wave.

FIGURE 32.1 An oscillating emf can be represented as a graph or as a phasor diagram.



Changing the frequency as work

Here the phasor is like the e𝕪 spoke of the -𝕠d and e𝕪 Pythagorean Triangle electron, it moves around in a rotational reference frame. If the -𝕆D torque of the axle is stronger then the -𝕆D kinetic difference or voltage is also stronger. That would contract the e𝕪 phasor, the alternating current would have a higher Hertz frequency. In the straight-line reference frame, the back-and-forth motion would be faster with the E𝕐/-𝕠d kinetic impulse over a shorter distance. This acts like a gearing change as with the pulley and lever.

AC current wavelength and frequency

The distance the alternating current moves back and forth on is reduced so the E𝕐 displacement is reduced, along with the faster frequency of the generator. This is like a reduction in the wavelength in the rotating wheel model, the wheel turns inversely faster with a higher frequency.

Transformers and voltage

A power station can also increase -𝕆D×e𝕪 kinetic work inversely to the E𝕐/-𝕠d kinetic impulse with a transformer. This has more coils on one side, that increases the -𝕆D kinetic torque at the same frequency.

Less electricity lost as work

There is less electricity lost because it moves mainly as a -𝕆D kinetic wave around the wire. At lower voltages the current has a greater E𝕐/-𝕠d kinetic impulse like a particle. This causes more scattering as electrons collide with each other, that loses some impulse as heat.

More impulse, less work in a home

When this higher voltage reaches a home, then it is stepped down with a transformer to have a higher E𝕐/-𝕠d kinetic impulse, the current has more power as a E𝕐/-𝕠d kinetic impulse. That allows it to push through a resistor such as an electric heater or light more.

Electron waves in a gas discharge lamp

In a fluorescent bulb the electron waves with -𝕆D×e𝕪 kinetic work make the electrons, in a thin gas, change their orbitals with a -𝕆D kinetic torque. This causes them to emit photons in a discrete spectrum. The -𝕆D kinetic difference or voltage can be fairly low, 90 volts for a neon lamp.

Cathode ray tubes

With the photoelectric effect the electrons can be emitted from a cathode, that requires a higher -𝕆D×e𝕪 kinetic work as the kinetic torque makes the electron waves leave the atom. Then the electrons act as particles in the straight-line reference frame, they move towards an anode screen in straight lines. They can be bent in different directions to form a picture in the rotational reference frame with magnets.

Incandescent bulbs and impulse

These are more efficient than an incandescent bulb with uses a E𝕐/-𝕠d kinetic impulse to create heat from electrons colliding with a resistor.

Voltage and gears

A change in voltage with a transformer is like gearing in a car, in the rotational reference frame first gear has the highest -𝕆D kinetic torque, that moves the car with the slowest velocity and lowest impulse. It is best for moving up a hill against +𝕀D×e𝕙 gravitational work. In top gear with the straight-line reference frame the E𝕐/-𝕠d kinetic impulse is maximized, there is the lowest -𝕆D kinetic torque. This is best for moving with an E𝕍/-𝕚d inertial impulse and lower +𝕀D×e𝕙 gravitational work in level roads.

A higher velocity with impulse

Because e𝕪 is larger with a E𝕐/-𝕠d kinetic impulse so is the kinetic velocity e𝕪/-𝕠d and the corresponding e𝕧/-𝕚d inertial velocity. The car can then go faster.

The phasor as a straight Pythagorean Triangle side

In this model the green arrow would be the e𝕪 spoke or phasor of a rolling wheel. That would be where the -𝕠d and e𝕪 Pythagorean Triangle electron had the -𝕠d kinetic axle pointing out of the page and the e𝕪 phasor in green rotating around it. This would not be a hypotenuse as in the diagram, it is a constant as the wheel rotates.

Observing and measuring the phase

When this is observed, in the vertical straight-line reference frame, the tip of the spoke seems to accelerate up and down with a E𝕐/-𝕠d kinetic impulse. The rotational frequency of this spoke as -𝕠d is ω, t gives the phase at an instant of time for the E𝕐/-𝕠d kinetic impulse. For -𝕆D×e𝕪 kinetic work the phase is measured with a position on the wire.

The Euler formula

The phasor here is like the hypotenuse in the Euler formula, that is written as $e^{\pi i}$ for a full circle. Then the axle, pointing out of the page through the origin, would rotate 360°. In the Euler formula the vertical axis is imaginary, and the horizontal axis would be real. This gives an equation e^{nx}=cosx+isinx.

The hypotenuse as the wheel spoke

In this model the hypotenuse is the e𝕪 spoke in the -𝕠d and e𝕪 Pythagorean Triangle, that rotates at a constant frequency -𝕠d. Here e𝕪 is not the hypotenuse as shown in the diagram below, it is the straight Pythagorean Triangle side.

The angle changing with the phase

In the diagram a second Pythagorean Triangle is drawn as the hypotenuse rotates. That would vary from 0 to 90° as the phase changes with the sine wave. That would have $e^{i\theta}$=cosθ+isinθ changing with the angle.

The rotating wheel does not change its angle θ

The -𝕠d and e𝕪 Pythagorean Triangle would not have its angle changing as long as the frequency stayed the same. This is because in its rotational reference frame time does not vary only the position of the wheel. When this -𝕠d and e𝕪 Pythagorean Triangle is observed with the vertical or horizontal axis, that changes with time. Changing from the rotational reference frame to the straight-line reference frame then allows for an observation of the E𝕐/-𝕠d kinetic impulse, the angle change in $e^{i\theta}$ gives the squared acceleration up and down.

Sine as torque, cosine as acceleration

With a cosine, cos x in the Euler formula, this changes with an acceleration back-and-forth. With the isinx term, that represents the changing torque of the sine wave. When this increases, the sine wave turns at the peak and trough.

The sine and cosine as inverses

With a constant area Pythagorean Triangle these are inverses, so as cosθ increases then isinθ decreases proportionally. The two forces cancel out so the wheel has no changing forces. Instead, it changes from torque to displacement over and over.

The rolling wheel

If a point was marked on the rolling wheel that would seem to vary up and down in the straight-line reference frame as an acceleration. It would also seem that something was pulling the point down from the peak and up from the trough as a torque. These forces come from the different reference frames, the wheel itself does not change.

No actual forces

In this model a Pythagorean Triangle can change its angle θ, opposite the spin Pythagorean Triangle side. This appears to be two kinds of forces, impulse in the straight-line reference frame and work in the rotational reference frame. Because these are inverses there are no actual forces, the angles change in the Pythagorean Triangle giving different physical phenomena.

Obscure instead of imaginary numbers

The vertical axis in the diagram is imaginary, with this model that would instead point out of the page as an Obscure number -𝕠d. That means it is still the square root of -1 like imaginary numbers, -𝕠d is the negative square root only. This is the -𝕠d and e𝕪 Pythagorean Triangle actually driving the sine wave, with a constant angle θ the observed impulse and measured work are inverses.

Changing the angle creates no forces

This means the act of observation creates the force of impulse. The act of measurement creates the force of work. But these forces only occur by using one reference frame. With a rolling wheel changing the angle θ would change its rolling frequency, and inversely the size of the spoke. It would continue to roll at the same velocity so there is no actual force change.

The reference frame creates the force

Photons as rolling wheel can then change their frequency, and inversely their wavelength, without changing their inertial velocity. The force of -𝔾D×e𝕪 light work appears with an angle change, then a photon wave can be absorbed into an electron. The force of the e𝕐/-𝕘d light impulse also appears with an angle change, then the photon particle can collide with an electron.

Changing the Pythagorean Triangle inversely

In the Euler formula, increasing the -𝕠d and e𝕪 Pythagorean Triangle angle θ, would increase the size of the spin Pythagorean Triangle side pointing out of the page. It would also contract the straight Pythagorean Triangle side as the hypotenuse in the diagram. This could be, for example, an electron absorbing a photon and changing its angle θ.

Impulse as the amplitude oscillates

With this -𝕠d and e𝕪 Pythagorean Triangle the Euler Formula works the same way as in conventional math. The inscribed Pythagorean Triangle in the diagram however has its angle oscillating from 0 to 90°. The sine wave amplitude goes up and down like the force of impulse, but the -𝕠d and e𝕪 Pythagorean Triangle electron itself has not changed its forces.

The electron is not created or destroyed

When this angle changes, the AC current can do measurable -𝕆D×e𝕪 kinetic work with a voltage, or be observed as power with its E𝕐/-𝕠d kinetic impulse. It does this with no actual forces, the electron only changes its angle and so it is not created or destroyed in the process.

Obscure and imaginary numbers

In this model -𝕠d is the negative square root of -1, that is the same as an imaginary -i. The Euler formula would then be written as $e^{i\theta}$=cosθ+sinθ because sinθ is -𝕠d/ζ where ζ is the hypotenuse. There is no need to write isin because every sine in this model has a spin Pythagorean Triangle side in it. Then cosθ would be e𝕪/ζ so this can be referred to as the real axis because e𝕪 is the straight not spin Pythagorean Triangle side.

Changing the angle θ

As the angle θ changes then cosθ+sinθ changes, it is not a constant even when they are inverses. This is because e𝕪+𝕠d are added not multiplied, that cannot be constant if -𝕠d×e𝕪 is a constant Pythagorean Triangle area. That gives the sine wave shape even though the two forces are inverses here.

Constant area trigonometry

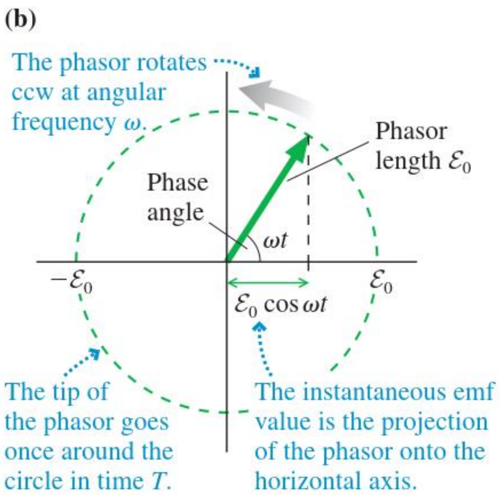
The angles, sines and cosines remain the same as in the Euler formula. This model uses constant area trigonometry, that is where the -𝕠d and e𝕪 Pythagorean Triangle has a constant area. A change in angle preserves the same Pythagorean Triangle area, but the hypotenuse is no longer constant. This would not affect the diagram because the e𝕪 straight Pythagorean Triangle side, as the hypotenuse, does not change.

The inscribed Pythagorean Triangle

The inscribed Pythagorean Triangle in the diagram does not have a constant area, this is because the hypotenuse is also e𝕪 which remains a constant. However this conventional trigonometry is the same as in constant area trigonometry, the sines and cosines are unchanged.

Rewriting the exponent

In this model $e^{i\theta}$ can be written as $e^{\text{e𝕪-𝕠d}}$ with the example of the -𝕠d and e𝕪 Pythagorean Triangle electron. Then -𝕠d is part of the sine -𝕠d/ζ and e𝕪 is part of the cosine e𝕪/ζ. This just substitutes the Pythagorean Triangle sides for the angle. It must also equal cosθ+sinθ. That allows for the Pythagorean Triangle sides to be squared to give -𝕆D×e𝕪 kinetic work and the E𝕐/-𝕠d kinetic impulse as inverses. Then the rolling wheel can be measured as work, also observed as the impulse.



Changing the circle in the Euler formula

In the Euler formula the -𝕠d and e𝕪 Pythagorean Triangle has a given angle θ, this gives a sine wave. Because the -𝕠d and e𝕪 Pythagorean Triangle has a constant area, when the -𝕠d kinetic frequency increases the e𝕪 spoke spins around faster. That spoke is the hypotenuse in the Euler formula.

Changing the spoke size

The spoke contracts in size to maintain a constant Pythagorean Triangle area, if the frequency doubles the spoke size halves. That allows the electron to be in different orbitals, then the -𝕆D kinetic torque increases as the orbital e𝕒 altitude gets higher.

Photon sine waves

The e𝕪 and -𝕘d Pythagorean Triangle photon is represented the same way, there is a e𝕪 spoke and the -𝕘d rotational frequency is the axis pointing out of the page. When this -𝕠d kinetic frequency increases so does the -𝔾D×e𝕪 light work of the photon wave. This leads to the photoelectric effect, a higher -𝔾D light torque as a work function can liberate electrons from an atom.

Photons as particles or waves

The different frequencies of electromagnetic waves, or photons, are used in various circuits. When the frequency is higher, the photons act more like waves in the rotational reference frame. When it is lower the photons are more like particles in the straight-line reference frame.

The emf as time

In (32.1) sinωt would be used instead of a cosine here for an $\boldsymbol{\mathcal{E}}$ voltage. If the straight-line reference frame is used, then this $\boldsymbol{\mathcal{E}}$ voltage is regarded as instantaneous. At a given instant, there is a voltage so in this model $\boldsymbol{\mathcal{E}}$ can be used as time itself. The rotation of the phasor is then like a clock gauge, the $\boldsymbol{\mathcal{E}}$ voltage would changing linearly like the rotation of the generator. When the generator turns at a constant rate, the $\boldsymbol{\mathcal{E}}$ can give a changing phasor like a constantly moving clock hand.

The emf as time is not visible

In this model time is not visible in the rotational reference frame so the clock gauge, or rotating $\boldsymbol{\mathcal{E}}$ is also not visible. Voltage in a wire is in the straight-line reference frame, it impels electron particles to accelerate or move with particle collisions along this circuit.

A circuit has voltage, a straight wire does not

When there is a circuit this is in the rotational reference frame, then the +𝕆D potential difference and -𝕆D kinetic difference change in the generator to give the current. When the wire is regarded as straight this is in the straight-line reference frame, but then there can be no connection to a +𝕆D potential and -𝕆D kinetic difference. The electron particles are accelerated through collisions, these transmit forces elastically with changes in their inertial velocities.

No voltage with time

Because of this, the $\boldsymbol{\mathcal{E}}$ instantaneous voltage can be referred to as time, there is no actual voltage because there are no instants in the rotational reference frame. In conventional physics the emf term is not well defined, here it sometimes refers to voltage as a force but also as an instantaneous value.

AC circuits are not limited to the use of 50 Hz or 60 Hz power-line voltages. Audio, radio, television, and telecommunication equipment all make extensive use of AC circuits, with frequencies ranging from approximately 10^2 Hz in audio circuits to approximately 10^9 Hz in cell phones. These devices use *electrical oscillators* rather than generators to produce a sinusoidal emf, but the basic principles of circuit analysis are the same.

You can think of an AC generator or oscillator as a battery whose output voltage undergoes sinusoidal oscillations. The instantaneous emf of an AC generator or oscillator, shown graphically in **FIGURE 32.1a**, can be written

$$\mathcal{E} = \mathcal{E}_0 \cos \omega t \qquad (32.1)$$

Radians per second as time

Here ω is in radians/second, because radians are an angle around a clock gauge this means ω can refer to time. This would be a duration such as where ω is a full rotation of 12 hours. In this model that would be a duration in the rotational reference frame so sinω would be used.

Clock hand positions

With ωt this is like a clock hand position, out of the 12 hours then 1/12 would be a position on the clock gauge in the rotational reference frame. That hour would be a duration where the clock hand started at 12, with an increasing torque started moving to reach 1 o'clock.

where $\mathcal{E}_0$ is the peak or maximum emf and $\omega = 2\pi f$ is the angular frequency in radians per second. Recall that the units of emf are volts. As you can imagine, the mathematics of AC circuit analysis are going to be very similar to the mathematics of simple harmonic motion.

The magnitude of the phasor

Here the magnitude of the phasor would be e𝕪 as a vector or spoke of the rolling wheel. This is referred to as $\mathcal{E}_0$ when the spoke points straight up, when $\mathcal{E}$ is time in the e𝕐/-𝕘d light impulse then this gives an instant when the impulse is strongest when projected in the straight-line reference frame.

$\mathcal{E}$ as a constant voltage

When the e𝕪 and -𝕘d Pythagorean Triangle is the rolling wheel, the rotation of the -𝕘d axle gives through different phases in a particle straight-line reference frame. The $\mathcal{E}$ value can be regarded as a constant voltage that comes from the angle θ of the e𝕪 and -𝕘d Pythagorean Triangle photon. When this time value as $\mathcal{E}$ is squared it gives -𝔾D×e𝕪 light work so -𝔾D as the light torque is a force as $\mathcal{E}^2$. That makes the emf an electromotive force that moves electrons in the rotational reference frame.

Changing $\mathcal{E}$ from a spin Pythagorean Triangle side to a hypotenuse

When the e𝕪 spoke becomes a hypotenuse in the above diagram, the $\mathcal{E}$ now changes linearly from a minimum to a maximum value. This is because as an instant there is no squared force, so there is no acceleration. In conventional physics this instantaneous voltage comes from a derivative, in this model only a straight Pythagorean Triangle side can be differentiated. Then $\mathcal{E}$ would come from integration as the area of the rolling wheel.

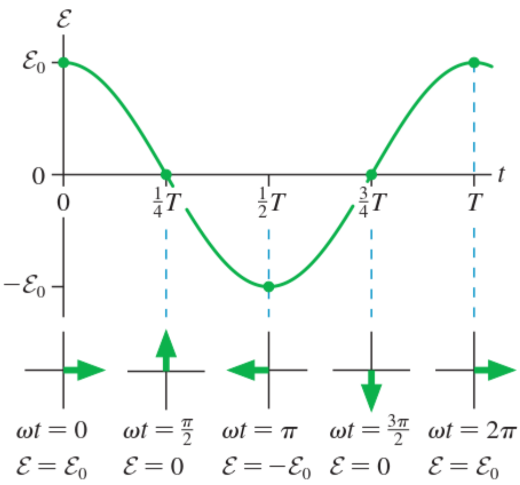
An alternative way to represent the emf and other oscillatory quantities is with the *phasor diagram* of **FIGURE 32.1b**. A **phasor** is a vector that rotates *counterclockwise* (ccw) around the origin at angular frequency ω. The length or magnitude of the phasor is the maximum value of the quantity. For example, the length of an emf phasor is $\mathcal{E}_0$. The angle ωt is the *phase angle*, an idea you learned about in Chapter 15, where we made a connection between circular motion and simple harmonic motion.

The quantity's instantaneous value, the value you would measure at time t, is the projection of the phasor onto the horizontal axis. This is also analogous to the connection between circular motion and simple harmonic motion. FIGURE 32.2 helps you visualize the phasor rotation by showing how the phasor corresponds to the more familiar graph at several specific points in the cycle.

A generator changing probabilities

Here the phase is projected onto the vertical straight axis, this gives a back-and-forth motion along it in the straight-line reference frame. The generator does -𝕆D×e𝕪 kinetic work as the magnets rotate, they change their distance from parts of the coiled armature. This gives a -𝕆D kinetic torque, changing the kinetic probability of where the armature is likely to be measured. That change in probability makes it rotate.

FIGURE 32.2 The correspondence between a phasor and points on a graph.



A stationary current in time

In this model the instant on a clock gauge is used to observe impulse. Then the change over a distance would be a displacement such as E𝕐 with a change in an electron's position as an electric charge. The instantaneous current as e𝕪/-𝕠d would make e𝕪 an infinitesimal and -𝕠d an instant.

Work and impulse as reference frames

In this model -𝔾D×e𝕪 light work for example combines a squared Pythagorean Triangle side with a linear one. This happens because they are in different reference frames. In the rotational reference frame there is a -𝔾D light torque, but e𝕪 positions are in the straight-line reference frame on a scale or ruler.

A position in the rotational reference frame

To measure work there is a squared duration of time, like a torque of moving a hand on a clock gauge. With the sine wave rolling wheel, this would be a torque on the axis pointing out of the page. A position is a point or infinitesimal, so it can be used in the rotational reference frame. A displacement cannot be, that would be a line from a starting to a final position. That requires a straight-line reference frame.

A photon in two reference frames

When a photon is in two reference frames, this comes from having a spin and straight Pythagorean Triangle side. A force must go from one position to another as a displacement in the straight-line reference frame, from one instant to another as a duration in the rotational reference frame.

A curve cannot exist in the straight-line reference frame

In the straight-line reference frame there can be an E𝕐 displacement with the photon, that can be observed at an instant which is like a point in time. There cannot be a duration of time this displacement occurs in, if there was then the point in time would become a starting point and an ending point. Because this is rotation that would be a curve, which cannot exist in the straight-line reference frame.

A stationary Zeno's arrow

In Zeno's arrow paradox, when the arrow is stationary no time is passing. But then while moving it would be stationary at any instant of time, so the arrow can never move. In the rotational reference frame the arrow can rotate to point in different directions, this is like the phasor of the sine wave.

A rotating arrow does not move

If Zeno's arrow rotates it has not moved from its position. Then the torque of this rotation can be measured at a single position, this is in the rotational reference frame.

An arrow's displacement is observed at an instant

The displacement of an arrow occurs at an instant in the straight-line reference frame. This displacement is a force from a starting to a final position, a clock gauge cannot rotate in the straight-line reference frame. So it cannot be used to observe a duration, as that would be a rotation, only an instant of time.

A constant Pythagorean Triangle area and reference frames

With Zeno's arrow, this can rotate with different -𝕀D inertial torque values in the same position. That means the angle θ of the -𝕚d and e𝕧 Pythagorean Triangle can change to give these different torque values. The e𝕧 position would have different e values, this does not affect the measurement because it is still a point.

Observing a photon

The photon can be observed in the straight-line reference frame with a e𝕐/-𝕘d light impulse, measured in the rotational reference frame with -𝔾D×e𝕪 light work. As a photon particle it is observed to have an E𝕐 displacement, for example it might collide with an electron giving or receiving an E𝕐 kinetic displacement from it. That occurs at a -𝕘d instant because the -𝕘d rotational frequency cannot rotate to be a duration in the straight-line reference frame.

Measuring a photon

The photon can be measured as -𝔾D×e𝕪 light work, this is like Zeno's arrow that can rotate in the same position without moving. In the rotational reference frame the photon wave is stationary, this is similar to Einstein's thought experiment of moving next to a photon so it appears stationary. The -𝔾D light torque is absorbed into an electron in its orbital, this cannot change its position because a line between points only exists in a straight-line reference frame.

Entangling photons in the straight-line reference frame

Here there is a displacement between the two entangled photons as they move apart. This can be observed at an instant of -𝕘d light time, the clock gauge cannot turn as it is not in the rotational reference frame. As two photon particles they preserve the spin they have from each other as this instant. When the first photon is measured with its -𝔾D light torque, the second photon retains this opposing instantaneous spin.

Two balls colliding

This is like two balls that collide, the first ball has a spin and transfers some of this to the second ball. Then when the first ball is measured a distance away, its spin is found to be opposed to the second ball. This assumes the first ball had a known spin value, then the amount of the second ball's spin would be known before that was measured. Because there is no time duration in the straight-line reference frame, the balls retain their spin over a long period of time.

Two entangled waves

In the rotational reference frame the two photons are entangled with -𝔾D×e𝕪 light work, this can be from an electron in an orbital. The -𝔾D light probability or torque means if one is measured with an up spin the second photon has a 100% light probability of having a down spin. This torque is measured at a single position, the photon appears as a stationary wave. Because there is no displacement in the rotational reference frame the photons retain their opposing spins over any distance.

Points on a line

Another paradox from Zeno referred to points with no length on a line. It said that no matter how many points there are there could always be points in between them, so they could never make up a line. If the points are in the rotational reference frame they can only rotate, they cannot move from a position to create a displacement as a line. Conversely a line is a displacement in the straight-line reference frame, it cannot change over time because a clock gauge cannot rotate. So no points can be removed from the line to make it a series of points.

Two reference frames in the tolling wheel model

These two reference frames are together in the rolling wheel photon model. The sine wave appears to have an acceleration up and down as a changing amplitude. This would be the e𝕐/-𝕘d light impulse, each instant of time cannot rotate to another instant in the straight-line reference frame. The ***Ɛ*** instantaneous voltage can appear as this time, with an electron current -𝕆D×e𝕪 kinetic work has -𝕆D as the kinetic difference or voltage.

Using ω and t

In the straight-line reference frame there can be no voltage as torque, it is referred to as ***Ɛ*** the instantaneous voltage like an instant of time. With ωt, this combines an instant of time as t with a rotational frequency as ω. It is like a particle wave duality where the straight-line reference frame and rotational reference frame are combined. This enables a cosωt and sinωt to connect to the straight-line reference frame and rotational reference frame respectively.

Oscillating in two reference frames

A spring can oscillate in both reference frames, there are no overall forces because an ideal spring does not emit or take in energy. When the length of the spring oscillates this is an E𝕍/-𝕚d inertial impulse in the straight-line reference frame. The coil winds and unwinds as the -𝕀D×e𝕧 inertial work drives the oscillation in the rotational reference frame. Because this is the inverse of the E𝕍/-𝕚d inertial impulse the overall energy is the same.

Division and multiplication

In this model division is the slope of a Pythagorean Triangle and multiplication is where the two Pythagorean Triangle sides form an area. This slope comes from a derivative with respect to the straight Pythagorean Triangle side, the area of the Pythagorean Triangle comes from an integral where the two Pythagorean Triangle sides are multiplied together.

Acceleration in a straight-line

In the straight-line reference frame, there are fractions such as meters/second, with the -𝕚d and e𝕧 Pythagorean Triangle as e𝕧/-𝕚d. They come from acceleration in a straight-line, with the E𝕍/-𝕚d inertial impulse this is in meters^2/second. The squared straight Pythagorean Triangle side is E𝕍, the spin Pythagorean Triangle cannot spin in the straight-line reference frame so it remains an instant of time.

Areas from rotation

In the rotational reference frame there is only multiplication, this is how an area must be constructed. Because of this the integrals are quantized as integers, they are the only numbers that are not fractions. The radius of a circle can sweep out an area as the frequency of the rolling wheel, like a phasor in a sine wave.

No time elapses in an orbital

These are measured like -𝕀D×e𝕧 inertial work at positions, there is a -𝕀D inertial torque like a wrench turning a bolt. This is in a fixed position, a motion in a straight-line can only occur in the straight-line reference frame. When an electron is in a circular orbital no time elapses, there can be a jump to another integer value as an orbital.

Time elapsing between electron orbitals

It has been shown a small amount of time elapses when an electron jumps between orbitals. In this model that would be in the straight-line reference frame, the electron moves partially in the straight-line up to a higher orbital.

A spiral in two reference frames

It also moves in a spiral where the -𝕆D kinetic torque of the electron is increasing. This is only measurable as quantized values, and in part with elliptical orbitals. In the rotational reference frame the spiral has its shape, in the straight-line reference frame it increases its distance from the center of the spiral with an acceleration.

An ellipse in two reference frames

Some atomic orbitals are ellipses, these also occur in Biv space-time in between quantized circles in a cone and parabolas. They combine the two reference frames, in the rotational reference frame there is a rotation around one of the foci. This is like Zeno's arrow that can spin around a point while not changing its position with -𝕀D×e𝕧 inertial work.

An ellipse with two axes

The electron in Roy electromagnetism rotates around this focus in the rotational reference frame, there is a torque as the electron has different -𝕆D kinetic torque values depending on its e𝕒 altitude above the proton. The -𝕆D×e𝕪 kinetic work of the electron has a major axis like the peak and trough of a sine wave. This is where the -𝕆D kinetic torque turns the electron around as a quantized value.

A faster acceleration at the minor axis

When this is observed at 90°, the edge of the ellipse appears as a line as in Zeno's paradox, from the point on this line as a focus the electron is observed to move with a faster acceleration around the minor axis. This is like the middle of the sine wave where its acceleration is at its maximum, also like the middle of a spring oscillating. Ocean waves are more elliptical as they come from +𝕀D×eh gravitational work.

The ellipse has no forces

The straight-line reference frame and the rotational reference frame are inverses of each other, so the impulse and work balance each other. Because the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy equation contains both the E𝕐/-𝕠d kinetic impulse and -𝕆D×e𝕪 kinetic work, these inverses give a constant energy value in an orbital whether circular or elliptical.

Resistors and protons

In this model a resistor is where the proton holds electrons more strongly, this is from how the electron orbitals are filled. It does +𝕆D×e𝕒 potential work reactively in the rotational reference frame, and the E𝔸/+𝕠d potential impulse in the straight-line reference frame. An instantaneous voltage would be the +𝕠d potential time, it is an instant in the E𝔸/+𝕠d potential impulse because there can be no turning in a clock gauge in the straight-line reference frame. That would give an e𝕒/+𝕠d potential current which is also instantaneous in the straight-line reference frame.

Resistor Circuits

In Chapter 28 you learned to analyze a circuit in terms of the current I, voltage V, and potential difference ΔV. Now, because the current and voltage are oscillating, we will use lowercase i to represent the *instantaneous* current through a circuit element and v for the circuit element's *instantaneous* voltage.

The resistor voltage in the rotational reference frame

When a current goes through a resistor this is in the straight-line reference frame, then there is an instantaneous e𝕒/+𝕠d potential current. The resistor voltage would be in the rotational reference frame as +𝕆D×e𝕒 potential work. The electrons would move with -𝕆D×e𝕪 kinetic work and a -𝕆D kinetic difference. This can come from a generator with AC current. The resistor reacts against the motion of the electron particles in both back-and-forth directions, its +𝕆D potential difference attracts the electrons like a friction as they try to move past it.

FIGURE 32.3 shows the instantaneous current i_R through a resistor R. The potential difference across the resistor, which we call the *resistor voltage* v_R, is given by Ohm's law:

Opposing kinetic differences

Here the sum of the +𝕆D potential differences from the resistor's protons, and the -𝕆D kinetic differences from the electrons, is zero. This would not be from the resistor, the +𝕆D×e𝕒 potential work would be less than the -𝕆D×e𝕪 kinetic work of the electrons or the current would not flow. The current reverses direction from the generator this means there is an equal -𝕆D kinetic difference in both direction which sum to zero.

Opposing voltages in an atom

In a Hydrogen atom the -𝕠d and e𝕪 Pythagorean Triangle electron has a -𝕆D kinetic torque or voltage that changes depending on its orbital. This is not equal to the +𝕆D potential torque or voltage from the electron, instead the -𝕆D×e𝕪 kinetic work and +𝕆D×e𝕒 potential work remain inverses.

Opposing voltages in the generator magnets

In the generator the -𝕆D kinetic torque from the magnets moves the electron waves around in a circle. There is also a reaction of a +𝕆D potential torque from the protons in the magnets trying to hold their electrons, also to try to capture the electrons flowing in the generator. These would be equal to each like in Kirchoff's law as the electrons in the magnets are not lost from their atoms.

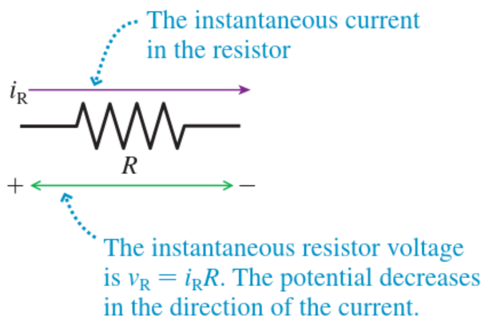
FIGURE 32.4 shows a resistor R connected across an AC emf $\mathcal{E}$. Notice that the circuit symbol for an AC generator is —⊙—. We can analyze this circuit in exactly the same way we analyzed a DC resistor circuit. Kirchhoff's loop law says that the sum of all the potential differences around a closed path is zero:

$$\sum \Delta V = \Delta V_{\text{source}} + \Delta V_{\text{res}} = \mathcal{E} - v_R = 0 \qquad (32.3)$$

Instantaneous current

In the rotational reference frame, the +𝕆D×e𝕒 potential work of the resistor reacts against the -𝕆D×e𝕪 kinetic work of the electron waves. This creates a -𝕆D kinetic gradient that decreases with e𝕪 positions along the resistor. With the instantaneous e𝕪/-𝕠d kinetic current, the instantaneous voltage comes from the phase at various -𝕠d kinetic times along the resistor.

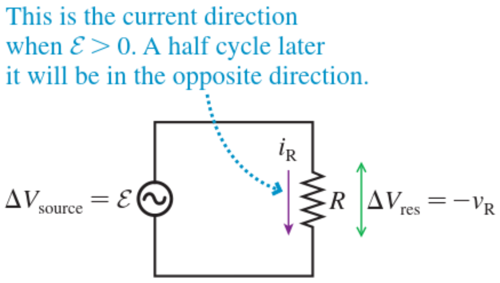
FIGURE 32.3 Instantaneous current i_R through a resistor.



A resistor and AC current

In the straight-line reference frame the current reverses direction with a 𝔼𝕐/-𝕠d kinetic impulse. Along the resistor there is a reaction with a 𝔼𝔸/+𝕠d potential impulse. That is pulling electrons directly towards the protons, to reduce their e𝕒 altitude.

FIGURE 32.4 An AC resistor circuit.



The kinetic difference varies with the phase angle

In this model the E𝕐/-𝕠d kinetic impulse decelerates through the resistor, This is because each proton reacts against any motion of the electrons in both directions. The E𝕐/-𝕠d kinetic impulse or kinetic voltage remains constant in this model, in the rotational reference frame. The direction changes, when the AC current moves as a sine wave the -𝕆D kinetic torque turns the current at the peak and trough of the sine wave.

Voltage and power as inverses

This is the maximum kinetic voltage, inversely to this the maximum power or impulse is in between the peak and trough. The voltage as a -𝕆D kinetic torque then varies with the phase angle.

The minus sign appears, just as it did in the equation for a DC circuit, because the potential *decreases* when we travel through a resistor in the direction of the current. We find from the loop law that $v_R = \mathcal{E} = \mathcal{E}_0 \cos \omega t$. This isn't surprising because the resistor is connected directly across the terminals of the emf.

The resistor voltage in an AC circuit can be written

$$v_R = V_R \cos \omega t \qquad (32.4)$$

Adding to the sine wave

This is divided by the resistor below, that would be +𝕆D×e𝕒 potential work adding to the kinetic sine wave through the resistor. This reduces the peaks and troughs of the sine wave as the electrons have a slower E𝕐/-𝕠d kinetic impulse back-and-forth.

Potential torque on the electrons

If electrons are moving slowly, they can be pulled closer to the protons and there is a stronger +𝕆D×e𝕒 potential work reacting to them. If they are moving faster, then they remain at a higher e𝕒 altitude and the +𝕆D×e𝕒 potential work is weaker. So when the electrons are faster, they have a stronger E𝕐/-𝕠d kinetic impulse, the E𝔸/+𝕠d potential impulse is vector added to them less.

An electromagnetic sphere of influence

The electrons tend to be closer to the resistor atoms without being absorbed into them. The effect is like a gravitational sphere of influence. That is proportional to the phase angle, when the current has a stronger E𝔸/+𝕠d potential impulse the phase angle means the electrons are turned less towards the protons. This is like a faster asteroid going past a planet, the slower ones are turned more towards the planet and may orbit it.

where V_R is the peak or maximum voltage. You can see that $V_R = \mathcal{E}_0$ in the single-resistor circuit of Figure 32.4. Thus the current through the resistor is

$$i_R = \frac{v_R}{R} = \frac{V_R \cos \omega t}{R} = I_R \cos \omega t \tag{32.5}$$

where $I_R = V_R/R$ is the peak current.

NOTE Ohm's law applies to both the instantaneous *and* peak currents and voltages of a resistor.

Reacting against the electron impulse

The resistor's instantaneous current and voltage are in phase, that is because they are reacting against both the E𝕐/-𝕠d kinetic impulse and -𝕆D×e𝕪 kinetic work. When the E𝕐/-𝕠d kinetic impulse is moving electrons back-and-forth, in the straight-line reference frame, then the E𝔸/+𝕠d potential impulse is reacting against this by vector adding to the E𝕐/-𝕠d kinetic impulse.

A reactive force has no active phase

The +𝕆D×e𝕒 potential work is also reacting against the -𝕆D×e𝕪 kinetic work of the electrons in the same phase because it is again a reaction. The protons in the resistor are adding to both the impulse and work of the electrons all the time and at every point. So they are both in phase. The effect is to narrow the amplitude of the kinetic AC current in the straight-line reference frame. It does not change the frequency of the AC current because it has no active forces.

Inertia has no phase

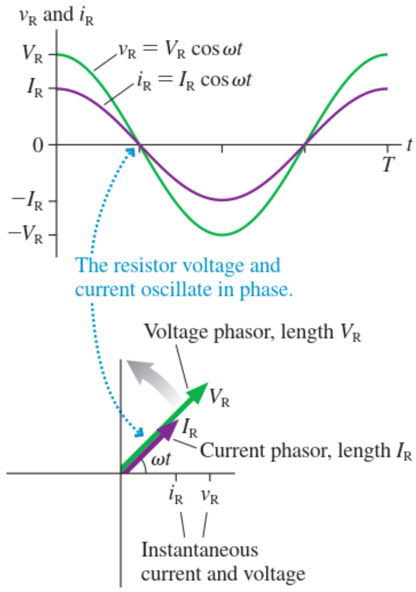
This is like inertia having no phase, a moon would move with an E𝕍/-𝕚d inertial impulse and do -𝕀D×e𝕧 inertial work in orbit around a planet. It cannot do more +𝕆D×e𝕒 potential work than have a E𝔸/+𝕠d potential impulse, these cannot separate because all they can do is be subtracted from the Eℍ/+𝕚d gravitational impulse and +𝕀D×e𝕙 gravitational work of the planet.

The resistor's instantaneous current and voltage are in phase, both oscillating as $\cos \omega t$. **FIGURE 32.5** shows the voltage and the current simultaneously on a graph and as a phasor diagram. The fact that the current phasor is shorter than the voltage phasor has no significance. Current and voltage are measured in different units, so you can't compare the length of one to the length of the other. Showing the two different quantities on a single graph—a tactic that can be misleading if you're not careful—illustrates that they oscillate in phase and that their phasors rotate together at the same angle and frequency.

Vector addition of the potential and kinetic displacement

In the diagram the amplitude of the E𝕐/-𝕠d kinetic impulse, in the straight-line reference frame is reduced. That is because the E𝔸/+𝕠d potential impulse is vector added to the E𝕐/-𝕠d kinetic impulse. The E𝔸 potential force vector is added to the E𝕐 kinetic force vector, as a reaction is this in the opposite direction like a subtraction. It is referred to as a vector addition here, that is because the +𝕠d potential magnetic field of the protons is positive and the -𝕠d kinetic magnetic field of the electrons is negative. In adding to the negative E𝕐/-𝕠d kinetic impulse, this reduces its amplitude.

FIGURE 32.5 Graph and phasor diagrams of the resistor current and voltage.



Flipped over magnets in a generator

In the generator a north and south pole are used, each uses the E𝕐/-𝕠d kinetic impulse but are flipped over in relation to each other. This is like a top, from above it might be rotating clockwise and from the bottom counterclockwise. The direction of the -𝕆D×e𝕪 kinetic work is at 90° to this -𝕆D kinetic torque of the magnet's electrons. When the spin appears to be reversed, so too is the direction the work is being done.

The wire closer to a pole

When the wire is closer to a magnetic pole, there is more -𝕆D×e𝕪 kinetic work done and the electrons move along the wire with a greater -𝕆D kinetic difference or voltage. The +𝕆D×e𝕒 potential work in the wire reacts against this from the wire's protons, but the -𝕆D×e𝕪 kinetic work is stronger. When the wire is further away from that pole there is less -𝕆D×e𝕪 kinetic work according to the inverse square law.

Creating the sine wave shape

That creates the sine wave shape, each magnetic pole moves the electrons in the opposing direction. This is because those electrons in the magnet appear flipped over to the electrons in the wire. It is like a top, the electrons are all spinning in the same direction in the magnet. From above the north pole they might appear to be clockwise, from above the south poles they would appear to be counterclockwise.

Acceleration in the straight-line reference frame

The acceleration of the electrons occurs in the straight-line reference frame along the wires, at 90° to this is the rotational reference frame of the turning armature. The magnet's do -𝕆D×e𝕪 kinetic work which is an inverse to the E𝕐/-𝕠d kinetic impulse of the power of the current.

Constructive interference between the generator poles

The electrons are moved in opposing directions by each pole, this is because they are flipped over when the wire is rotated to the other magnetic pole. There is a constructive interference between the north and south poles in the generator, this makes them more likely to be closer to each other so they are attracted.

Flipped over electrons

When the electrons are moved closer to the opposing magnetic pole they are flipped over by the rotation of the wire armature. For example, when they are next to the north pole, the electrons would line up with the same spin direction as the electrons in the magnet. When the armature is rotated, the electrons are flipped over. But now they have an opposing spin to the south pole, the -𝕆D×e𝕪 kinetic work pushes them in the opposite direction that the north pole did.

Sound waves as an alternating current

This allows both magnetic poles to do the same -𝕆D×e𝕪 kinetic work on the electrons, they move them with the same -𝕆D×e𝕪 kinetic work in opposing directions. The effect is like sound waves moving through the air with a compression and expansion. This is in the straight-line reference frame where the electrons are like particles.

Maximum kinetic torque closer to the magnet

When the electron current reverses direction, this is from the opposing magnetic pole doing -𝕆D×e𝕪 kinetic work in the opposing direction. The maximum -𝕆D kinetic torque is done on the electron current, in the rotational reference frame, when the wire is closest to either the north or south generator pole.

Inversely higher kinetic impulse

In between magnets the -𝕆D×e𝕪 kinetic work is lower, so the E𝕐/-𝕠d kinetic impulse is higher as the inverse. The -𝕆D×e𝕪 kinetic work starts the electrons moving which compresses them closer together, this is like a gas being compressed in a sound wave. The repulsion of the electrons moves them apart with destructive interference, the same happens with the gas molecules in the expansion phase of the sound wave.

Compression and expansion

When the wire approaches the opposite magnetic pole in the generator, the -𝕆D×e𝕪 kinetic work done is in the opposite direction. This slows the expansion phase of the electrons in the straight-line reference frame, then the compression phase is where the -𝕆D×e𝕪 kinetic work of that magnetic pole moves the electrons in the opposite direction. The expansion phase follows when the wire moves further away from the magnetic pole again and the cycle continues. These opposing directions are like the push and pull motion of a speaker.

A wire loop in the rotational reference frame

The wire is a loop because -𝕆D×e𝕪 kinetic work is in the rotational reference frame. That requires a loop of wire to have a change -𝕆D kinetic torque around it. This is like the kinetic torque in a loop in an electromagnet. The impulse of the electron particles is in the straight-line reference frame, at 90° to the work done in the rotational reference frame.

The rotational reference frame of the generator

In the rotational reference frame, the wires are not visible, only as points or positions. These would move closer and further away to the magnets as snapshots or movie frames, not continuously. In each snapshot there is a -𝕆D kinetic probability of where the electrons would be measured, the force is an inverse square.

The electrons destructively interfere with the poles

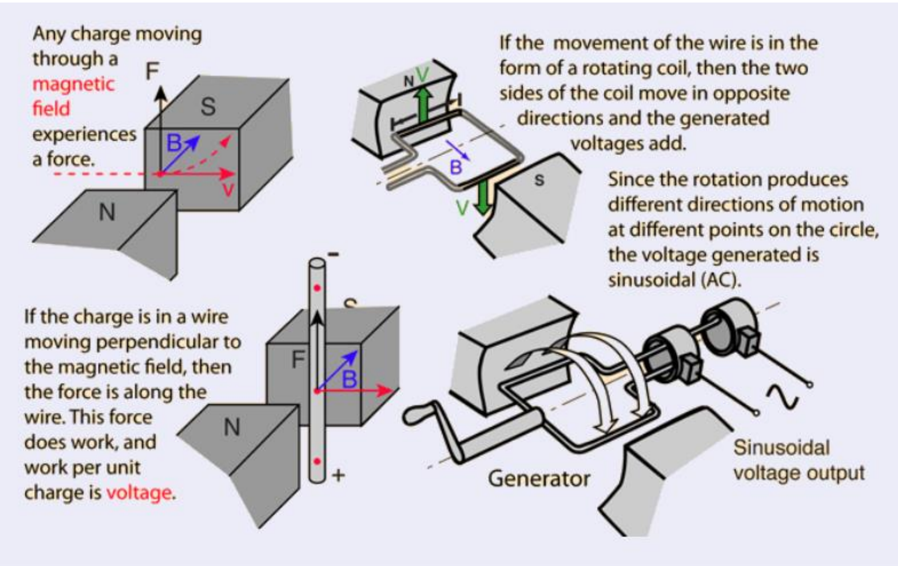
The electron waves have a destructive interference with the electrons in the magnet, they would be less likely to be measured there and so they move out one side of the wire away from the magnetic pole. When the armature rotates they are also less likely to be measured close to the other pole, so they move out the other end of the wire in the opposite direction. This is like electrons in orbitals destructively interfering, separating them more from each other.

The straight-line reference frame of the generator

In the straight-line reference frame at 90°, the wires are observed as straight lines and the rotation is not visible. The electron particles are accelerated in one direction along the wire. It is not both directions because the spins of the poles are opposed. The wire is observed to accelerate closer to a magnetic pole and get further away.

No magnetic field

There is no magnetic field in the straight-line reference frame, the motion comes from the compressed electron particles expanding up the wire as they collide. This is like gas molecules colliding and expanding in a sound wave. The electrons move like a pump or in a diode in one direction, as a rolling wheel this is in opposing directions when the spin is flipped for the other magnetic pole. The wire moving closer to the magnet compresses the electron particles and they are released in one direction.



Coulombs as momentum

Here ±q would be the -𝕠d×e𝕪/-𝕠d kinetic momentum in Coulombs for the electrons, the +𝕠d×e𝕒/+𝕠d potential momentum in Coulombs for the protons. When the electrons are compressed in a position this gives a higher E𝕐/-𝕠d kinetic impulse, when they expand there is a higher E𝔸/+𝕠d potential impulse from the positive charge.

Capacitance times voltage is a constant

Rearranging (32.6) gives the voltage times the capacitance equals the -𝕠d×e𝕪/-𝕠d kinetic momentum which is a constant, that comes from the constant Pythagorean Triangle area. The voltage here would be -𝕠d and times the capacitance would be e𝕪. When the voltage is measured it would be the -𝕆D kinetic difference, times E𝕐 as the kinetic displacement that is still equal to a constant. With +𝕠d×e𝕒=constant this also comes from the constant Pythagorean Triangle area.

Compression and expansion of the electron particles

The capacitor fills with electron particles in a compression phase, then as the AC current flows in the opposite direction this becomes positive from the E𝔸/+𝕠d potential impulse of atoms stripped of their electrons. This is in the straight-line reference frame, the back-and-forth would use cosωt. In the rotational reference frame this would be looking down either wire onto a plate, the -𝕠d kinetic magnetic field surround the wire where the -𝕆D kinetic probability is higher. At other positions there would be a +𝕆D potential probability where the electrons waves are less likely to be measured.

Positions and instants

This +𝕆D potential difference or voltage, and the -𝕆D kinetic difference or voltage, is at 90° to cosωt and so it can be sinωt. The instantaneous voltage would instead be the +𝕆D×e𝕒 potential work or -𝕆D×e𝕪 kinetic work at an infinitesimal position. It is equivalent here because at an instant the current can be regarded as being at this infinitesimal point.

32.2 Capacitor Circuits

FIGURE 32.7a shows current i_C charging a capacitor with capacitance C. The instantaneous capacitor voltage is $v_C = q/C$, where $\pm q$ is the charge on the two capacitor plates at this instant. It is useful to compare Figure 32.7a to Figure 32.3 for a resistor.

FIGURE 32.7b, where capacitance C is connected across an AC source of emf $\mathcal{E}$, is the most basic capacitor circuit. The capacitor is in parallel with the source, so the capacitor voltage equals the emf: $v_C = \mathcal{E} = \mathcal{E}_0 \cos\omega t$. It will be useful to write

$$v_C = V_C \cos\omega t \qquad (32.6)$$

Current varying over time

In (32.7) q would be the -𝕠d×e𝕪/-𝕠d kinetic momentum, Cv would be -𝕠d×e𝕪. When q is the E𝕐/-𝕠d kinetic impulse then this would vary as the amplitude E𝕐 with the turning of the phasor. The derivative is taken here with respect to time as -𝕠d, that makes this a sine wave as -𝕆D×e𝕪 kinetic work. The kinetic current e𝕪/-𝕠d would change with respect to -𝕠d to give the e𝕪/-𝕆D kinetic work.

where V_C is the peak or maximum voltage across the capacitor. You can see that $V_C = \mathcal{E}_0$ in this single-capacitor circuit.

To find the current to and from the capacitor, we first write the charge

$$q = Cv_C = CV_C \cos\omega t \qquad (32.7)$$

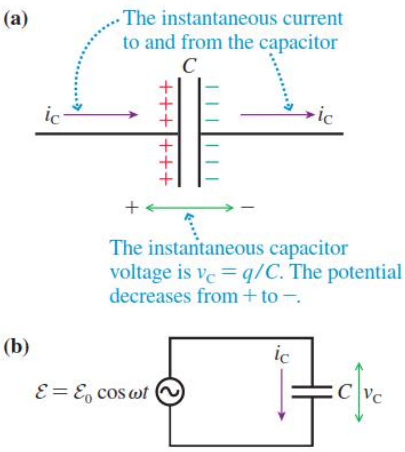
The current is the *rate* at which charge flows through the wires, $i_C = dq/dt$, thus

$$i_C = \frac{dq}{dt} = \frac{d}{dt}(CV_C \cos\omega t) = -\omega CV_C \sin\omega t \qquad (32.8)$$

The instantaneous current

The instantaneous current here would be the E𝕐/-𝕠d kinetic impulse and the E𝔸/+𝕠d potential impulse.

FIGURE 32.7 An AC capacitor circuit.



Current and voltage as a Pythagorean Triangle

In this model the current is in a straight-line reference frame, the voltage is in a rotational reference frame. Because of this they are at 90° to each other, they form a Pythagorean Triangle with the voltage as the spin side and the current as the straight side.

In contrast to a resistor, a capacitor's current and voltage are *not* in phase. In **FIGURE 32.8a**, a graph of the instantaneous voltage v_C and current i_C, you can see that the current peaks one-quarter of a period *before* the voltage peaks. The phase angle

A current changing direction

The current has its highest E𝕐/-𝕠d kinetic impulse when it is moving most in a straight-line, as with an oscillator. As it changes direction this is like the reversal of the back-and-forth motion of the straight-line reference frame. The rolling wheel of the rotational reference frame comes from the rotating generator, it slows the current to make it change direction because of the commutator switching the current's direction.

No derivative velocity at the turning position

This comes from -𝕆D×e𝕪 kinetic work, the velocity is a minimum as it changes direction. At that position there is no derivative velocity as a fraction, then it is in the rotational reference frame only. The generator turns at the same rate, the -𝕆D kinetic torque starts the E𝕐/-𝕠d kinetic impulse acceleration in the opposite direction.

of the current phasor on the phasor diagram of **FIGURE 32.8b** is $\pi/2$ rad—a quarter of a circle—larger than the phase angle of the voltage phasor.

We can summarize this finding:

The AC current of a capacitor *leads* the capacitor voltage by $\pi/2$ rad, or 90°.

The current reaches its peak value I_C at the instant the capacitor is fully discharged and $v_C = 0$. The current is zero at the instant the capacitor is fully charged.

Kinetic velocity and kinetic current

A spring, as an oscillator, also has its e𝕪/-𝕠d kinetic velocity leading the -𝕆D kinetic torque of the spring. This kinetic velocity is the same as the kinetic current, it comes from the motion here of electrons in the spring's atoms. As the spring expands the spring also uncoils, this is -𝕆D×e𝕪 kinetic work that increases until the spring slows and reaches its maximum size.

Coiling and uncoiling the spring with torque

Then this kinetic torque forces the spring to contract, that is to resume the original shape of the spring. It overshoots this at a maximum e𝕪/-𝕠d kinetic velocity again, at 90° to this the -𝕆D×e𝕪 kinetic work reaches a maximum as the spring's direction reverses again. This also happens from the -𝕆D kinetic torque as the spring is now coiled more the opposite way to when it was expanded.

Protons in the spring and the wire

That also occurs from the +𝕠d and e𝕒 Pythagorean Triangle protons in the spring, the -𝕆D×e𝕪 kinetic work has the +𝕆D×e𝕒 potential work of the protons pulling the spring's electrons back to their original configuration. The spring also moves with an e𝕒/+𝕠d potential speed, that is the inverse of the kinetic velocity.

Speed and velocity

In this model the +𝕠d and e𝕒 Pythagorean Triangle protons, and +𝕚d and e𝕙 Pythagorean Triangle gravity are referred to as speed or brevity. This is because they are in circular geometry, electrons move around in orbitals and satellites around a planet in orbits. The potential speed as e𝕒/+𝕠d has +𝕠d as the orbital period in potential time, the e𝕒 altitude would be constant with a circular orbital. This is like a rotating wheel where the radius is constant. The satellite's orbit as e𝕙/+𝕚d also has an orbital period, this time in +𝕚d gravitational time.

The potential speed is the inverse of the kinetic velocity

So when the kinetic velocity is at its maximum, when the spring is in the middle of the contraction and expansion cycle, the potential velocity is at a minimum. This is because the spring's atoms are closest to their original shape in the middle. Then the spring overshoots to a compression phase, the +𝕆D×e𝕒 potential work is pulling on the electrons to untwist back to their original shape again.

Kinetic and potential current

This also happens with current and voltage, the kinetic current is strongest in the middle of the sine wave. The potential current as e𝕒/+𝕠d is weakest there because the electrons are closest to where they would normally be. Then the current moves to a turning point where the +𝕆D×e𝕒 potential work is weakest, this is because the electrons are furthest from being influenced by the wire's proton.

A simple harmonic oscillator provides a mechanical analogy of the 90° phase difference between current and voltage. You learned in Chapter 15 that the position and velocity of a simple harmonic oscillator are

$$x = A\cos\omega t$$

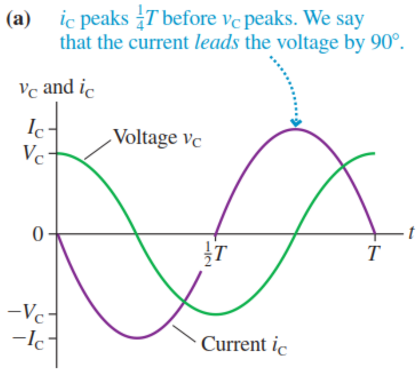
$$v = \frac{dx}{dt} = -\omega A \sin\omega t = -v_{max}\sin\omega t = v_{max}\cos\left(\omega t + \frac{\pi}{2}\right)$$

You can see that the velocity of an oscillator leads the position by 90° in the same way that the capacitor current leads the voltage.

The cosine in the straight-line reference frame

In this model the voltage would be a sine wave in the rotational reference frame. The current would not be a cosine wave here, at 90° would be up and down on the vertical axis with a back-and-forth motion.

FIGURE 32.8 Graph and phasor diagrams of the capacitor current and voltage.



Current and voltage from Pythagorean Triangle sides

The e𝕪/-𝕠d kinetic current here is e𝕪 as the straight Pythagorean Triangle side. The voltage is -𝕠d from the kinetic magnetic field, that comes from the magnets in the generator, in a derivative it is an instant of time as a fluxion. The current vector can change with a E𝕐/-𝕠d kinetic impulse, accelerating and decelerating. The voltage changes, not as a straight vector, but as a scalar that turns around a position with -𝕆D×e𝕪 kinetic work.

Voltage and spin

The voltage here represents spin, like time passing on a clock gauge moves the hands. Because of this the voltage would point out of the page, that is still at 90° to the e𝕪 current vector. Then the rotation of e𝕪 around the -𝕠d axle is in the rotational reference frame. When looking from the above or to the side, the current phasor appears to move back-and-forth with no rotation observed.

A changing torque

In the straight-line reference frame, the rotation of the e𝕪 phasor is observed on the vertical axis by convention. The end of the phasor appears to accelerate up and down with a E𝕐/-𝕠d kinetic impulse, this corresponds to the current accelerating back-and-forth in the wire. From this point of view, the rotation of the axle seems to change its torque inversely to the acceleration.

The point of view from a piston

This is like a piston connected to a flywheel at 90°, the wheel appears to have a constant angular momentum while in the straight-line reference frame the piston seems to accelerate and decelerate with a force. From the point of view of the piston the rotation seems to vary, it is connected to the end of the radius on the flywheel.

Rotation in the straight-line reference frame

When the piston is halfway through its cycle, its velocity is at a maximum but the wheel seems to be turning only in the same direction. At the end of the piston's motion it is stationary, but the wheel now seems to have a strong torque to one side that was not visible before. This is like the voltage, it increasingly appears in the straight-line reference frame at the peak and trough of the piston's motion.

The two reference frames are incompatible

This comes from the contradiction between the straight-line reference frame and the rotational reference frame. The piston moves with a E𝕐/-𝕠d kinetic impulse in a straight-line, but it cannot turn to one side. The time is observed as an instant in the straight-line reference frame because the clock gauge cannot rotate there. If it could then that would be a duration of time, that would be in work only.

The torque at 90° at the end of the piston's motion

When the piston needs to reverse, this is the end of the straight-line reference frame, the torque cannot be measured however because the piston can only move back-and-forth. This torque as the voltage is then at 90° because the wheel is now turning, not along with the motion of the piston, but at 90° as the direction reverses.

The flywheel cannot change its position

In the rotational reference frame the opposite happens, the wheel cannot move back-and-forth, it can only rotate around a single position. Its -𝕀D×e𝕧 inertial work as the flywheel turns is at a single position, it cannot be measured with changing positions like the piston's displacement, only the torque of the flywheel can have a force.

Combining reference frames

To do so would combine the straight-line reference frame and the rotational reference frame, then the Pythagorean Triangles in this model would no longer have a straight and a spin Pythagorean Triangle side. This is what happens in energy, such as with the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy. Work and impulse are combined into one equation, this creates uncertainty which means h must be used.

Schrodinger's equation

Schrodinger's equation is based on this energy equation, so it also combines the two reference frames of work and impulse. The h value in Schrodinger's equation comes from the area of the e𝕪 and -𝕘d Pythagorean Triangle photon, this is the minimum amount of change an electron can have. That is quantized in the wave function as -𝔾D×e𝕪 light work, with the observation of the e𝕐/-𝕘d light impulse it can change continuously.

An observation like the piston

When there is an observation the E𝕐/-𝕠d kinetic impulse part of Schrodinger's equation is like the piston, in the straight-line reference frame. This appears as a particle at an instant of time, the clock gauge cannot turn past this instant.

The rotational reference frame as the wave function collapsing

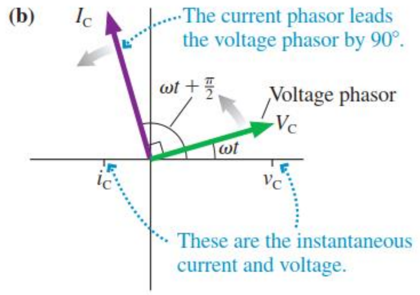
This is referred to as a wave function, from -𝕆D×e𝕪 kinetic work, collapsing from many interfering waves of -𝕆D kinetic probabilities into a particle at this instant. Then as the position of the particle becomes less certain, this is like the rotating flywheel where the position of the work being done is in the straight-line reference frame.

A displacement cannot be measured

The displacement of the piston cannot be measured in the rotational reference frame, and so the wave function cannot be used to observe the particle. It spreads apart with a normal curve shape from the -𝕆D kinetic probabilities.

An instant cannot be used with probabilities

The instant the particle is observed in cannot be used to measure the probabilities of the wave function, in the rotational reference frame. They come from the clock gauge turning with a duration of time, that can only happen in the rotational reference frame.

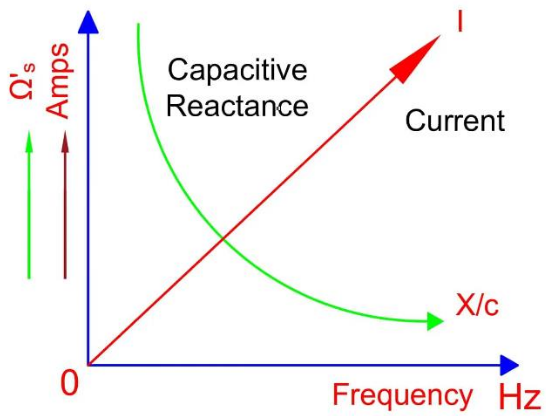


The capacitive reactance from impulse

Here the capacitive reactance comes from the E𝕐/-𝕠d kinetic impulse, this is because the capacitor is in the straight-line reference frame. As the frequency of the alternating current increases, so does the -𝕆D×e𝕪 kinetic work and the E𝕐/-𝕠d kinetic impulse decreases inversely to this. Conversely when the -𝕆D×e𝕪 kinetic work decreases, the capacitor is less likely to have a current move in between the plates so the reactance goes up.

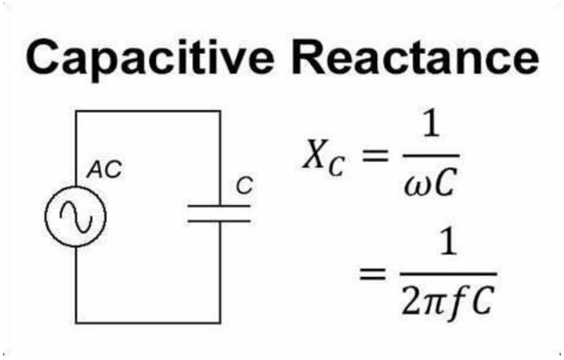
The current moves as a field

The current must be transmitted by a field, in this model there is only a magnetic field not an electric field. With a larger 1/-𝕠d frequency, there is more -𝕆D×e𝕪 kinetic work done by the faster generator in the rotational reference frame. Conversely there is less of a E𝕐/-𝕠d kinetic impulse in the straight-line reference frame, the current jumps over the capacitor gap as -𝕠d×e𝕪 instead of e𝕪/-𝕠d.



Constant Pythagorean Triangle area

The equation below can be rearranged as $X_c \times \omega = 1/c$ as a constant. So as the reactance X increases the frequency ω decreases inversely to it, that comes from the constant area of the -𝕠d and e𝕪 Pythagorean Triangle.



The capacitive reactance

In (32.11) the current is -𝕠d/e𝕪 so the voltage V_C is -𝕠d and the reactance is 1/e𝕪. The voltage would be -𝕠d as the kinetic magnetic field, this equals the current as -𝕠d/e𝕪 times e𝕪 as the reactance. These are 90° out of phase because the two sides of the -𝕠d and e𝕪 Pythagorean Triangle are 90° to each other.

Capacitive Reactance

We can use Equation 32.9 to see that the peak current to and from a capacitor is $I_C = \omega C V_C$. This relationship between the peak voltage and peak current looks much like Ohm's law for a resistor if we define the **capacitive reactance** X_C to be

$$X_C \equiv \frac{1}{\omega C} \qquad (32.10)$$

With this definition,

$$I_C = \frac{V_C}{X_C} \quad \text{or} \quad V_C = I_C X_C \qquad (32.11)$$

The units of reactance, like those of resistance, are ohms.

NOTE Reactance relates the *peak* voltage V_C and current I_C. But reactance differs from resistance in that it does *not* relate the instantaneous capacitor voltage and current because they are out of phase. That is, $v_C \neq i_C X_C$.

The reactance and the photoelectric effect

The reactance is related to the photoelectric effect in this model. The electrons move with a greater -𝕆D kinetic torque in higher orbitals. To move an electron over the ionization barrier gap, -𝔾D×e𝕪 light work is done by photons. When they have a higher -𝕘d light frequency they do more -𝔾D×e𝕪 light work and move the electrons more easily. A higher frequency also moves electrons over the gap in a capacitor.

Quantum tunneling

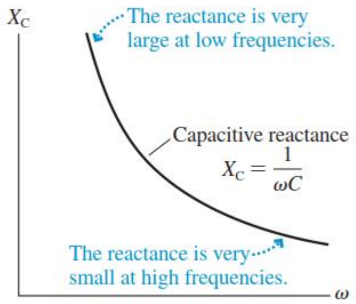
In quantum tunneling a higher frequency means electrons are more likely to be measured on the other side of a barrier. In this model these are all the same -𝕆D×e𝕪 kinetic work.

Resistance and frequency

In this model the resistance comes from the E𝔸/+𝕠d potential impulse and +𝕆D×e𝕒 potential work. These do not change with the AC current frequency as that is from the -𝕠d and e𝕪 Pythagorean Triangle electrons. When the E𝕐/-𝕠d kinetic impulse is larger, electron particles can move faster around the +𝕠d and e𝕒 Pythagorean Triangle protons. However when they reverse direction they encounter the same protons again, overall this makes little difference to the resistance.

A resistor's resistance R is independent of the emf frequency. In contrast, as FIGURE 32.9 shows, a capacitor's reactance X_C depends inversely on the frequency. The reactance becomes very large at low frequencies (i.e., the capacitor is a large impediment to current). This makes sense because $\omega = 0$ would be a nonoscillating DC circuit, and we know that a steady DC current cannot pass through a capacitor. The reactance decreases as the frequency increases until, at very high frequencies, $X_C \approx 0$ and the capacitor begins to act like an ideal wire. This result has important consequences for how capacitors are used in many circuits.

FIGURE 32.9 The capacitive reactance as a function of frequency.



RC circuits with an alternating current

In this model a time constant comes from the straight-line reference frame, the -𝕠d kinetic time is in the E𝕐/-𝕠d kinetic impulse. The capacitor is in the straight-line reference frame, this is reacted against by the E𝔸/+𝕠d potential impulse in the resistor.

32.3 RC Filter Circuits

You learned in Chapter 28 that a resistance R causes a capacitor to be charged or discharged with time constant $\tau = RC$. We called this an RC circuit. Now that we've looked at resistors and capacitors individually, let's explore what happens if an RC circuit is driven continuously by an alternating current source.

An RC circuit at lower frequencies

Here the -𝕆D×e𝕪 kinetic work drives the -𝕆D kinetic probabilities of electrons tunneling across the capacitor gap. When the -𝕆D×e𝕪 kinetic work is low the generator turns more slowly, this corresponds to a lower -𝕆D kinetic probability of the electron waves being measured across the capacitor gap. Because -𝕠d is smaller, the -𝕠d kinetic voltage through the resistor is also lower.

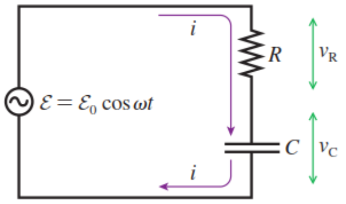
FIGURE 32.10 shows a circuit in which a resistor R and capacitor C are in series with an emf $\mathcal{E}$ oscillating at angular frequency ω. Before launching into a formal analysis, let's try to understand qualitatively how this circuit will respond as the frequency is varied. If the frequency is very low, the capacitive reactance will be very large, and thus the peak current I_C will be very small. The peak current through the resistor is the same as the peak current to and from the capacitor (just as in DC circuits, conservation of charge requires $I_R = I_C$); hence we expect the resistor's peak voltage $V_R = I_R R$ to be very small at very low frequencies.

An RC circuit at higher frequencies

If the frequency is higher then more -𝕆D×e𝕪 kinetic work is done, more -𝕆D kinetic voltage gets across the plates. This forces more electrons through the positions in the resistor with -𝕆D×e𝕪 kinetic work, that is against the +𝕆D×e𝕒 potential work being done by the resistor protons.

On the other hand, suppose the frequency is very high. Then the capacitive reactance approaches zero and the peak current, determined by the resistance alone, will be $I_R = \mathcal{E}_0/R$. The resistor's peak voltage $V_R = IR$ will approach the peak source voltage $\mathcal{E}_0$ at very high frequencies.

FIGURE 32.10 An *RC* circuit driven by an AC source.



Kinetic and potential voltage

In this model the resistor does +𝕆D×e𝕒 potential work on electrons near it. When the frequency is higher so is the -𝕆D×e𝕪 kinetic work, but this reduces the kinetic velocity of the electrons. That allows them to be attracted more around the resistor's protons, increasing their resistance. The capacitor voltage decreases because it cannot build up on one side of the capacitor.

This reasoning leads us to expect that V_R will *increase* steadily from 0 to $\mathcal{E}_0$ as ω is increased from 0 to very high frequencies. Kirchhoff's loop law has to be obeyed, so the capacitor voltage V_C will *decrease* from $\mathcal{E}_0$ to 0 during the same change of frequency. A quantitative analysis will show us how this behavior can be used as a *filter*.

The goal of a quantitative analysis is to determine the peak current I and the two peak voltages V_R and V_C as functions of the emf amplitude $\mathcal{E}_0$ and frequency ω. Our analytic procedure is based on the fact that the instantaneous current i is the same for two circuit elements in series.

Phasors as the straight Pythagorean Triangle sides

When phasors are used for analysis, these would only be straight Pythagorean Triangle sides here as the capacitance C, the reactance, X, the current as e𝕪. Voltage would not be a phasor as it would come from the spin Pythagorean Triangle side, the voltage would turn the phasors around in a rotational reference frame. In the diagram below, the voltage phasor would point out of the page at 90° like the generator axle.

Potential work and impulse

The resistor has its e𝕒 altitude, or straight Pythagorean Triangle side pointing directly at the protons in circular geometry. This E𝔸/+𝕠d potential impulse reacts against the E𝕐/-𝕠d kinetic impulse of the current, its +𝕆D×e𝕒 potential work also reacts against the -𝕆D×e𝕪 kinetic work of the voltage. That means the E𝔸/+𝕠d potential impulse reduces the E𝕐/-𝕠d kinetic impulse or amplitude of the current. +𝕆D×e𝕒 potential work reduces the -𝕆D×e𝕪 kinetic work of the voltage.

Current and voltage in different reference frames

The capacitor current leads the voltage by 90°, this is because they are in different reference frames. When the current is high as the E𝕐/-𝕠d kinetic impulse, the -𝕆D×e𝕪 kinetic work being done must be low. This follows from the inverse relationship of -𝕆D×e𝕪 kinetic work and the E𝕐/-𝕠d kinetic impulse.

Phase and Pythagorean Triangles

Being 90° out of phase, the voltage is in the rotational reference frame and the current is in the straight-line reference frame. They must then be 90° out of phase as the right angle of the -𝕠d and e𝕪 Pythagorean Triangle electron.

Ocean waves and torque

The capacitor gap is analogous to a sea wall that regular ocean waves can partially cross They climb and go over the barrier when the -𝕀D×e𝕧 inertial work of the wave is at its strongest. The -𝕀D inertial torque enables the wave to curl over the sea wall.

Quantum tunneling and torque

This is analogous to quantum tunneling, the -𝕆D kinetic torque of electron waves can move through a barrier as a -𝕆D kinetic probability. The same happens across a capacitor gap, the higher frequency increases the -𝕆D kinetic torque and probability. The electron waves can move across the gap like the ocean waves can move over the sea wall.

The photelectric effect and torque

This is also like the photoelectric effect, higher -𝔾D×e𝕪 light work can be done with a faster frequency. As a -𝔾D light torque the electrons receive a higher -𝔾D light probability to be able to tunnel across the ionization boundary.

Impulse and ocean waves

With a lower wave frequency, there is less -𝕀D inertial torque in the waves and they are further apart. Then the wave hits the sea wall with an E𝕍/-𝕚d inertial impulse and reflects more off it. This is where the ocean wave moves back-and-forth more rather than curling with the -𝕀D inertial torque.

Quantum tunneling and impulse

In quantum tunneling a lower frequency means the electron is more like a particle, with a higher E𝕐/-𝕠d kinetic impulse. It would then reflect off a barrier like the ocean waves do. In the photoelectric effect, the intensity of low frequency photons with their e𝕐/-𝕘d light impulse can be increased. This is like directing more low torque ocean waves at the sea wall, no more can get over it. With quantum tunneling the electron particles reflect off the barrier because they still have a low -𝕆D kinetic probability of getting through it, even though there are more electrons.

High torque waves at the sand bar

When the higher -𝕀D inertial torque waves get over the sea wall, they can come to a sandbar like the resistor. Because the waves do more -𝕀D×e𝕧 inertial work they have less of an E𝕍/-𝕚d inertial impulse, the waves tend to break more on the sandbar rather than traveling across it with an E𝕍/-𝕚d inertial impulse.

Tsunamis and impulse

Conversely if the waves had a greater E𝕍/-𝕚d inertial impulse, they could move more across the sandbar. An example of this is a tsunami, it has a higher impulse force but a low height and can move a longer way inland. Some parts of the world have higher -𝕀D inertial torque in their The higher E𝕍/-𝕚d inertial impulse waves get reflected more off the sea wall, so they don't get to the resistor like sandbar.

Current and voltage forming a Pythagorean Triangle

The ratio of the -𝕀D×e𝕧 inertial work and E𝕍/-𝕚d inertial impulse would form the -𝕚d and e𝕧 Pythagorean Triangle with an angle θ, the e𝕧 length in the e𝕧/-𝕚d ocean current is analogous to e𝕪 in the kinetic current. The -𝕚d inertial mass of the ocean wave is analogous to the -𝕠d kinetic voltage, both act as a pushing force with work.

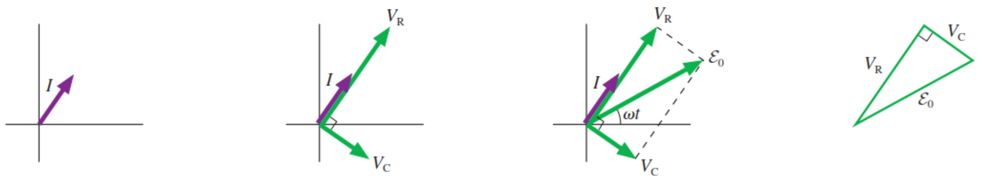
A stronger resistor and sandbar

The inverse of this is the +𝕠d and e𝕒 Pythagorean Triangle in the resistor, this is like the size of the sandbank in resisting the ocean waves. A longer sandbar would react against the ocean wave with an E𝕍/-𝕚d inertial impulse, this is like a longer resistor in Ohm's law.

Scattering ocean waves like particles

If the sand was more granular it can have more friction, scattering the ocean waves as if they were particles, like a stronger resistor. This comes from the +𝕚d and e𝕙 Pythagorean Triangle and gravity, when the sand is heavier the ocean waves cannot push it aside as easily. This is seen in some sandbars that can be moved, that is analogous to weaker resistors. The +𝕚d and e𝕙 Pythagorean Triangle as gravity is analogous to the +𝕠d and e𝕒 Pythagorean Triangle with protons, both are in circular geometry. Both electrons and inertia are in hyperbolic geometry.

Using phasors to analyze an *RC* circuit



Begin by drawing a current phasor of length I. This is the starting point because the series circuit elements have the same current i. The angle at which the phasor is drawn is not relevant.

The current and voltage of a resistor are in phase, so draw a resistor voltage phasor of length V_R parallel to the current phasor I. The capacitor current leads the capacitor voltage by 90°, so draw a capacitor voltage phasor of length V_C that is 90° behind [i.e., clockwise (cw) from] the current phasor.

The series resistor and capacitor are in parallel with the emf, so their *instantaneous* voltages satisfy $v_R + v_C = \mathcal{E}$. This is a *vector* addition of phasors, so draw the emf phasor as the vector sum of the two voltage phasors. The emf is $\mathcal{E} = \mathcal{E}_0 \cos \omega t$, hence the emf phasor is at angle ωt.

The length of the emf phasor, $\mathcal{E}_0$, is the hypotenuse of a right triangle formed by the resistor and capacitor phasors. Thus $\mathcal{E}_0^2 = V_R^2 + V_C^2$.

Squaring two voltages

Here the squared voltage of the resistor is +𝕆D, that is added to the squared voltage of the capacitor as -𝕆D, then the hypotenuse squared is $\mathcal{E}^2$. Taking R as 1/e𝕒 then e𝕒/+𝕠d × e𝕒 is 1/+𝕠d and squared is the voltage 1/+𝕆D. With e𝕪/-𝕠d as the kinetic current, multiplying this by 1/e𝕪 as the capacitive reactance gives 1/-𝕆D. Squaring this is 1/-𝕆D. This can be inverted because the current e𝕪/-𝕠d is the same as -𝕠d/e𝕪 like meters/second is the same as seconds/meter. That gives -𝕆D +𝕆D as the hypotenuse squared.

Varying with the frequency

The capacitive reactance as e𝕪 varies inversely with the -𝕠d kinetic frequency, that gives the constant area of the -𝕠d and e𝕪 Pythagorean Triangle. The resistive reactance or resistivity reacts against the +𝕠d/e𝕒 potential current in the straight-line reference frame. The resistance here does not vary with the frequency, the capacitive reactance does because the current needs to move across the capacitor gap. The peak voltage values are where the sine wave turns with a maximum torque.

The relationship $\mathcal{E}_0^2 = V_R^2 + V_C^2$ is based on the peak values, not the instantaneous values, because the peak values are the lengths of the sides of the right triangle. The peak voltages are related to the peak current I via $V_R = IR$ and $V_C = IX_C$, thus

$$\begin{aligned}\mathcal{E}_0^2 &= V_R^2 + V_C^2 = (IR)^2 + (IX_C)^2 = (R^2 + X_C^2)I^2 \\ &= (R^2 + 1/\omega^2 C^2)I^2\end{aligned} \tag{32.12}$$

Taking the resistance and reactance separately

If the capacitive reactance squared as E𝕐 is large, taken separately, then the frequency is lower. $\mathcal{E}_0$ as the spin Pythagorean Triangle side would be proportional to the frequency as time. That would be -𝕠d/√E𝕐 or -𝕠d/e𝕪. If the resistor reactance is larger as E𝔸, then the square root is taken, this gives +𝕠d/e𝕒 as the potential current. When -𝕆D +𝕆D are in the denominator, and the square root is taken, this is the hypotenuse which is referred to here as the emf.

Making a Pythagorean Triangle

In this model the hypotenuse would not be a spin Pythagorean Triangle side, in an atom +𝕠d and -𝕠d would be orthogonal to each other so that implies a hypotenuse value. In the circuit +𝕠d and -𝕠d as used as from the resistor reactance and capacitor reactance, these are not inverses and so the Pythagorean Triangle formed need not have a constant area.

Consequently, the peak current in the RC circuit is

$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + 1/\omega^2 C^2}} \tag{32.13}$$

Knowing I gives us the two peak voltages:

$$\begin{aligned}V_R &= IR = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + 1/\omega^2 C^2}} \\ V_C &= IX_C = \frac{\mathcal{E}_0 X_C}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0/\omega C}{\sqrt{R^2 + 1/\omega^2 C^2}}\end{aligned} \tag{32.14}$$

The crossover frequency

Here the voltage changes for different reasons. With the ocean wave analogy, the increased -𝕀D inertial torque of the waves allows them to break over a sea wall like crossing a capacitor gap. The lower frequency allows the E𝕐/-𝕠d kinetic impulse of the current to cross a resistor more easily, this is like the E𝕍/-𝕚d inertial impulse of the ocean waves crossing a sand bar. The two together crossover at a frequency, this can be used as a filter.

Frequency Dependence

Our goal was to see how the peak current and voltages vary as functions of the frequency ω. Equations 32.13 and 32.14 are rather complex and best interpreted by looking at graphs. FIGURE 32.11 is a graph of V_R and V_C versus ω.

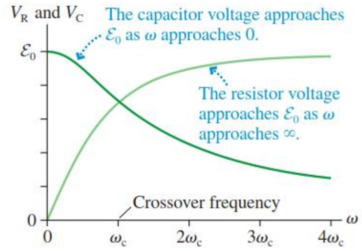
You can see that our qualitative predictions have been borne out. That is, V_R increases from 0 to $\mathcal{E}_0$ as ω is increased, while V_C decreases from $\mathcal{E}_0$ to 0. The explanation for this behavior is that the capacitive reactance X_C decreases as ω increases. For low frequencies, where $X_C \gg R$, the circuit is primarily capacitive. For high frequencies, where $X_C \ll R$, the circuit is primarily resistive.

The frequency at which $V_R = V_C$ is called the **crossover frequency** ω_c. The *crossover* frequency is easily found by setting the two expressions in Equations 32.14 equal to each other. The denominators are the same and cancel, as does $\mathcal{E}_0$, leading to

$$\omega_c = \frac{1}{RC} \qquad (32.15)$$

In practice, $f_c = \omega_c/2\pi$ is also called the crossover frequency.

FIGURE 32.11 Graph of the resistor and capacitor peak voltages as functions of the emf angular frequency ω.



Peak voltage and torque

In this model the peak voltage is where the sine wave turns, this is the strongest -𝕆D kinetic torque. There is a maximum +𝕆D potential torque in the resistor as a reaction, so the two squares are added. The instantaneous values are -𝕠d for electrons in the capacitor and +𝕠d for protons in the resistor.

We'll leave it as a homework problem to show that $V_R = V_C = \mathcal{E}_0/\sqrt{2}$ when $\omega = \omega_c$. This may seem surprising. After all, shouldn't V_R and V_C add up to $\mathcal{E}_0$?

No! V_R and V_C are the *peak values* of oscillating voltages, not the instantaneous values. The instantaneous values do, indeed, satisfy $v_R + v_C = \mathcal{E}$ at all instants of time. But the resistor and capacitor voltages are out of phase with each other, as the phasor diagram shows, so the two circuit elements don't reach their peak values at the same time. The peak values are related by $\mathcal{E}_0^2 = V_R^2 + V_C^2$, and you can see that $V_R = V_C = \mathcal{E}_0/\sqrt{2}$ satisfies this equation.

NOTE It's very important in AC circuit analysis to make a clear distinction between instantaneous values and peak values of voltages and currents. Relationships that are true for one set of values may not be true for the other.

Low pass filter

In the low pass filter, a higher frequency moves -𝕆D×e𝕪 kinetic work through the resistor back-and-forth more quickly. This increases the resistance as +𝕆D×e𝕒 potential work because the electron moves through the resistor more often. The -𝕆D×e𝕪 kinetic work across the capacitor allows electrons waves to cross over the plates. When the frequency is low, the -𝕆D×e𝕪 kinetic work moves through the resistor more like a DC current. There is less +𝕆D×e𝕒 potential work done as a reaction so the resistance is lower.

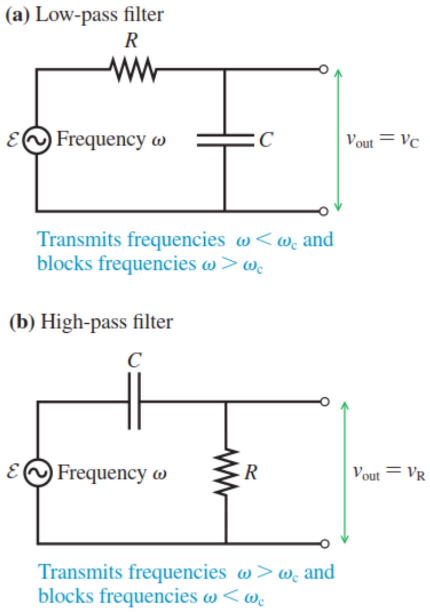
Filters

FIGURE 32.12a is the circuit we've just analyzed; the only difference is that the capacitor voltage v_C is now identified as the *output voltage* v_{out}. This is a voltage you might measure or, perhaps, send to an amplifier for use elsewhere in an electronic instrument. You can see from the capacitor voltage graph in Figure 32.11 that the peak output voltage is $V_{out} \approx \mathcal{E}_0$ if $\omega \ll \omega_c$, but $V_{out} \approx 0$ if $\omega \gg \omega_c$. In other words,

- If the frequency of an input signal is well below the crossover frequency, the input signal is transmitted with little loss to the output.
- If the frequency of an input signal is well above the crossover frequency, the input signal is strongly attenuated and the output is very nearly zero.

This circuit is called a **low-pass filter.**

FIGURE 32.12 Low-pass and high-pass filter circuits.



High pass filter

When the frequency is high, so is the -𝕆D×e𝕪 kinetic work. The electron waves move across the plates more easily. When the frequency is low, the electrons move more in the straight-line reference frame as particles. Because of this they cannot jump across the capacitor gap.

The circuit of **FIGURE 32.12b**, which instead uses the resistor voltage v_R for the output v_{out}, is a **high-pass filter.** The output is $V_{out} \approx 0$ if $\omega \ll \omega_c$, but $V_{out} \approx \mathcal{E}_0$ if $\omega \gg \omega_c$. That is, an input signal whose frequency is well above the crossover frequency is transmitted without loss to the output.

Filter circuits are widely used in electronics. For example, a high-pass filter designed to have $f_c = 100$ Hz would pass the audio frequencies associated with speech ($f > 200$ Hz) while blocking 60 Hz “noise” that can be picked up from power lines. Similarly, the high-frequency hiss from old vinyl records can be attenuated with a low-pass filter, allowing the lower-frequency audio signal to pass.

A simple *RC* filter suffers from the fact that the crossover region where $V_R \approx V_C$ is fairly broad. More sophisticated filters have a sharper transition from off ($V_{out} \approx 0$) to on ($V_{out} \approx \mathcal{E}_0$), but they’re based on the same principles as the *RC* filter analyzed here.

Inductor circuits

Here the voltage decreases as the frequency 1/dt or 1/-𝕠d increases. This is because the E𝕐/-𝕠d kinetic impulse increases in the straight-line reference frame, the electron particles encounter more resistance in moving through the inductor loops. When the frequency is higher then in the straight-line reference frame the electron waves more with a -𝕆D kinetic torque through the loops more easily.

32.4 Inductor Circuits

FIGURE 32.13a shows the instantaneous current i_L through an inductor. If the current is changing, the instantaneous inductor voltage is

$$v_L = L\frac{di_L}{dt} \tag{32.16}$$

The potential reacts against changes

+𝕆D×e𝕒 potential work reacts against a change in -𝕆D×e𝕪 kinetic work, so if -𝕆D×e𝕪 kinetic work is increasing then +𝕆D×e𝕒 potential work reacts against this. If -𝕆D×e𝕪 kinetic work is decreasing, then +𝕆D×e𝕒 potential work also reacts against it. This is like Lenz's law.

Sines as voltage

Here the cosine is used of ωt, that gives the frequency and the phase. In this model the -𝕠d and e𝕪 Pythagorean Triangle itself rotates, the frequency comes from the size of the -𝕠d Pythagorean Triangle side. The phase comes from where the e𝕪 spoke or Pythagorean Triangle side is during this rotation. In this model the sine would be used with voltage as this comes from the spin Pythagorean Triangle side.

You learned in Chapter 30 that the potential decreases in the direction of the current if the current is increasing ($di_L/dt > 0$) and increases if the current is decreasing ($di_L/dt < 0$).

FIGURE 32.13b is the simplest inductor circuit. The inductor L is connected across the AC source, so the inductor voltage equals the emf: $v_L = \mathcal{E} = \mathcal{E}_0 \cos \omega t$. We can write

$$v_L = V_L \cos \omega t \qquad (32.17)$$

The inductor current

In this model the current would be e𝕪/-𝕠d, taking (32.18) as dt gives 1/-𝕠d.

where V_L is the peak or maximum voltage across the inductor. You can see that $V_L = \mathcal{E}_0$ in this single-inductor circuit.

We can find the inductor current i_L by integrating Equation 32.17. First, we use Equation 32.17 to write Equation 32.16 as

$$di_L = \frac{v_L}{L} dt = \frac{V_L}{L} \cos \omega t \, dt \qquad (32.18)$$

Cosine as the current

Here integrating gives sinωt, this is like changing 90° to the rotational reference frame. In this model the current would be the cosine in the straight-line reference frame, that is at 90° because the sine and cosine are at 90° to each other in the -𝕠d and e𝕪 Pythagorean Triangle. Here ωt has π/2 or 90° subtracted from it, that turns the sine into a cosine. It also changes the rotational reference frame into a straight-line reference frame.

Integrating gives

$$\begin{aligned} i_L &= \frac{V_L}{L} \int \cos \omega t \, dt = \frac{V_L}{\omega L} \sin \omega t = \frac{V_L}{\omega L} \cos\left(\omega t - \frac{\pi}{2}\right) \\ &= I_L \cos\left(\omega t - \frac{\pi}{2}\right) \end{aligned} \qquad (32.19)$$

where $I_L = V_L/\omega L$ is the peak or maximum inductor current.

Inductive reactance

The inductive reactance is in the rotational reference frame, at 90° to the capacitor reactance in the straight-line reference frame. In this rotational reference frame an increase in frequency also increases the -𝕆D kinetic torque, the reactance then increases with the frequency. More of the -𝕆D×e𝕪 kinetic work being done in the AC current is canceled out with destructive interference. This reduces the amount of current through the inductor in the straight-line reference frame.

NOTE Mathematically, Equation 32.19 could have an integration constant i_0. An integration constant would represent a constant DC current through the inductor, but there is no DC source of potential in an AC circuit. Hence, on physical grounds, we set $i_0 = 0$ for an AC circuit.

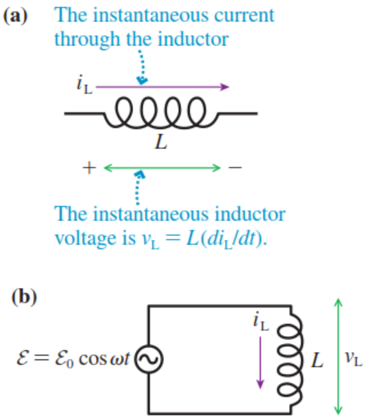
We define the **inductive reactance,** analogous to the capacitive reactance, to be

$$X_L \equiv \omega L \tag{32.20}$$

Instantaneous current

The instantaneous current is the inverse of the frequency here as 1/-𝕠d, with e𝕪/-𝕠d as -𝕠d increases then e𝕪 decreases inversely because of the constant Pythagorean Triangle areas. This instantaneous current as e𝕪/-𝕠d is divided by -𝕠d to leave e𝕪. That is referred to as the instantaneous voltage, here this would be the amount of current moving in a -𝕠d instant.

FIGURE 32.13 Using an inductor in an AC circuit.



Impulse and work as inverses

Here the peak current is e𝕪 in the straight-line reference frame, that is where the acceleration of the E𝕐/-𝕠d kinetic impulse is largest in the middle of the oscillation. The peak voltage is 1/-𝕠d, that is where the -𝕆D×e𝕪 kinetic work has the maximum -𝕆D kinetic torque that turns around the impulse to the opposite direction.

Changing frequencies

These two are inverses of each other, if not then a larger E𝕐/-𝕠d kinetic impulse would keep increasing its displacement which would be slowing the frequency. That would be like a guitar string that was continuously reduced in tension.

Inductive reactance increases with frequency

The reactance goes up here as the frequency increases, so a large reactance in the denominator means there is less current. This is because the reversal of -𝕆D×e𝕪 kinetic work in the inductor has more destructive interference in the rotational reference frame.

Then the peak current $I_L = V_L/\omega L$ and the peak voltage are related by

$$I_L = \frac{V_L}{X_L} \quad \text{or} \quad V_L = I_L X_L \qquad (32.21)$$

Inductive reactance

The inductive reactance is two directions of -𝕆D×e𝕪 kinetic work, these interfere with each other destructively. When the direction of the AC current changes more quickly, the -𝕆D×e𝕪 kinetic work is interfering inside the inductor more. This cancels out the -𝕆D×e𝕪 kinetic work and so it is less likely for the electron waves to be measured there. That makes the inductor more resistant to the current in the straight-line reference frame.

Reactance and inductive resistance

This is similar to in a resistor, the changing direction of the -𝕆D×e𝕪 kinetic work means the +𝕆D×e𝕒 potential work of the resistor protons reacts against it more.

Zeno's arrow

In the rotational reference frame a spinning iota does not move, this is like Zeno's arrow that can spin but remains stationary. The straight-line reference frame is only motion, so the current moves more as the torque decreases. Zeno's arrow would move the more its forces are in a straight-line rather than in turning the arrow.

FIGURE 32.14 shows that the inductive reactance increases as the frequency increases. This makes sense. Faraday's law tells us that the induced voltage across a coil increases as the time rate of change of $\vec{B}$ increases, and $\vec{B}$ is directly proportional to the inductor current. For a given peak current I_L, $\vec{B}$ changes more rapidly at higher frequencies than at lower frequencies, and thus V_L is larger at higher frequencies than at lower frequencies.

Current lags the voltage

Here the voltage comes first in the inductor from -𝕆D×e𝕪 kinetic work, the current is in the straight-line reference frame and so is at 90° to the -𝕆D×e𝕪 kinetic work. This comes after the work as the rotational reference frame is driving the voltage through the loops of the inductor. This is the opposite to a capacitor which is in the straight-line reference frame, then the impulse comes first and the voltage comes after it. The voltage as -𝕠d and current as e𝕪 are at 90° to each other in the -𝕠d and e𝕪 Pythagorean Triangle.

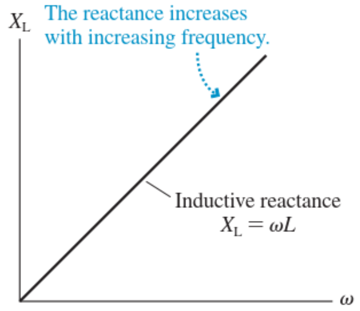
FIGURE 32.15a is a graph of the inductor voltage and current. You can see that the current peaks one-quarter of a period *after* the voltage peaks. The angle of the current phasor on the phasor diagram of **FIGURE 32.15b** is $\pi/2$ rad less than the angle of the voltage phasor. We can summarize this finding:

The AC current through an inductor *lags* the inductor voltage by $\pi/2$ rad, or 90°.

Reactance goes up linearly

Here the reactance goes up linearly as the frequency increases. It is regarded an an instantaneous voltage so it is not a square.

FIGURE 32.14 The inductive reactance as a function of frequency.



The voltage drives the current

Here the current is in the straight-line reference frame, the voltage comes from -𝕆D×e𝕪 kinetic work so it is the driving force.

The spin axle of the voltage

The voltage here would be a sine wave, the current would be at 90° to this but moving up and down as an amplitude. The sine wave is driven in the rotational reference frame by a spin axle pointing out of the page as the electron wave moves.

In the capacitor the current comes first

With the capacitor the amplitude would change first, because the current fills the capacitor along a straight wire in the straight-line reference frame. As this current and E𝕐/-𝕠d kinetic impulse increases, then the inverse voltage decreases. The change in the current occurs first so the time-based voltage occurs later as it uses time here.

The torque increases as the current decreases

With the inductor the voltage is changing with -𝕆D×e𝕪 kinetic work so this drives the force here. The stronger -𝕆D kinetic torque points out of the page in the rotational reference frame, this increases and decreases in torque. As it increases to maintain a constant Pythagorean Triangle area the e𝕪 current decreases.

After in position not time

The current does not come after the voltage here in the rotational reference frame with respect to time, this refers to changes in position. It comes after when observed from above the coil in the straight-line reference frame, then there are not visible curves in the coil. The electron particles would move by collisions up and down the zigzag of the coil. Above the coil there are no visible loops, only straight lines.

Electrons accelerating then decelerating

From this straight-line reference frame the electrons would appear to accelerate, then reach the end of the zigzag that they bounce off, then back in the other direction. First there would be the E𝕐/-𝕠d kinetic impulse in the straight part of the coil, then at the turn there would be the -𝕆D kinetic torque not visible in the straight-line reference frame.

Electron waves in a spiral

In the rotational reference frame looking at 90° to this, the coil is seen by looking along the wire. It appears as a spiral. The electron waves go around the spiral with a -𝕆D kinetic torque that is even, so there is a gradient where the torque can decrease through the inductor. This decrease occurs after the highest voltage as the -𝕆D kinetic difference or torque. The after here then refers to a distance, in the straight-line reference frame from above, the after refers to time in the zigzag of the coil.

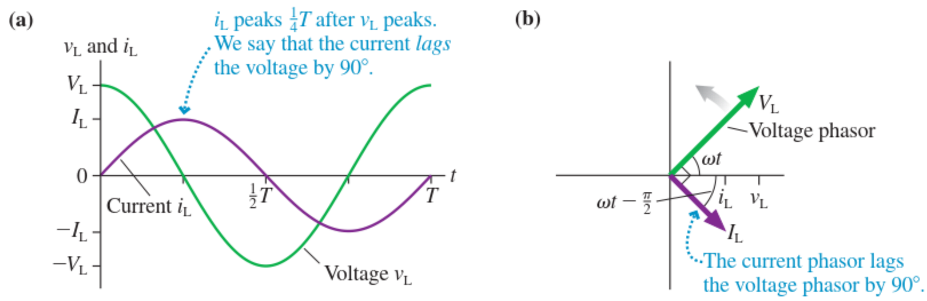
Electrons spin in circles

In this spiral a higher frequency means the -𝕆D×e𝕪 kinetic work increases after smaller changes in position. It becomes more like an atom where the electrons spin in circles, rather than moving in straight-lines with a current. This is not just in one loop, each loop is interfering with the others.

Electrons colliding in the inductor

In the straight-line reference frame the electron particles are moving through zig zags rather than loops. If the frequency increases, then the electrons reverse direction more often. This loses some E𝕐/-𝕠d kinetic impulse with scattering, some electron particles are moving forward and collide with others moving backwards. That is like electron waves interfering destructively in the rotational reference frame. The electron particles have less time to get out of the inductor before these collisions, and so the current is slowed down.

FIGURE 32.15 Graph and phasor diagrams of the inductor current and voltage.



Three reference frames

The capacitor works mainly in the straight-line reference frame, the inductor in the rotational reference frame. The resistor is mainly reactionary from the proton while the capacitor and inductor use the electron.

32.5 The Series *RLC* Circuit

The circuit of **FIGURE 32.16**, where a resistor, inductor, and capacitor are in series, is called a **series *RLC* circuit.** The series *RLC* circuit has many important applications because, as you will see, it exhibits resonance behavior.

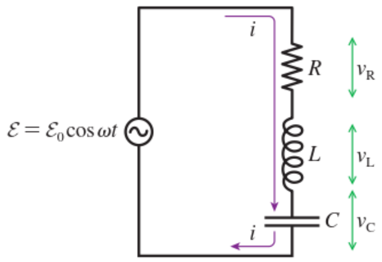
The analysis, which is very similar to our analysis of the *RC* circuit in Section 32.3, will be based on a phasor diagram. Notice that the three circuit elements are in series with each other and, together, are in parallel with the emf. We can draw two conclusions that form the basis of our analysis:

1. The instantaneous current of all three elements is the same: $i = i_R = i_L = i_C$.
2. The sum of the instantaneous voltages matches the emf: $\mathcal{E} = v_R + v_L + v_C$.

Three components in the straight-line reference frame

Here the three components are in series, which is in the straight-line reference frame. The instantaneous current refers to an instant of time, so the force would be the E𝕐/-𝕠d kinetic impulse. In the resistor there would be the E𝔸/+𝕠d potential impulse against this.

FIGURE 32.16 A series *RLC* circuit.



Current and voltage of a resistor

The current and voltage of a resistor are in phase, this is because it has no active forces. When there is -𝕆D×e𝕪 kinetic work, the +𝕆D×e𝕒 potential work of the resistor reacts against this. The E𝔸/+𝕠d potential impulse reacts against the E𝕐/-𝕠d kinetic impulse in the straight-line reference frame. This is like in an atom, the proton cannot exert forces against an electron. It can only add to the negative -𝕆D kinetic probability of the electron wave. If there was a lagging force it would unbalance the circular orbitals.

The phasor as a hand on a clock

Here the capacitor voltage lags the current over time, that would be drawn as a sine wave so the spin Pythagorean Triangle side points out of the page. This is also like the pivot of a clock gauge, the phasor moves around like hands on the clock. This clock is not visible in the straight-line reference frame because it is a rotation, only the instant of time observes the E𝕐 displacement of the current.

Observing electron particles in the gap

When the frequency is high enough, the capacitor gap is bridged with -𝕆D×e𝕪 kinetic work in the rotational reference frame. In the straight-line reference frame this would appear as if the electron particles are jumping across the gap. The electron waves have a -𝕆D kinetic probability of being measured in different positions. These wave functions can collapse to observe electron particles in the gap.

An accelerating current

When the current accelerates through a capacitor this is in the straight-line reference frame, the clock gauge is like a rolling wheel that is being accelerated. This appears to be collisions of electron particles in the straight-line reference frame because there are no fields.

Electrons waves as points

The electrons act like wheels rather than point particles in the rotational reference frame. That is where they only have a -𝕆D kinetic torque at a point or position. In the rotational reference frame there are no lines and distances, only points as position. Because of this the electron is only measured as a point particle.

Electron particles as lines not points

In the straight-line reference frame the electron particles cannot appear as points, only as lines or diameters. Times cannot appear as a torque or probability, only as instants as the rotation on a clock cannot exist there.

Two balls deforming

When two electrons collide this is like two tennis balls colliding, the continuous deformation of the balls is the E𝕍 inertial displacement as a line not a quantized series of points. For example, the balls might be deformed 1 centimeter each in the collision, then rebound 1 centimeter.

Elastic collisions

The electrons also collide elastically and continuously, the e𝕪 spoke is deformed as an E𝕐 kinetic displacement. This spoke is the same in all directions, it cannot be referred to as pointing in one direction because that would be a rotation. It appears more like the many spokes of a wheel, but is observed only where the wheel is being displaced or deformed.

An instant of kinetic time

When this happens the -𝕠d kinetic time of the electron particle is observed as a kinetic instant. This E𝕐 displacement changes continuously, not as a quantization as in the rotational reference frame. The changes in the displacement are as different kinetic instants.

A progressive formation

This is like observing the progressive deformation of the tennis balls colliding at different instants of time, such as with a high-speed camera. The collision slows the e𝕧/-𝕚d inertial velocity of the electrons as with the balls. The E𝕍 displacement of the deformations comes off the e𝕧 length in the e𝕧/-𝕚d inertial velocity of the balls as an E𝕍/-𝕚d deceleration in meters^2/second.

Reduced velocity in the collision

The electrons also reduce their e𝕪/-𝕠d kinetic velocity in the collision, according to the changing E𝕐 kinetic displacement in their E𝕐/-𝕠d kinetic impulse. As this kinetic velocity decreases as e𝕪/-𝕠d, e𝕪 decreases and -𝕠d increases to be the slower velocity. That maintains the constant Pythagorean Triangle areas.

A minimum velocity

When this reaches a minimum velocity, the electron particles reverse direction and move apart with an increasing E𝕐 kinetic displacement. This corresponds to a faster e𝕪/-𝕠d kinetic velocity as e𝕪 increases and -𝕠d decreases. The distance apart cannot go to zero as that would be a point in the straight-line reference frame, also the -𝕠d and e𝕪 Pythagorean Triangle would have no area. Usually the limit of the electron's inertial velocity would be c, so the deformation would have a limit of 1/c.

An appearance of an electric field

In this model there is no electric field, at 90° in the rotational reference frame the field effects always come from the magnetic field. The E𝕐/-𝕠d kinetic impulse must synchronize with the magnetic field, so it can be approximately regarded as an electric field.

Gravitational displacement

This duality also occurs with gravity in this model. The +𝕀D×e𝕙 gravitational work around a planet gives a -𝕠d gravitational field and a geodesic. Moving directly towards a planet, there is an acceleration but these would be a Eℍ/+𝕚d gravitational impulse and Eℍ displacement not the gravitational field.

Planets and moons

A planet and a moon would have this mutual Eℍ/+𝕚d gravitational impulse in between them as an attraction in circular geometry. This is like the E𝕐/-𝕠d kinetic impulse and E𝕐 kinetic displacement between the electron particles as a repulsion in hyperbolic geometry.

A collision in the rotational reference frame

In the rotational reference frame, the collision occurs with -𝕆D×e𝕪 kinetic work. The -𝕆D kinetic probabilities of each electron interfere destructively, this makes it less likely the electrons will be measured closer to each other. These change in quantized snapshots of positions, the electrons exchange virtual e𝕪×-𝕘d photons in between them like electrons changing orbits. That destructive interference moves the electrons away from each other, so that the two reference frames synchronize.

Vector addition

Here the $\boldsymbol{\mathcal{E}}$ is added from all three components, in this model only the straight Pythagorean Triangle sides can be vector added. As their inverses, the spin Pythagorean Triangle sides can be vector added with the same answer.

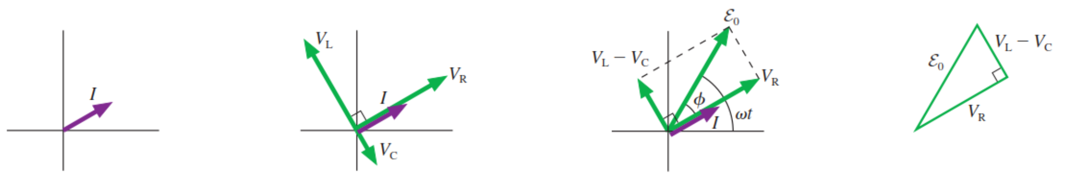
Summing the changes in time

Spin would usually only be added and subtracted, with impulse this would be summing the increase or decrease in the current times through the circuit. With a higher frequency the e𝕪/-𝕠d kinetic current would move faster so -𝕠d as kinetic time contracts as a Pythagorean Triangle side. This current slows through the inductor, so e𝕪 contracts and -𝕠d as the kinetic time dilates. In the resistor the current again slows as the E𝔸/+𝕠d potential impulse of the protons tends to hold onto the electron particles more.

The emf as a voltage or time

The $\boldsymbol{\mathcal{E}}$ as a voltage would be a square, so these are added in the Pythagorean Triangle here. As an instantaneous voltage $\boldsymbol{\mathcal{E}}$ is kinetic times, their inverses can be vector added.

Using phasors to analyze an *RLC* circuit



Begin by drawing a current phasor of length I. This is the starting point because the series circuit elements have the same current i.

The current and voltage of a resistor are in phase, so draw a resistor voltage phasor parallel to the current phasor I. The capacitor current leads the capacitor voltage by 90°, so draw a capacitor voltage phasor that is 90° behind the current phasor. The inductor current *lags* the voltage by 90°, so draw an inductor voltage phasor 90° ahead of the current phasor.

The instantaneous voltages satisfy $\mathcal{E} = v_R + v_L + v_C$. In terms of phasors, this is a *vector* addition. We can do the addition in two steps. Because the capacitor and inductor phasors are in opposite directions, their vector sum has length $V_L - V_C$. Adding the resistor phasor, at right angles, then gives the emf phasor $\mathcal{E}$ at angle ωt.

The length $\mathcal{E}_0$ of the emf phasor is the hypotenuse of a right triangle. Thus $\mathcal{E}_0^2 = V_R^2 + (V_L - V_C)^2$

Capacitor and inductor

When there is a capacitor and inductor in the same circuit, then there are two separate forces of impulse and work respectively. With the capacitor, the E𝕐/-𝕠d kinetic impulse has the E𝕐 kinetic displacement force in the middle of the sine wave. This drives the oscillation like in the middle of a spring. With an inductor this looks like a spring, the force comes from the twisting or torque in the rotational reference frame.

Two forces in the spring

These are different forces but are combined in the spring, each comes from a side of a Pythagorean Triangle. When the spring has a force, this comes first and then the other comes second as a reaction. With the E𝕐/-𝕠d kinetic impulse the spring is displaced to move with an expansion and compression.

Action/reaction pair

There is a reaction against this from the +𝕆D potential torque of the protons in the spring, they react against the spring's molecules being twisted out of shape. That makes an action/reaction pair.

Reactions from protons

In the circuit, the capacitor acts like the spring displacement in the straight-line reference frame, there is a reaction against this from the protons in the plates and wire. This also comes from the generator, as it spins the E𝕐/-𝕠d kinetic impulse of the current increases in part of the cycle.

Protons holding onto their electrons

When the generator turns to move the current in the opposite direction, the protons in the magnets and coils pull on the electrons to move in this other direction. If they did not, then the atoms in the generator would lose their electrons and become positively charged. This is the same as when the spring's protons hold onto their electrons and reverse the displacement direction, from an expansion to a compression, over and over.

Turning of the generator

The turning of the generator, before the change in current direction, is at 90° to where the current was at its maximum. This exerts the maximum deceleration of the current in the rotational reference frame, so this is the -𝕆D kinetic voltage. In this 90° direction there is no displacement, so there is no current.

Twisting the spring as torque

In the spring, it can also be oscillated by twisting it in a clockwise or counterclockwise direction. This looks at the spring through the coil, at 90° to seeing it from the side in the straight-line reference frame. When this is the initial force, the change in current comes after this as with the inductor.

After as position or time

That is not after as in time, but as positions, the twisting of the spring changes the positions in its length. With no twist or torque, the position does not change.

Quantized torque

These changes in the torque are not continuous but are quantized from snapshots. The twisting of the spring occurs in one snapshot, in the next one as a sequence of move like frames there is a change in the length of the spring as a series of different frozen positions. This happens in each atom, the -𝕆D×e𝕪 kinetic work of each electron changes the -𝕆D kinetic probability of what e𝕪 position it will be measured.

Two forces in the spring

The spring can have two difference forces applied together, as impulse in the straight-line reference frame and work in the rotational reference frame. The circuit can be regarded as a first spring being the capacitor and the second spring being the inductor.

A dampened spring as a resistor

The resistor can be a heavily damped spring that resists both forces.

The capacitor spring's current

The first spring, as the capacitor, is moved back-and-forth with an impulse in the straight-line reference frame. Then the voltage or maximum torque comes after this when the spring reaches its minimum or maximum length.

The inductor spring's voltage

The second spring, as the inductor, is twisted clockwise and counterclockwise in the rotational reference frame. Then the torque comes first, after this as a position comes the change in displacement. This torque is equivalent to voltage in this model.

Frequency of the dampened spring

The resistor spring can also change according to the oscillations of the first and second spring. When this oscillation is fast, then the resistor spring vibrates more and dampens both forces more each cycle. These are both in phase because the resistor spring is not creating any forces.

A slower frequency in the dampened spring

A slower frequency approaches a DC current, this is expanding or compressing the dampened spring in one direction. The dampening is less because the force do not collide or interfere in it. The dampened spring is inelastic, so there are more forces dissipated in the third orthogonal direction as work or impulse as it wobbles more.

Current and displacement, voltage and torque

The current here is the displacement of the first spring, the $\mathcal{E}$ is the torque or twisting of the second spring. Overall, one force will lag the other. In the straight-line reference frame, the first spring as the capacitor might have its force come first in time. In the rotational reference frame the second spring, as the inductor, might have its force come first in position which is equivalent to being first in time. Overall, one will lag the other.

If $V_L > V_C$, which we've assumed, then the instantaneous current i lags the emf by a phase angle ϕ. We can write the current, in terms of ϕ, as

$$i = I\cos(\omega t - \phi) \qquad (32.22)$$

Squaring two Pythagorean Triangle sides

In (32.23) the lag between the capacitor and the inductor comes from the E𝕐 and -𝕆D squared Pythagorean Triangle sides respectively. Because these are at 90° to each other they give the -𝕠d and e𝕪 Pythagorean Triangle electron. The lag will be proportional to the angle θ opposite the spin Pythagorean Triangle side.

Squaring sides as a convention

Here they are subtracted from each other, in this model they would not be comparing a single -𝕠d and e𝕪 Pythagorean Triangle because the capacitor and inductor can be different sizes. With a Pythagorean Triangle the subtraction implies the positive term would be the hypotenuse and subtracting the negative term would give the remaining squared Pythagorean Triangle side.

This would be a convention and does not relate to the lag, it is not used in this model. The dampened spring as the resistor squared would also not be used, the hypotenuse as a square represents a value for comparisons only.

Using the reactance as a reaction

The squared Pythagorean Triangle sides are also substituted by the reactance, the capacitive reactance here is the inverse of the inductor reactance. Because the capacitive reactance decreases with the frequency, this would be e𝕪 as the kinetic electric charge and the straight Pythagorean Triangle side. The inductive reactance increases with the frequency inversely, this would be -𝕠d as the spin Pythagorean Triangle side.

The capacitive reactance as a reaction

The reactance can also refer to the +𝕠d and e𝕒 Pythagorean Triangle protons in the wire and capacitor plates. Then they would be the inverse, the capacitive reactance would come from +𝕆D×e𝕒 potential work reacting against the increased frequency. This would be decreasing because -𝕆D×e𝕪 kinetic work would be moving electrons more easily along the wire and across the gap.

The inductive reactance as a reaction

The inductive reactance can refer to the protons in the inductor coil. These react with +𝕆D×e𝕒 potential work against the -𝕆D×e𝕪 kinetic work done by the AC current. When the frequency increases the electrons move with a slower acceleration with their E𝕐/-𝕠d kinetic impulse in the straight-line reference frame. This is because they change direction more, the e𝕧/-𝕚d inertial velocity and e𝕪/-𝕚d kinetic velocity of the electrons remains lower.

Electron capture

They would then tend to fall in towards the protons more with a E𝔸/+𝕠d potential impulse. Also there is more of a -𝕆D destructive interference in between the electrons, the +𝕆D potential probabilities of the protons does not change and so they sum to the -𝕆D kinetic probabilities more. This tends to capture the electrons reducing the current.

A lag in either reference frame

In (32.23) the hypotenuse is a square and this is multiplied by the current squared. Taking the square root of both, the overall value can be E𝕐 from the capacitive reactance or -𝕆D from the inductive reactance. With the square root that would give e𝕪 or -𝕠d depending on the direction of the lag. Multiplied by the e𝕪/-𝕠d current, this would give the E𝕐/-𝕠d kinetic impulse in the straight-line reference frame when the capacitor was stronger, or e𝕪/-𝕆D kinetic work in the rotational reference frame when the inductor was stronger.

Of course, there's no guarantee that V_L will be larger than V_C. If the opposite is true, $V_L < V_C$, the emf phasor is on the other side of the current phasor. Our analysis is still valid if we consider ϕ to be negative when i is ccw from $\mathcal{E}$. Thus ϕ can be anywhere between $-90°$ and $+90°$.

Now we can continue much as we did with the RC circuit. Based on the right triangle, $\mathcal{E}_0^2$ is

$$\mathcal{E}_0^2 = V_R^2 + (V_L - V_C)^2 = \left[R^2 + (X_L - X_C)^2\right] I^2 \qquad (32.23)$$

Three terms from the protons

This square root is taken below, that gives the lag in either reference frame while adding the resistor as a square. If all three used the +𝕠d and e𝕒 Pythagorean Triangle protons, the resistance comes from how tightly the electrons are bonded into the resistor's atoms. The capacitive reactance come from the straight-line reference frame, how the wire and plate protons hold the electron current. The inductor reactance would come from the protons capturing the electrons in a related way to in the resistor.

Geometric ratios

On the right-hand side the size L of the inductor is the number of loops, the C capacitance comes from the size of the plates. These are both geometric, the number of loops increases linearly. The size of the plates can also be regarded as increasing linearly, for example taking them as rectangles and increasing the number of these as the area. The plates are flat in the straight-line reference frame, the loops are curves in the rotational reference frame.

where we wrote each of the peak voltages in terms of the peak current I and a resistance or a reactance. Consequently, the peak current in the RLC circuit is

$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \qquad (32.24)$$

The three peak voltages are then found from $V_R = IR$, $V_L = IX_L$, and $V_C = IX_C$.

Impedance as a reaction

Here the impedance from the +𝕠d and e𝕒 Pythagorean Triangle protons represents the three kinds of reactions. The R^2 is the resistance from the nucleus of the resistor's atoms. The X_L inductive reactance is similar to the resistance, the coil's protons hold onto the electrons more as they change direction more quickly. The X_C capacitive reactance also relates to the protons in the wire and plates, these tend to hold onto the electrons preventing them from jumping over the gap.

Impedance from three orthogonal Pythagorean Triangles

These can be regarded as three orthogonal +𝕠d and e𝕒 Pythagorean Triangles, then the three spin Pythagorean Triangle sides squared can give a hypotenuse squared.

The work function

Electrons moving across the capacitor gap are like the work function in the photoelectric effect, in it the -𝔾D×e𝕪 light work of photons can impart enough -𝔾D light torque to the electrons to liberate them from atoms. When the electrons are moved with an increasing frequency they emit and absorb photons in the rotational reference frame. This liberates them more from the sphere of influence around the protons, so they can jump over the gap more easily.

Quantum tunneling

It is also like quantum tunneling where a higher frequency leads to an increased probability of the electron waves not being trapped or reflected in a barrier wall.

Impedance

The denominator of Equation 32.24 is called the **impedance** Z of the circuit:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad (32.25)$$

Impedance, like resistance and reactance, is measured in ohms. The circuit's peak current can be written in terms of the source emf and the circuit impedance as

$$I = \frac{\mathcal{E}_0}{Z} \qquad (32.26)$$

Equation 32.26 is a compact way to write I, but it doesn't add anything new to Equation 32.24.

Phase angle

The phase angle would correspond to an -𝕠d and e𝕪 Pythagorean Triangle, but not an electron itself. The geometry of the capacitor plates and the inductor coils give E𝕐 and -𝕆D respectively, they can also be regarded as an +𝕠d and e𝕒 Pythagorean Triangle with E𝔸 and +𝕆D.

Phase Angle

It is often useful to know the phase angle ϕ between the emf and the current. You can see from **FIGURE 32.17** that

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{(X_L - X_C)I}{RI}$$

The current I cancels, and we're left with

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \qquad (32.27)$$

Current as impulse, voltage as work

In this model the current moves with a E𝕐/-𝕠d kinetic impulse in the straight-line reference frame, the resistance comes from the E𝔸/+𝕠d potential impulse. The voltage comes from -𝕆D×e𝕪 kinetic work in the rotational reference frame, the resistance reacts against this with +𝕆D×e𝕒 potential work.

The spring's reference frames

In the spring analogy, the capacitor is represented by pulling on the spring so it expands and compresses in the straight-line reference frame. It can be twisted clockwise and counterclockwise to this at 90°, looking through the spring in the rotational reference frame. The protons in the spring have a E𝔸/+𝕠d potential impulse and +𝕆D×e𝕒 potential work together, this is because they are only reactive forces.

Potential work and impulse

The E𝔸/+𝕠d potential impulse cannot be observed directly, it can only be vector added as E𝔸 to the E𝕐/-𝕠d kinetic impulse as E𝕐. +𝕆D×e𝕒 potential work cannot be measured directly, it can only be summed as +𝕆D to -𝕆D in -𝕆D×e𝕪 kinetic work.

We can check that Equation 32.27 agrees with our analyses of single-element circuits. A resistor-only circuit has $X_L = X_C = 0$ and thus $\phi = \tan^{-1}(0) = 0$ rad. In other words, as we discovered previously, the emf and current are in phase. An AC inductor circuit has $R = X_C = 0$ and thus $\phi = \tan^{-1}(\infty) = \pi/2$ rad, agreeing with our earlier finding that the inductor current lags the voltage by 90°.

Other relationships can be found from the phasor diagram and written in terms of the phase angle. For example, we can write the peak resistor voltage as

$$V_R = \mathcal{E}_0 \cos\phi \qquad (32.28)$$

Notice that the resistor voltage oscillates in phase with the emf only if $\phi = 0$ rad.

The two reference frames still exist

In the diagram the current lags the $\boldsymbol{\mathcal{E}}$ because the -𝕆D×e𝕪 kinetic work and +𝕆D×e𝕒 potential work in the inductor is stronger in the rotational reference frame. The two reference frames still exist in this model, in the straight-line reference frame there is still a E𝕐/-𝕠d kinetic impulse jumping over the capacitor gap. This is still reacted against by the E𝔸/+𝕠d potential impulse of the protons in the wire and plates.

Summing to a single rotational reference frame

Because the impulse and work are in the same Pythagorean Triangle as inverses, the sum gives a single reference frame further down the wire as being stronger. This is reacted against by the resistor, whether it is the E𝔸/+𝕠d potential impulse or +𝕆D×e𝕒 potential work. If the inductor is stronger, then the electron waves crossing the capacitor gap also have their voltage before the current. This adds to the voltage before the current in the inductor.

Summing to a single straight-line reference frame

If the frequency was lower, then the E𝕐/-𝕠d kinetic impulse of the electron particles would be the stronger force. Then the current or impulse would come before the voltage or work at 90° in the straight-line reference frame. The inductor would have electron particles being accelerated in the zig zag of the wire instead of in the coil. With the particle/wave duality the frequency or inversely the wavelength gives the stronger force.

Blackbody curve

A similar duality happens in the blackbody curve, with a higher frequency -𝔾D×e𝕪 light work is a stronger force which is quantized. That liberates, or moves between orbitals, electrons from the atoms in the blackbody. Because these are quantized there are a limited number of -𝕆D×e𝕪 kinetic work levels and the curve goes downwards on the left.

The photoelectric effect

The -𝔾D light voltage drives the current in the sense that liberated electrons can make a current such as in a photocell. The photoelectric effect is limited in that there are a limited number of orbitals the electrons can be moved up to, as well as a limited number which can leave the atoms.

The blackbody curve

On the right-hand side of the blackbody curve the frequency is lower, the photons act more with a e𝕐/-𝕘d light impulse in the straight-line reference frame. This would give an exponential curve, on the left-hand side it becomes an inverse exponential or normal curve. this lower frequency means the electrons cannot jump the gaps between the electron orbitals as easily. It is like their not being able to jump the capacitor gap.

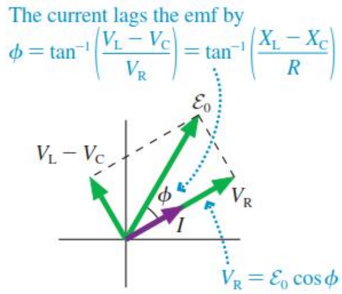
Electron waves in between orbitals

The higher frequency then allows for the photoelectric effect, quantum tunneling, jumping across the capacitor gap, and jumping as electron waves in between orbitals. When this frequency is lower the e𝕐/-𝕘d light impulse is stronger as a light particle, When electrons are in their orbitals they are like being in an inductor coil.

The Fermi energy

A low frequency does not move electrons easily in the photoelectric effect, even if the intensity goes up with more photon particles. The e𝕐/-𝕘d light impulse creates a higher temperature like in the blackbody, the Fermi energy needed to liberate electrons is much higher because temperature is in the straight-line reference frame.

FIGURE 32.17 The current is not in phase with the emf.



The spring resonance

With the spring analogy, there is a frequency that creates the most oscillations, this is the resonance of the spring. It is a balance between the E𝕐/-𝕠d kinetic impulse back-and-forth in the straight-line reference frame and -𝕆D×e𝕪 kinetic work in the rotational reference frame where the spring is twisted clockwise and counterclockwise.

Lower and higher frequencies

When the frequency is lower, the spring moves back-and-forth more slowly and there is less -𝕆D kinetic torque in the spring's molecules. When the frequency is too high, the -𝕆D kinetic torque is faster and so the molecular bonds are vibrated more elastically.

Resonance

Suppose we vary the emf frequency ω while keeping everything else constant. There is very little current at very low frequencies because the capacitive reactance $X_C = 1/\omega C$ (and thus Z) is very large. Similarly, there is very little current at very high frequencies because the inductive reactance $X_L = \omega L$ becomes very large.

If I approaches zero at very low and very high frequencies, there should be some intermediate frequency where I is a maximum. Indeed, you can see from Equation 32.24 that the denominator will be a minimum, making I a maximum, when $X_L = X_C$, or

$$\omega L = \frac{1}{\omega C} \qquad (32.29)$$

The angles θ of the electrons in the circuit

Here LC is like -𝕆D×E𝕐 and the square root is -𝕠d×e𝕪 as an -𝕠d and e𝕪 Pythagorean Triangle area. This is not an electron, the ratio of the inductor and capacitor can correspond to the angles θ in the -𝕠d and e𝕪 Pythagorean Triangle electrons in the circuit. That will make the electrons more like particles if the impulse is stronger in the straight-line reference frame, or more like waves with work in the rotational reference frame.

The spring's inertial resonance

With the spring the L value would be -𝕀D as the inertial torque in twisting the spring, C would be the E𝕍 inertial displacement in compressing and expanding the spring. A ratio of these in the -𝕚d and e𝕧 Pythagorean Triangle would give an angle θ as the resonance.

The frequency ω_0 that satisfies Equation 32.29 is called the **resonance frequency:**

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{32.30}$$

Current, emf and resistance

Here the current I comes from the E𝕐/-𝕠d kinetic impulse, this varies according to the $\mathcal{E}_0$ from -𝕆D×e𝕪 kinetic work. The resistor reduces both like a dampened spring.

This is the frequency for *maximum current* in the series *RLC* circuit. The maximum current

$$I_{max} = \frac{\mathcal{E}_0}{R} \tag{32.31}$$

The dampened spring and resonance

At the resonance frequency the impedance only comes from the resistor. With the spring analogy, if it was dampened then the resonant frequency would have both the E𝕍/-𝕚d inertial impulse in the straight-line reference frame and -𝕀D×e𝕧 inertial work in the rotational reference frame being reduced.

is that of a purely resistive circuit because the impedance is $Z = R$ at resonance.

You'll recognize ω_0 as the oscillation frequency of the *LC* circuit we analyzed in Chapter 30. The current in an ideal *LC* circuit oscillates forever as energy is transferred back and forth between the capacitor and the inductor. This is analogous to an ideal, frictionless simple harmonic oscillator in which the energy is transformed back and forth between kinetic and potential.

Resonant driving frequency

When the oscillation frequency of the spring matches the resonance, the spring will move with a maximum E𝕐/-𝕠d kinetic impulse in between the maximum expansion and compression. Also there will be maximum -𝕆D×e𝕪 kinetic work at the turning points, this is because more -𝕆D kinetic torque is created by the spring having been twisted more.

Adding a resistor to the circuit is like adding damping to a mechanical oscillator. The emf is then a sinusoidal driving force, and the series *RLC* circuit is directly analogous to the driven, damped oscillator that you studied in Chapter 15. A mechanical oscillator exhibits *resonance* by having a large-amplitude response when the driving frequency matches the system's natural frequency. Equation 32.30 is the natural frequency of the series *RLC* circuit, the frequency at which the current would

Maximum current and impulse

In the diagram the maximum current comes from the maximum E𝕐/-𝕠d kinetic impulse. This also appears like a particle in the straight-line reference frame. The turning points of -𝕆D×e𝕪 kinetic work are like wave functions, the -𝕆D kinetic probabilities are higher for the spring end to be measured there because it is slowest.

The spring’s turning points

Looking through the spring in the rotational reference frame, the turning points are where the spring is stationary. The -𝕆D×e𝕪 kinetic work is measured as snapshots, these are denser around the turning points representing a higher -𝕆D kinetic probability.

The maximum displacement in the middle

In the straight-line reference frame the spring is observed at 90° to looking through it. The maximum displacement per second is in the middle. If the middle of the spring is regarded as a particle, it seems to move back-and-forth with an acceleration. This is more concentrated at the resonant frequency, that gives the higher spike in the diagram. The spring moves faster there, but it also decelerates faster away from this area.

A boson standing wave

Taking an electron as being like the spring, when it oscillates in an orbital there is -𝕆D×e𝕪 kinetic work done. There may be two electrons in a Helium orbital for example, these would have opposing spins and so the oscillations of the springs cancel out to be like a standing wave as a boson.

Two springs as a boson

This is like two springs connected together each taking up half a circle. When they expand they push each other at one position, the second position is where they are pulling apart. This cycle continues over and over. There is no torque in one direction, the springs like a boson can then occupy a lower orbital.

A fermion as a single spring

A single fermion electron in Lithium would occupy its own orbital, this oscillates with -𝕆D×e𝕪 kinetic work in the rotational reference frame. When it is observed the resonant frequency produces a sharp spike as the collapse of the -𝕆D×e𝕪 kinetic work wave function.

An electron spring in a box

This is observed in the straight-line reference frame as a particle moving quickly back-and-forth, also like an electron in a box. At the ends of the box the spring more likely to be measured as it is slower. In the middle of the box the spring’s midpoint moves faster, oscillating from side to side like a particle.

Observing the spring in the box

When the spring is measured, there are oscillations coming from the box, for example it might jump around as the spring hits each side of the box. To observe the spring’s midpoint, time is needed because its position changes most in the middle over time and its position is most uncertain. At the ends of the box time is less useful because the midpoint of the spring is close to stationary.

like to oscillate. Consequently, the circuit has a large current response when the oscillating emf matches this frequency.

FIGURE 32.18 shows the peak current I of a series RLC circuit as the emf frequency ω is varied. Notice how the current increases until reaching a maximum at frequency ω_0, then decreases. This is the hallmark of a resonance.

As R decreases, causing the damping to decrease, the maximum current becomes larger and the curve in Figure 32.18 becomes narrower. You saw exactly the same behavior for a driven mechanical oscillator. The emf frequency must be very close to ω_0 in order for a lightly damped system to respond, but the response at resonance is very large.

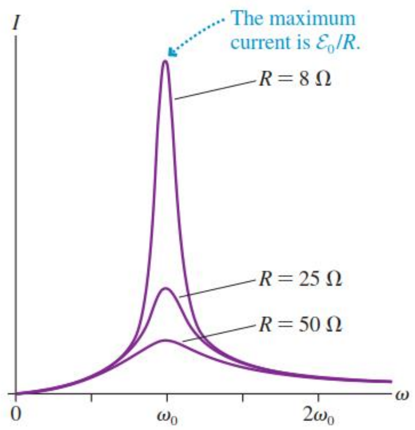
A dampened spring

When the spring is dampened, the E𝕐/-𝕠d kinetic impulse is most concentrated in the middle. This is because the spring slows more away from the middle, the turning points become closer to the middle. This is like the spring slowing down from the dampening, the distance in between the expansion and compression is smaller.

AC current in a resistor

With an AC current in a resistor, the electrons are observed more in a spike around the protons and their E𝔸/+𝕠d potential impulse. This is because they are most strongly attracted here, E𝔸 vector subtracts E𝕐 more as force vectors.

FIGURE 32.18 A graph of the current I versus emf frequency for a series RLC circuit.



Resonant frequencies and reference frames

When the spring is oscillated at above the resonant frequency, -𝕆D×e𝕪 kinetic work is stronger because the frequency -𝕠d in kinetic time is larger. This makes the current lag after the -𝕆D kinetic voltage. When the oscillation is below the resonant frequency the E𝕐/-𝕠d kinetic impulse is larger, e𝕪 increases and -𝕠d decreases because the electrons in the spring have constant Pythagorean Triangle areas.

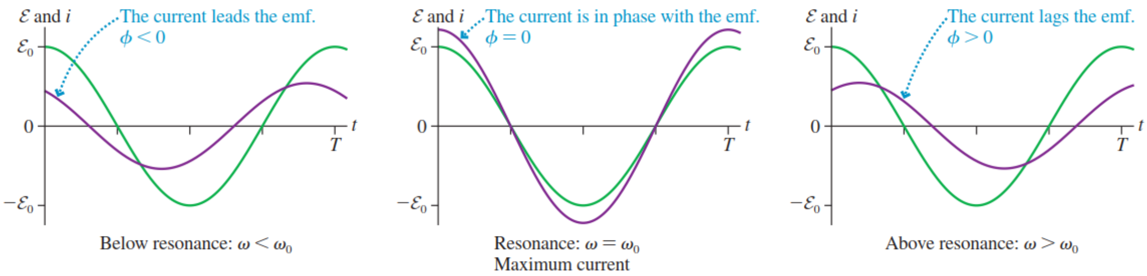
For a different perspective, **FIGURE 32.19** graphs the instantaneous emf $\mathcal{E} = \mathcal{E}_0 \cos \omega t$ and current $i = I\cos(\omega t - \phi)$ for frequencies below, at, and above ω_0. The current and the emf are in phase at resonance ($\phi = 0$ rad) because the capacitor and inductor essentially cancel each other to give a purely resistive circuit. Away from resonance, the current decreases *and* begins to get out of phase with the emf. You can see, from Equation 32.27, that the phase angle ϕ is negative when $X_L < X_C$ (i.e., the frequency is below resonance) and positive when $X_L > X_C$ (the frequency is above resonance).

Resonance circuits are widely used in radio, television, and communication equipment because of their ability to respond to one particular frequency (or very narrow range of frequencies) while suppressing others. The selectivity of a resonance circuit improves as the resistance decreases, but the inherent resistance of the wires and the inductor coil keeps R from being 0 Ω.

Impulse the inverse of work at the resonant frequency

At the resonant frequency the E𝕐/-𝕠d kinetic impulse is the inverse of -𝕆D×e𝕪 kinetic work, this comes from the constant Pythagorean Triangle areas. Also the +𝕆D×e𝕒 potential work and the E𝔸/+𝕠d potential impulse of the protons are inverses, as they are in phase which each other there is only a dampening of the spring.

FIGURE 32.19 Graphs of the emf $\mathcal{E}$ and the current i at frequencies below, at, and above the resonance frequency ω_0.



Power as impulse

In this model, power would be the E𝕐/-𝕠d kinetic impulse in the straight-line reference frame. In 32.32) this would be -𝕆D×e𝕪 kinetic work in the rotational reference frame. When this is instantaneous it is in the straight-line reference frame, an instantaneous voltage would be the -𝕠d kinetic time. The instantaneous current would be e𝕪 as in an infinitesimal. This would then represent the -𝕠d and e𝕪 Pythagorean Triangle electrons with no forces.

Power and the Pythagorean Triangle angle θ

These Pythagorean Triangles can vary with their angles θ, when θ is small then so is -𝕠d opposite it and e𝕪 is larger. The E𝕐/-𝕠d kinetic impulse would be stronger as power.

32.6 Power in AC Circuits

A primary role of the emf is to supply energy. Some circuit devices, such as motors and lightbulbs, use the energy to perform useful tasks. Other circuit devices dissipate the energy as an increased thermal energy in the components and the surrounding air. Chapter 28 examined the topic of power in DC circuits. Now we can perform a similar analysis for AC circuits.

The emf supplies energy to a circuit at the rate

$$p_{source} = i\mathcal{E} \qquad (32.32)$$

where i and $\mathcal{E}$ are the instantaneous current from and potential difference across the emf. We've used a lowercase p to indicate that this is the instantaneous power. We need to look at the power losses in individual circuit elements.

Power referred to as energy

In (32.33) power is referred to as energy, when the current is squared that is E𝕐/-𝕀D from the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy. The resistor reacts against the kinetic energy as the ½×+e𝔸/+𝕆d ×+𝕠d rotational potential energy. The potential current e𝕒/+𝕠d comes from the protons in the resistor.

Ocean waves on a beach

The instantaneous power loss would change with the angle of the sine wave, for example ocean waves can break on a beach. Their motion is reacting with inertia against the kinetic forces driving the waves as an action/reaction pair. This is like the action forces of the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy and the reaction forces of the ½×+e𝔸/+𝕆d ×+𝕠d rotational potential energy.

Changing angles in the waves

The ocean waves have a -𝕆D kinetic torque driving them, at different angles this pushes against the sand under them. In between the waves there is less force on the sand, when the waves reverse and go out this is like the AC current reversing direction.

Impulse of the waves

The waves have a E𝕐/-𝕠d kinetic impulse in the straight-line reference frame, this pushes the water up the beach. It is reacted against by the E𝕍/-𝕚d inertial impulse of the sand. The waves also do -𝕆D×e𝕪 kinetic work in the rotational reference frame, this twists the sand and often stirs it up. The sand reacts against this with its cohesion as -𝕀D×e𝕧 inertial work.

Torque of the waves

The torque of the waves is like the -𝕆D kinetic torque of the sine wave going through a resistor, this stirs up and breaks free some electrons in the resistor. In a conductor this is much easier like loose sand. When the waves change direction the sand settles down again, when the AC current changes direction the electrons can return to the resistor's atoms.

Looser sand and electrons

Other electrons are more loosely held in a sphere of influence around the atoms, this is like looser sand on the beach. The sand, and the electrons, are moved partially by collisions with the particles and E𝕐/-𝕠d kinetic impulse in the straight-line reference frame. They are also moved as a probability density, the sand is like a field when suspended in the water. The electrons are also a field of higher probabilities of where they are measured in the resistor.

Resistors

A resistor dissipates energy at the rate

$$p_R = i_R v_R = i_R^2 R \qquad (32.33)$$

We can use $i_R = I_R \cos \omega t$ to write the resistor's instantaneous power loss as

$$p_R = i_R^2 R = I_R^2 R \cos^2 \omega t \qquad (32.34)$$

Power loss

Here the power loss is at a maximum when the amplitude of the sine wave is a maximum. This would be where -𝕆D×e𝕪 kinetic work is at a maximum turning the current, also +𝕆D×e𝕒 potential work in the resistor atoms is at a maximum.

The turning point of the waves

With the ocean wave analogy, when the waves turn to go out the E𝕐/-𝕠d kinetic impulse is reduced more by the E𝕍/-𝕚d inertial impulse reactions of the sand, In this model $\sin^2$ would be used as the changing -𝕆D kinetic torque of the electrons and ocean waves. The answer is the same, in conventional physics the rolling wheel has the hypotenuse coming out of the origin and rotating.

A rotating Pythagorean Triangle

In this model the -𝕠d and e𝕪 Pythagorean Triangle is itself rotating, the +𝕠d and e𝕒 Pythagorean Triangle proton would be reacting against this with a counter rotation. The proton is not actually rotating, but its forces are exerted against the rotating -𝕠d and e𝕪 Pythagorean Triangle.

Rotations in the waves

With an ocean wave, the -𝕠d and e𝕪 Pythagorean Triangle would have its axis pointing at 90° to the wave's motion. The -𝕆D kinetic torque of the wave would be turning along with eddy currents. With the electrons, the -𝕠d and e𝕪 Pythagorean Triangles are also rotating with the -𝕆D kinetic torque as an axle like a rolling wheel.

A Pythagorean Triangle side instead of the hypotenuse

The hypotenuse is here the e𝕪 straight Pythagorean Triangle side instead. When the energy dissipation is maximized at the turning points, this is -𝕆D×e𝕪 kinetic work as the -𝕆D kinetic torque of the axle forces the e𝕪 spoke to change direction. If the spoke is moving upwards, or forwards in a current then this torque forces it to go backwards. This happens with the ocean waves as well as the electron.

Sines in the rotational reference frame

In this model the sine is used as the sine wave is in the rotational reference frame. The cosine is used for back-and-forth motion in the straight-line reference frame with a E𝕐/-𝕠d kinetic impulse. Together these make up the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy.

Torque is lowest with the highest velocity

There is more energy dissipation in this energy at the turning points from the point of view of power. Because voltage is at 90° to the power of the current, in the rotational reference frame, this is lowest around the middle of the sine wave. With the ocean waves the torque is lowest when the wave is moving forward or backwards the fastest on the beach.

Combining work and impulse in energy

The actual energy of the electrons would be conserved when there are no forces. This is because the -𝕠d and e𝕪 Pythagorean Triangles are rotating, there are no squared Pythagorean Triangle sides. They can continue to rotate with no loss of work or impulse. In a resistor the +𝕠d and e𝕒 Pythagorean Triangles also have no forces, but in the ½×+e𝔸/+𝕆d ×+𝕠d rotational potential energy both Pythagorean Triangle sides are squared as a combination of work and impulse.

FIGURE 32.20 shows the instantaneous power graphically. You can see that, because the cosine is squared, the power oscillates twice during every cycle of the emf. The energy dissipation peaks both when $i_R = I_R$ and when $i_R = -I_R$.

In practice, we're more interested in the *average power* than in the instantaneous power. The **average power** P is the total energy dissipated per second. We can find P_R for a resistor by using the identity $\cos^2(x) = \frac{1}{2}(1 + \cos 2x)$ to write

$$P_R = I_R{}^2 R \cos^2 \omega t = I_R{}^2 R\left[\tfrac{1}{2}(1 + \cos 2\omega t)\right] = \tfrac{1}{2} I_R{}^2 R + \tfrac{1}{2} I_R{}^2 R \cos 2\omega t$$

Average power loss

The average power loss is a half of the two turning points and their -𝕆D×e𝕪 kinetic work summed. These are not positive and negative in this model so they do not themselves sum to zero. Because the normal curve comes from the -𝕆D kinetic probabilities of the rotational reference frame, the average is in between the edges as the turning points.

Integrating to give a ½ factor

The ½ in the average also comes from taking the power loss as an integral. This squares the current e𝕪/-𝕠d to become E𝕐/-𝕆D in the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy. In this model only the spin Pythagorean Triangle sides can be integrated, only the straight Pythagorean Triangle sides can be differentiated. An integral gives a probability distribution and so there is an average. With an exponential there would be no average, that comes from impulse in the straight-line reference frame with derivatives only.

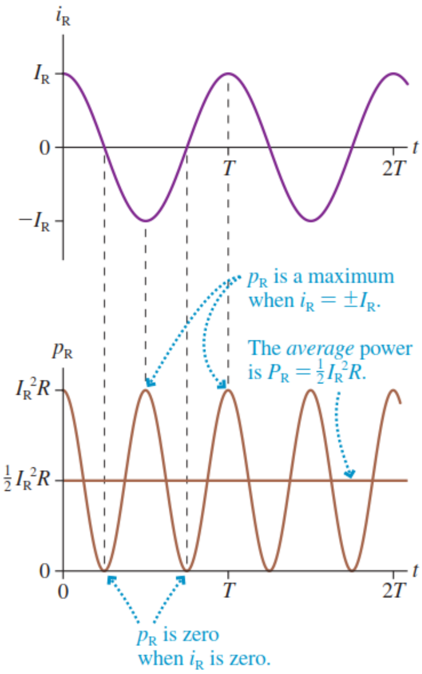
The $\cos 2\omega t$ term oscillates positive and negative twice during each cycle of the emf. Its average, over one cycle, is zero. Thus the average power loss in a resistor is

$$P_R = \tfrac{1}{2} I_R{}^2 R \qquad \text{(average power loss in a resistor)} \qquad (32.35)$$

Power loss from the turning points

The power loss comes from -𝕆D×e𝕪 kinetic work and +𝕆D×e𝕒 potential work when the wave turns.

FIGURE 32.20 The instantaneous power loss in a resistor.



Root means square

Here the square root is taken of the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy, that leaves out -𝕠d which would be proportional to the -𝕚d inertial mass. That gives the root means square.

It is useful to write Equation 32.25 as

$$P_R = \left(\frac{I_R}{\sqrt{2}}\right)^2 R = (I_{rms})^2 R \qquad (32.36)$$

where the quantity

$$I_{rms} = \frac{I_R}{\sqrt{2}} \qquad (32.37)$$

AC and DC current

At the turning points the -𝕆D×e𝕪 kinetic work is like DC current with the -𝕆D kinetic difference or voltage. There is no acceleration of the DC current in the straight-line reference frame, The electron particles collide with each other and are slowed by the E𝔸/+𝕠d potential impulse of the wire's protons.

is called the **root-mean-square current,** or rms current, I_{rms}. Technically, an rms quantity is the square root of the average, or mean, of the quantity squared. For a sinusoidal oscillation, the rms value turns out to be the peak value divided by $\sqrt{2}$.

The rms current allows us to compare Equation 32.36 directly to the energy dissipated by a resistor in a DC circuit: $P = I^2R$. You can see that the average power loss of a resistor in an AC circuit with $I_{rms} = 1$ A is the same as in a DC circuit with $I = 1$ A. **As far as power is concerned, an rms current is equivalent to an equal DC current.**

Root means square voltage

Here the root mean square is associated with the -𝕆D kinetic torque as the kinetic difference of the voltage.

Similarly, we can define the root-mean-square voltage and emf:

$$V_{rms} = \frac{V_R}{\sqrt{2}} \qquad \mathcal{E}_{rms} = \frac{\mathcal{E}_0}{\sqrt{2}} \tag{32.38}$$

Multiplying the two Pythagorean Triangles

Here the power is from the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy as E𝕐/-𝕆D, this is multiplied by E𝔸/+𝕆D from the ½×+e𝔸/+𝕆d ×+𝕠d rotational potential energy. That would give a fraction of the current dissipated by the resistor. In this model E𝔸 would be vector added to E𝕐 and +𝕆D would be added to -𝕆D as squared Pythagorean Triangle sides. That gives a different definition of resistance to a fraction.

Multiplying the rms values

Here the two rms values are multiplied together, the √2 factor is squared to give ½. Then e𝕪/-𝕠d is multiplied by e𝕒/+𝕚d to give (e𝕪×e𝕒)/(-𝕠d×+𝕠d), this would reduce the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy as a different definition of resistance to this model.

The resistor's average power loss in terms of the rms quantities is

$$P_R = (I_{rms})^2 R = \frac{(V_{rms})^2}{R} = I_{rms} V_{rms} \tag{32.39}$$

Average power from work and impulse

Here the power would be the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy, that is taking the kinetic current as e𝕪/-𝕠d and the voltage at 90° as a different e𝕪/-𝕠d. In this model E𝕐/-𝕆D would be made up of the E𝕐/-𝕠d kinetic impulse as power giving E𝕐, and -𝕆D×e𝕪 kinetic work giving 1/-𝕆D.

and the average power supplied by the emf is

$$P_{source} = I_{rms} \mathcal{E}_{rms} \tag{32.40}$$

Power loss in the resistor

The instantaneous voltage would be -𝕠d, that would also be the instantaneous **ℰ**. These would both be divided by 1/√2.

The single-resistor circuit that we analyzed in Section 32.1 had $V_R = \mathcal{E}$ or, equivalently, $V_{rms} = \mathcal{E}_{rms}$. You can see from Equations 32.39 and 32.40 that the power loss in the resistor exactly matches the power supplied by the emf. This must be the case in order to conserve energy.

Line voltage

With the AC current sine wave, the turning point would have a -𝕠d instantaneous value. When divided by √2 this is the square root from the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy.

NOTE Voltmeters, ammeters, and other AC measuring instruments are calibrated to give the rms value. An AC voltmeter would show that the "line voltage" of an electrical outlet in the United States is 120 V. This is $\mathcal{E}_{rms}$. The peak voltage $\mathcal{E}_0$ is larger by a factor of $\sqrt{2}$, or $\mathcal{E}_0 = 170$ V. The power-line voltage is sometimes specified as "120 V/60 Hz," showing the rms voltage and the frequency.

Instantaneous current of a capacitor

This gives the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy except that here the E𝕐/-𝕠d kinetic impulse jumps over the capacitor gap in the straight-line reference frame.

Capacitors and Inductors

In Section 32.2, we found that the instantaneous current to a capacitor is $i_C = -\omega C V_C \sin \omega t$. Thus the instantaneous energy dissipation in a capacitor is

$$p_C = v_C i_C = (V_C \cos \omega t)(-\omega C V_C \sin \omega t) = -\tfrac{1}{2}\omega C V_C^2 \sin 2\omega t \qquad (32.41)$$

The capacitor as a spring

The capacitor is also like a spring, the E𝕐/-𝕠d kinetic impulse accumulates at one side of the capacitor and rebounded elastically. On the other side there is a E𝔸/+𝕠d potential impulse which also oscillates. When the frequency is low there is little -𝕆D×e𝕪 kinetic work being transferred across the gap.

where we used $\sin(2x) = 2\sin(x)\cos(x)$.

FIGURE 32.22 on the next page shows Equation 32.41 graphically. Energy is transferred into the capacitor (positive power) as it is charged, but, instead of being dissipated, as it would be by a resistor, the energy is stored as potential energy in the capacitor's electric field. Then, as the capacitor discharges, this energy is given back to the circuit. Power is the rate at which energy is *removed* from the circuit, hence p is negative as the capacitor transfers energy back into the circuit.

Using a mechanical analogy, a capacitor is like an ideal, frictionless simple harmonic oscillator. Kinetic and potential energy are constantly being exchanged,

Capacitor and inductor storage

The capacitor stores the E𝕐/-𝕠d kinetic impulse with elastic collisions between the e𝕪/-𝕠d kinetic current and the plate. The inductor stores -𝕆D×e𝕪 kinetic work with a -𝕆D kinetic torque or probability of electrons being measured in it. With no resistor, both are mainly from the -𝕠d and e𝕪 Pythagorean Triangle electrons and so there are no reactive forces.

but there is no dissipation because none of the energy is transformed into thermal energy. The important conclusion is that **a capacitor's average power loss is zero: $P_C = 0$.**

The same is true of an inductor. An inductor alternately stores energy in the magnetic field, as the current is increasing, then transfers energy back to the circuit as the current decreases. The instantaneous power oscillates between positive and negative, but **an inductor's average power loss is zero: $P_L = 0$.**

Minimum losses

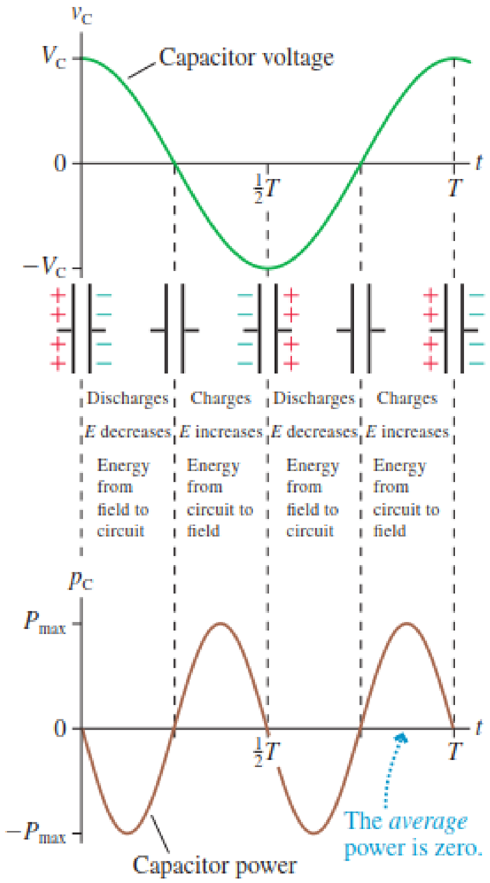
There would always be a minimum amount of loss because the +𝕠d and e𝕒 Pythagorean Triangle protons are in the wire and plates.

NOTE We're assuming ideal capacitors and inductors. Real capacitors and inductors inevitably have a small amount of resistance and dissipate a small amount of energy. However, their energy dissipation is negligible compared to that of the resistors in most practical circuits.

Power flow as back and forth

Here the power flow is a sine wave, in this model it would move back-and-forth in the straight-line reference frame. With a higher frequency -𝕆D×e𝕪 kinetic work increases in the rotational reference frame giving a sine wave shape across the gap.

FIGURE 32.22 Energy flows into and out of a capacitor as it is charged and discharged.



Instantaneous current and voltage

The instantaneous current would be e𝕪, the instantaneous voltage here is -𝕠d at 90° forming the -𝕠d and e𝕪 Pythagorean Triangle.

The Power Factor

In an *RLC* circuit, energy is supplied by the emf and dissipated by the resistor. But an *RLC* circuit is unlike a purely resistive circuit in that the current is not in phase with the potential difference of the emf.

We found in Equation 32.22 that the instantaneous current in an *RLC* circuit is $i = I\cos(\omega t - \phi)$, where ϕ is the angle by which the current lags the emf. Thus the instantaneous power supplied by the emf is

$$p_{source} = i\mathcal{E} = (I\cos(\omega t - \phi))(\mathcal{E}_0\cos\omega t) = I\mathcal{E}_0\cos\omega t\cos(\omega t - \phi) \qquad (32.42)$$

Part of the Pythagorean Triangle

Here the power can be from one side of the -𝕠d and e𝕪 Pythagorean Triangle, the other is set to zero using -𝕆D×e𝕪 kinetic work at the turning points of the sine wave. The average of this uses ½ from the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy.

We can use the expression $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$ to write the power as

$$p_{\text{source}} = (I\mathcal{E}_0\cos\phi)\cos^2\omega t + (I\mathcal{E}_0\sin\phi)\sin\omega t\cos\omega t \qquad (32.43)$$

In our analysis of the power loss in a resistor and a capacitor, we found that the average of $\cos^2\omega t$ is $\frac{1}{2}$ and the average of $\sin\omega t\cos\omega t$ is zero. Thus we can immediately write that the *average* power supplied by the emf is

$$P_{\text{source}} = \tfrac{1}{2}I\mathcal{E}_0\cos\phi = I_{\text{rms}}\mathcal{E}_{\text{rms}}\cos\phi \qquad (32.44)$$

Spring energy

With the spring analogy the E𝕐/-𝕠d kinetic impulse is the change of the displacement in the straight-line reference frame. -𝕆D×e𝕪 kinetic work is looking down the coils in the rotational reference frame. These are at 90° to each other, when taking them together as energy they are not aligned as in a DC current or a resistor.

Cycling between work and impulse in energy

In the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy then the E𝕐/-𝕠d kinetic impulse as E𝕐 is stronger when the 1/-𝕆D is weaker, cycling over and over. Using either of these misses out on part of the spring's work and impulse.

The power here is a square of the angle ϕ, that comes from the changing -𝕆D kinetic torque.

The rms values, you will recall, are $I/\sqrt{2}$ and $\mathcal{E}_0/\sqrt{2}$.

The term $\cos\phi$, called the **power factor,** arises because the current and the emf in a series *RLC* circuit are not in phase. Because the current and the emf aren't pushing and pulling together, the source delivers less energy to the circuit.

We'll leave it as a homework problem for you to show that the peak current in an *RLC* circuit can be written $I = I_{\max}\cos\phi$, where $I_{\max} = \mathcal{E}_0/R$ was given in Equation 32.31. In other words, the current term in Equation 32.44 is a function of the power factor. Consequently, the average power is

$$P_{\text{source}} = P_{\max}\cos^2\phi \qquad (32.45)$$

Generators and work

Generators work in the rotational reference frame, there is then a changing -𝕆D kinetic torque in the AC current. The E𝕐/-𝕠d kinetic impulse is at 90° to this, so capacitors are used to line this up with -𝕆D×e𝕪 kinetic work.

where $P_{max} = \frac{1}{2} I_{max} \mathcal{E}_0$ is the *maximum* power the source can deliver to the circuit.

The source delivers maximum power only when $\cos\phi = 1$. This is the case when $X_L - X_C = 0$, requiring either a purely resistive circuit or an *RLC* circuit operating at the resonance frequency ω_0. The average power loss is zero for a purely capacitive or purely inductive load with, respectively, $\phi = -90°$ or $\phi = +90°$, as found above.

Motors of various types, especially large industrial motors, use a significant fraction of the electric energy generated in industrialized nations. Motors operate most efficiently, doing the maximum work per second, when the power factor is as close to 1 as possible. But motors are inductive devices, due to their electromagnet coils, and if too many motors are attached to the electric grid, the power factor is pulled away from 1. To compensate, the electric company places large capacitors throughout the

Energy loss in the resistor

As with a dampened spring, the ½×e𝕐/-𝕆d ×-𝕠d linear kinetic energy would be dissipated by the ½×+e𝔸/+𝕆d ×+𝕠d rotational potential energy of the spring's protons. This occurs in both the E𝕐/-𝕠d kinetic impulse and -𝕆D×e𝕪 kinetic work.

transmission system. The capacitors dissipate no energy, but they allow the electric system to deliver energy more efficiently by keeping the power factor close to 1.

Finally, we found in Equation 32.28 that the resistor's peak voltage in an *RLC* circuit is related to the emf peak voltage by $V_R = \mathcal{E}_0 \cos\phi$ or, dividing both sides by $\sqrt{2}$, $V_{rms} = \mathcal{E}_{rms} \cos\phi$. We can use this result to write the energy loss in the resistor as

$$P_R = I_{rms} V_{rms} = I_{rms} \mathcal{E}_{rms} \cos\phi \quad (32.46)$$

But this expression is P_{source}, as we found in Equation 32.44. Thus we see that the energy supplied to an *RLC* circuit by the emf is ultimately dissipated by the resistor.